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Noticing and understanding children's everyday mathematics during play in early childhood classrooms

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ABSTRACT

Children's everyday mathematics has been well evidenced by a growing body of research and is widely acknowledged as fundamental to meaningful learning in early childhood education. However, literature supports that drawing upon children's mathematical resources for further learning requires that the range of these resources is valued and noticed at school. Considering that up to now little is known about the way children express their mathematical ideas during naturally occurring activities within classrooms, this study sought to enhance our understanding of this issue. Informed by previous research on the emergence of early mathematics during play and based on the funds of knowledge framework, we investigated the content, the processes and the origin of children's mathematical knowledge in a kindergarten classroom, with the aim to understand through a sociocultural lens where and how this knowledge is situated and how this information is revealed during play activities. Evidence foregrounds the importance of examining all the different facets of children's everyday mathematics for creating responsive curricula and confirms the critical role of play for investigating this type of cultural knowledge within schools.

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Introduction

Mathematics is a cultural construct, which includes symbolic systems, rigorous structure, rules and artefacts and despite its formal and abstract nature permeates many of our everyday activities and therefore children's life. Using money for buying things, calculating quantities during cooking, indicating important dates in the calendar, and playing board games are some of the family and cultural activities children participate in, and through which they have already experienced a wealth of everyday mathematics by the time they arrive at early childhood education (ECE) settings. This perspective reflects an image of children as capable learners of mathematics, which is not new. It has been well established by the ongoing research on children's mathematical experience appropriated in family context (e.g. Anderson and Anderson 2018; Aubrey, Bottle, and Godfrey 2003). For Ginsburg et al. (2003) everyday mathematics is mostly 'untaught' and includes

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> whatever mathematics children acquire in their ordinary physical and social environments. It may be formal and intuitive; it may be based in social and cultural experience. (236)

Nowadays building meaningful mathematical learning in ECE on children's out-of-school mathematics is of paramount importance (e.g. van Oers 2010), which, however, is a process that requires noticing and acknowledging the range of this knowledge within classrooms (Dockett and Goff 2013). Children-initiated play has been suggested as a key activity for both the documentation and the development of early mathematics (Wager and Parks 2016), as within play, children bring, share and extend bodies of cultural knowledge and interests (Hedges, Cullen, and Jordan 2011; Riojas-Cortez 2001), which among others include aspects of everyday mathematics (Worthington and van Oers 2016).

Research on children's mathematics as expressed in natural occurring activities at preschool settings is not extensive and has focused mainly on the mathematical topics integrated in children's talk (Ginsburg et al. 2003) and actions (Cook 1996). Aspects such as the origin of this knowledge (Worthington and van Oers 2016) and the way they use it (Papandreou 2019; Sumpter and Hedefalk 2015), have scarcely been researched upon. Thus, drawing upon the sociocultural theory and especially the funds of knowledge (FofK) framework (Moll et al. 1992) as well as new play pedagogies (e.g. Fleer 2017), the present study seeks to enhance our knowledge in this field by investigating the different aspects of children's mathematics during the routine of free play in an ECE classroom.

Theoretical background

The funds of knowledge framework

The FofK construct, established upon central ideas of the Vygotskian theory (i.e. mediation, ZPD, everyday and scientific concepts), constitutes a conceptual framework both for understanding the wealth of knowledge children bring at school, and for providing them with meaningful learning in school (Moll 2014). FofK have been described as 'culturally developed bodies of knowledge and skills', which children appropriate naturally by participating with more experienced others in context specific events of their family (Moll et al. 1992, 134), functioning as psychological tools (Vygotsky 1978), 'that mediate human behaviour' (Esteban-Guitart and Moll 2014, 36).

Although FofK research has been evolving for over than 35 years, it continues to feed pioneering educational approaches seeking to counteract the deficit thinking model (Hogg 2011), quite prevalent among teachers, concerning students coming from deprived environments. Considering that this approach aims to increase children's meaningful participation in classrooms (Hogg 2011), which cannot be taken for granted in ECE (e.g. Correia et al. 2019), young children could be considered more broadly as a deprived group altogether. However, having a broader conceptualisation of the FofK approach, Esteban-Guitart and Moll (2014, 43) claim that,

> FofK approach is based on a simple premise: regardless of any socioeconomical and sociocultural 'deficit' that people may or may not have, all families accumulate bodies of beliefs, ideas, skills, and abilities based on their experiences.

This view is recently reflected in early childhood studies with children from non-typical deprived groups (Chesworth 2016; Hill and Wood 2019).

By identifying different areas and sources of knowledge in the various conceptualizations of FofK, Hogg (2011, 669) argues that it is a 'contested' term. Although the original definition emphasised family knowledge (Moll et al. 1992), González, Moll, and Amanti (2005, 19) afterwards highlighted the strong link between families and community, suggesting that 'social networks are central to any household's functioning within its particular environments'. As research has extended in various cultural contexts and children's ages, later studies enriched the FofK construct revealing the diversity of influences on children's lifeworld beyond family life. Multiple sources of knowledge such as popular culture, peers, schools and teachers are encompassed within the FofK concept (e.g. Hedges, Cullen, and Jordan 2011), while aspects of knowledge not previously discussed such as household pedagogy (Velez-Ibanez and Greenberg 2005), funds of identity (Esteban-Guitart and Moll 2014), and FofK-based interests (Chesworth 2016; Hedges 2015) are endorsed. However, given that FofK are 'highly contextualized' and 'personalized' (Hogg 2011, 670), and have a dynamic and complex nature (Hedges 2015), acknowledging and even challenging the sometimes-inconsistent descriptions of FofK, not only among cultures, but also among peers, ages and even within an individual child may well be the very essence of the meaning of this term. As a child gains new experience and enacts the old experiences in novel situations, her FofK constantly change, while they remain highly differentiated from her peers' FofK, in similar or different contexts.

FofK research, albeit not quite extended in ECE, has revealed the variety of children's cultural knowledge (Riojas-Cortez 2001), the different contexts it originates from (Hedges, Cullen, and Jordan 2011), and the way it influences children's interests (Chesworth 2016; Hedges 2015) and working theories (Hill and Wood 2019). An area that is under-researched is preschoolers' mathematical FofK. Although this issue has been investigated for older children (e.g. Civil 2016; González, Moll, and Amanti 2005), only one study has focused on young learners' cultural ways of knowing mathematics (Worthington and van Oers 2016).

The ways for investigating children's FofK is another issue of concern in this research area. Although home visits have been traditionally considered as a key strategy for looking for these specific bodies of children's knowledge, this practice has some limitations. First, parents may hold reservations towards sharing their everyday life with teachers and researchers, especially in contexts like the Greek ECE in which this study took place and where building strong school-family relations is not among the teachers' priorities (Rentzou and Ekine 2017). Second, as Moll (2005, 279) underlines 'interviews with adults and participant observations in households, may inform us about adults and their social worlds but not necessarily about their children'. Research in ECE classrooms confirms this argument as it has demonstrated that besides their family knowledge, children co-create FofK and interests with their peers (Chesworth 2016; Hedges 2015; Hill and Wood 2019), often beyond 'the adult's social life' (Esteban-Guitart and Moll 2014, 43). However, a lot of knowledge children bring and use in schools often goes unnoticed by teachers (Birbili 2019; Hedges, Cullen, and Jordan 2011). This renders the documentation of children's mathematics and the related FofK in ECE classrooms a challenging task that this study had to deal with.

Modern conceptualisations of play

Nowadays, scholars challenge the mainstream understandings of play which are permeated either by an idealised idea of freedom and free choice in ECE settings or by curricular guidelines which restrict children's play (Wood 2014). Moving away from taken-for-granted assumptions of what free play includes, new play pedagogies (e.g. Fleer 2017) take into account the children's differentiated play experiences (Riojas-Cortez 2001; Trawick-Smith 2010), the value attributed to play in various families and cultures (Wood 2014), and the way children's play is transforming in our rapidly changing world (Trawick-Smith 2010). The teacher's role is being reshaped as she does not only provide play material, but also seeks to understand children's perspectives, interests and FofK expressed during play (Chesworth 2016; Hedges, Cullen, and Jordan 2011; Hill and Wood 2019). Thus, the teacher enriches both the material environment and peer interactions while also creating shared intentions targeting children's conceptual learning (Fleer 2017; van Oers 2010). In other words, rather than dichotomising play and learning, modern conceptualizations of play reinforce the dialectical relation between them and propose a continuum ranging from free or child-initiated play to mutually directed play -by both the teacher and the child- and ending in teacher-directed play (Pyle and Danniels 2017).

Research has also questioned psychological classifications of play (e.g. Piaget 1962), revealing the complexity of children's play behaviours (e.g. playing in unexpected locations, merging materials and different activities), the subtle limits between traditional types of play activities (e.g. socio-dramatic play may emerge during block play) and activities that usually remain unseen or undervalued (e.g. tumble and rough play, language play) in ECE settings (Trawick-Smith 2010).

Modern conceptualisations of play have strongly influenced learning and teaching mathematics in ECE (e.g. Ginsburg et al. 2003; Perry and Dockett 2008; van Oers 2010; Wager and Parks 2016). Incorporating features that are integral to mathematical learning and thinking, such as representation and communication (van Oers 2010), investigation and problem-solving (Perry and Dockett 2008), and providing opportunities for linking everyday life with scientific concepts (Vygotsky 1978), when child-initiated play is mediated by adults is suggested as a key context for mathematical learning. However, such mediating role requires valuing children's cultural mathematical understandings, and viewing their free play as a serious activity in which they express related experiences and knowledge (Dockett and Goff 2013; van Oers 2010; Worthington and van Oers 2016).

The emergence of mathematics during free play

The identification of young children's informal mathematics within play context in ECE settings is disputable, as the contradictory research evidence has demonstrated. For example, Gifford's informal observations in ECE classrooms (2005) revealed that despite the provision of play material with mathematical content (e.g. coins and priced goods in shop-corner), children did not take advantage of them and seemed to be more interested in other topics than in numbers and quantities. A recent large-scale research (Hendershot et al. 2016) in five settings also found that children's mathematical talk was rather low.

These findings have raised questions and drew researchers' attention to issues such as (a) in which play instances are children likely to use mathematics (Gifford 2005), (b) which are the 'dominant discourses and practices of play' that may prevent us from identifying children's mathematics embedded in play (Worthington and van Oers 2016, 52) and (c) how our specific expectations may influence the way we observe children or limit the scope of what we value as children's competence and strengths (Dockett and Goff 2013).

Contrary to the abovementioned non-positive evidence were the findings of Cook's study (1996). Under the emergent numeracy perspective and after enriching the play areas of two ECE settings with numerical material, she documented a wealth of numerical words and actions embedded in children's play. These findings were confirmed by later studies. Ginsburg et al. (2003), in a large-scale research, found that both US and Chinese toddlers' mathematical talk was quite frequent during play activities, while it included not only number topics, but also space, measurement, shape, time, sharing, pattern and classification. Similar findings were also reported by Lee (2012), who focused on outdoor play in a New Zeeland preschool setting.

By arguing that children's mathematics is not characterised only by its specific content, Sumpter and Hedefalk (2015) investigated the ways young learners used their knowledge for creating collective mathematical reasoning and problem-solving during outdoor play. They found that children used the mathematical ideas for structuring, supporting and disputing arguments, which were expressed verbally and by using concrete materials.

A variety of children's math graphics (i.e. self-invented signs and formal symbols), produced spontaneously during child-initiated activities, was revealed in the research of Carruthers and Worthington (2006). Similarly, a recent study demonstrated that children conducted self-initiated data investigations during which they created and used a wealthy repertoire of math graphics (Papandreou 2019). In another study, although focusing only on pretend play, Worthington and van Oers (2016) found that children's interactions, narratives and graphics reflected their early understandings of various mathematical subjects, which are social, contextual and communicative in nature, and, more importantly, that the cultural knowledge of the selected children continued to influence their mathematical thinking throughout the year 'by providing coherent contextual and mathematical meanings within their chosen play narratives' (64).

The above studies, though few, show that different aspects of children's mathematics can be revealed during play; this in turn suggests the need for further investigation in this field, by considering all these facets for understanding the emergence of their mathematical knowledge, which was ultimately the purpose of the present study.

Purpose and research questions

Considering mathematical learning as a cultural activity, which starts to be formed from birth through participation in family and cultural practices, this study sought to enhance our understanding of children's everyday mathematics expressed during free time in ECE settings. Research questions were formulated as follows:

(1) What mathematical knowledge do children express and how do they use it?
(2) In which free play activities are children's mathematical experiences situated within ECE settings?
(3) What are the sociocultural influences on children's mathematics and how are they revealed during their free play?

Method

Following Parker-Jenkins's terminology (2018), this qualitative research can be considered as an '*ethno-case study*', that is, a case study that employs ethnographic techniques, but 'is limited in terms of research time, engagement with the data and the extent of the findings' (29). Since the study focused on children's experiences and knowledge, and the ways they share this kind of information in school, the method of participant observation during free time, (i.e. a daily routine scheduled in ECE settings for child-directed activities, otherwise free play) was chosen as the most appropriate (Creswell 2012). During this period, children can exercise agency and express their perspectives by making play choices, interacting and discussing naturally with each other without being controlled by adults, which is a key for understanding aspects of their lives (Chesworth 2018).

Data were collected through audio-recording, keeping field notes, and taking photos (e.g. the different phases of block building). However, taking into consideration the complexity of researching with children, and trying to reduce the risk of simplistic and subjective interpretations (Chesworth 2018), while at the same time being careful not to disturb children's play, the researcher in some cases integrated short discussions for illuminating specific events. As this study intended to understand the community and family influences on children's mathematics, we followed the FofK research tradition, which considers parents as key informants for their children's FofK (e.g. Hedges, Cullen, and Jordan 2011; Moll et al. 1992). Therefore, through an open parent questionnaire, we collected information about parents' occupation, out-of-school, and family activities, children's play behaviour and whatever activities and practices with mathematical content the children engage in at home.

Participants and procedure

The study was conducted in the 2nd Experimental Kindergarten[1] of Thessaloniki in Greece, which welcomes children from the whole city. A total of 13 children participated in the study (six girls and seven boys with mean age: 5,2 years). Aiming to develop a detailed understanding on the central phenomenon (Creswell 2012), our primary criterion for selecting this particular classroom was to gather rich information and 'learn the most' (Stake 2005, 153). The teacher in this classroom is highly qualified holding a master's degree, while she implements the new proposed Greek curriculum framework (IEP 2014), which values children's diversity and child-initiated play encouraging a negotiated curriculum based on students' interests and queries. As the policy of this school is to participate in research and innovative projects, the first author had established a close cooperation with the teacher and knew her attitude towards children's play. In her classroom, children have plenty of time for free play (45–60 mi per day), and are allowed to play without restrictions regarding the material provided or the number of children per play centres, while they are familiarised with participant observation and documentation (i.e. taking notes, photographs and discussing), and the teacher's subtle involvement into their play (Fleer 2017).

Upon receipt of ethical clearance from the scientific committee of the school and the teacher's permission, we requested parents' consent and invited them to answer the questionnaire, while we asked the teacher to discuss with the children and invite them to draw their response in a consent form. Prior to data collection the researcher spent two weeks for establishing respectful relationships with children, and familiarising herself with the classroom culture (Garvis, Odegaard, and Lemon 2015), while she was constantly attentive to children's indications of discomfort or annoyance.

Considering the cultural meanings of artefacts and their role in children's life (Chesworth 2016), and after examining the information obtained by the parents' questionnaires, the researcher equipped the play centres with materials related to children's out-of-school activities (e.g. a baking book, maps, images of children's hobbies) and parents' occupations (e.g. medical playset and measurement tools).

Data collection and analysis

Six weeks were spent for observing freely chosen activities during the morning sessions, according to the daily schedule of this kindergarten. The research data included 22 h of transcribed audio material, complemented by 223 photos, 58 graphic material (e.g. drawings, writings) and detailed field notes (e.g. gestures, play materials and group synthesis), which were oganised in 122 activities. Recording was initiated once the children to be observed started playing in an area, for example making a road with blocks, and was completed when they ended this particular play episode. Thus, the amount of actions that took place during that period was defined as one activity. Initiation of play in another area by the same children or by some of them, for example, painting in the art centre, was considered as a new activity. Data were collected from all play areas (i.e. a playhouse, two block play areas, a library and literacy centre, an art centre, and a math area), as well as the circle time area and the tables, which were available, according to the children's will, for various activities during free time (e.g. sociodramatic play, board games and play dough). The researcher ensured that each child was observed at least twice.

The research questions guided the analysis as follows. First, we examined data for detecting in children's expressions, verbal and non-verbal (e.g. gestures, actions, graphic material), specific mathematical topics, and assigned descriptors, such as *numeral recognition*, *direct comparison of distances, length measurement, counting, shape recognition, shape construction*. This analysis was informed by the related literature on early mathematics (e.g. NCCA 2014) and the Greek proposed curriculum for ECE (IEP 2014). Coding was deductive, but it was adjusted to the children's observed actions and references in order to avoid misinterpretations. In the final step, we grouped the topics in four widely accepted areas of mathematics: *numbers and operations, measurement, space and geometry, algebraic thinking*. Afterwards, the reexamination of the whole data revealed three mathematical processes that children enacted:

problem solving, representation and *reasoning,* which also drew on contemporary classifications of mathematics (e.g. IEP 2014; NCCA 2014). Given that the unit of analysis was the activity, a specific mathematical topic was coded once in each activity even though a related word, phrase or action was mentioned more than one times. If, for example during block play, children discussed about the quantity of blocks each of them had, and used phrases such as 'more than', 'less than', 'two yellow cubes more than' etc., then all of them were coded once as *comparison of quantities* (i.e. a topic of *number and operations* category). However, since children during an activity often used a variety of mathematics, more than one topic and/or process were coded in most activities.

In the second phase of data analysis, the 122 activities were classified according to seven mutually exclusive types of activities, presented in Table 1. Although we drew on previous classifications of play (e.g. Trawick-Smith 2010; Whitebread et al. 2017), our categories and the corresponding terms were adapted to the particular play behaviours observed and/or the terminology used in this classroom. For example, two of our categories, 'discussions' and 'graphic-activities', are included in the 'symbolic-play' category in Whitebread et al. (2017) classification. However, as graphic activities are considered critical for early mathematical learning (Carruthers and Worthington 2006) and were quite common during children's play in this study, they formed a distinct category, differentiated from the 'discussions' category. Additionally, although different play types may be embedded in a particular play activity (Trawick-Smith 2010), we coded each activity once. For example, when instances of socio-dramatic play were identified during an extended block construction, only the later activity was taken in consideration and was coded.

Finally, we looked for prior family or cultural experiences and factors that seemed to impact upon children's mathematical knowledge. First, the observational data were re-examined, since this time we looked for links between children's mathematical references or activities and their corresponding references to specific contexts (e.g. family life, pop-culture) and previous related experiences. These references were isolated, and coding drew on existing literature (Hedges, Cullen, and Jordan 2011; Riojas-Cortez 2001), while it was enriched according to the present data. Additionally, data from the parents' questionnaires were organised and cross-checked with the observational data for each of the related children. The themes that resulted from this analysis enhanced the initial coding. The overall agreement for the categories coded reached to 85%, while all disagreements were jointly reexamined and resolved.

Table 1. Types of free-time activities.

Types of activities	Description and examples
a. Discussions	Making jokes or discussing experiences (e.g. a family trip)
b. Graphic activities	Drawing (e.g. a month calendar), painting (e.g. symmetrical shapes) and writing (e.g. telephone numbers)
c. Block play	Making constructions (e.g. a tower, a treehouse)
d. Object play	Exploring the function or other features of objects (e.g. coins and measuring cups)
e. Socio-dramatic play	Playing everyday scenarios (e.g. organise a birthday party, pretend pop culture characters)
f. Book reading	Picture storybooks and comics
g. Games with rules	Board games or puzzles

Table 2. The distribution of children's mathematics across the main mathematical areas.

Main areas of mathematics	Mathematical topics	N	%
Numbers & operations	Counting one by one, rote counting & backward rote counting, cardinal & ordinal number, multiplicative number, symbol recognition, writing numbers, identifying quantities at once, sharing quantities, addition, estimation & comparison of quantities, sorting numbers & quantities	223	44%
Measurement	Estimation & comparison of size, length, area, capacity, volume, time, velocity, investigating measurement tools & techniques	170	33,5%
Space & Geometry	Position, direction and movement, spatial terms, 2D &3D shape recognition & construction, shape terms & properties, sorting 3D shapes, associating 2D & 3D geometric shapes, symmetry recognition & construction	104	20,5%
Algebraic thinking	Patterns recognition & construction/creation, exploring correlation between changing dimensions.	9	1,8%
Total		506	100%

Results

The children's mathematical knowledge

The range and the quantity of the mathematics involved in children's play is shown in Table 2. Evidence of mathematics were identified in the total of the observed activities ($N = 122$). The children included in their play various topics related to number and operations (44%), measurement (33,5%), space and geometry (20,5%) quite frequently, while evidence of algebraic thinking (1,8%) was rarely identified.

Besides the above-mentioned topics, three mathematical processes were identified (Table 3). Children often used representations related to mathematics ($N = 37$), which were evident in all mathematical areas. A closer look at representations revealed two main attitudes: a. creating math related representations with blocks, drawing, writing and body movements or gestures and b. interpreting or reading math representations (e.g. numerals on measurement tools or within books). Mathematical reasoning ($N = 20$), though identified in topics from all math areas, was mainly expressed in measurement activities. Finally, only 10 problem-solving cases were detected in measurement and algebraic topics.

The way children use their mathematical knowledge

A closer look at our data allowed us to examine deeper the way children used their mathematics during free time, which fell into two distinct types. The most common type was

Table 3. Mathematical processes involved in children's play.

Mathematical processes	Examples	N	%
Representation	Using blocks for making a pattern, drawing symmetrical figures, using body & gestures for showing geometrical properties (size and shape), writing numerals, reading numerals in a book or on a measuring cup	37	55,2
Reasoning	Explaining why a figure is symmetrical, reasoning about quantities, ages, volume and length, with or without conventional tools, explaining the way a pattern is made	20	29,9
Problem solving	Finding ways to extend their construction in block play, enacting comparisons and choosing the right size of blocks, finding and describing strategies for making a puzzle	10	14,9
Total		67	100

the use of momentary mathematical words, phrases or gestures (e.g. showing numerical quantity) for serving the ongoing activity and the communication between peers without direct involvement in mathematical meanings. For example, while drawing a school bus, Nicolas began rote counting backwards from nine to one and eagerly announced: 'school finished!' Probably, rote counting backwards represented time measurement, which in conjunction with the bus drawing served to express his expectation or prevision that the school day would end soon. Nicolas, in this case, did not elaborate further on the mathematical content apparent in this activity, but he used rote counting to complement the meaning of his drawing.

On the other hand, we identified instances, which were labelled as mathematical events (MEs), in which a child or a group of children were engaged in and elaborated further on specific mathematical meanings. In these cases, the children's actions were directly focused on mathematical topics and/or processes. For example, during a block construction, Alexandros suggested helping Daniel finish his building by providing him with extra blocks. Daniel described in detail the features of his red-blue patterned construction and emphasised that not all colours would fit in with it, asking Alexandros to check over for the right coloured block each time. In another case, Eva tried to build a tower with blocks by using those that had similar size and shape. Thus, for adding a new block in her construction, she would check very carefully each time both the shape and the size of each area. Eva made several direct comparisons, while she made comments such as 'we have to measure it', 'it should be like the other'. In both block-play events, children's attention was on a precise goal with specific mathematical content. In the former, the boys stuck to the red-blue pattern construction. In the latter, Eva was, for some time, preoccupied with explicit size and shape comparisons as a strategy to deal with her block-building plan. In total, 51 MEs indicating such conscious use of and engagement in mathematical ideas by the children were identified within the 122 activities observed, which included more measurement ($N = 25$), space and geometry ($N = 20$) topics compared to number ($N = 13$) and algebraic ($N = 9$) ones.

Types of the activities

Children's mathematics was expressed within seven types of activities, common during free time in the specific class. As we already mentioned a wide range of mathematical ideas were evident in the whole data ($N = 122$ activities), but only in MEs ($N = 51$) children were explicitly involved with or elaborated on one or more mathematical topics. In Figure 1, we see that although the most common activity was natural occurring discussions about various themes between peers, MEs emerged more frequently during graphic activities and object-play (e.g. with coins and measuring tools). However, block play (e.g. making castles and boats) and sociodramatic play (e.g. hospital and school scenarios), although to a lesser extent, also favoured the emergence of mathematics, and its further elaboration by the children. Finally, book reading and especially games with rules, although rarely occurred during the observation period, included MEs.

Sociocultural influences on children's mathematical knowledge

Further data analysis provided evidence about the underpinning of children's informal mathematics. More specifically, we found that children referred explicitly to influential

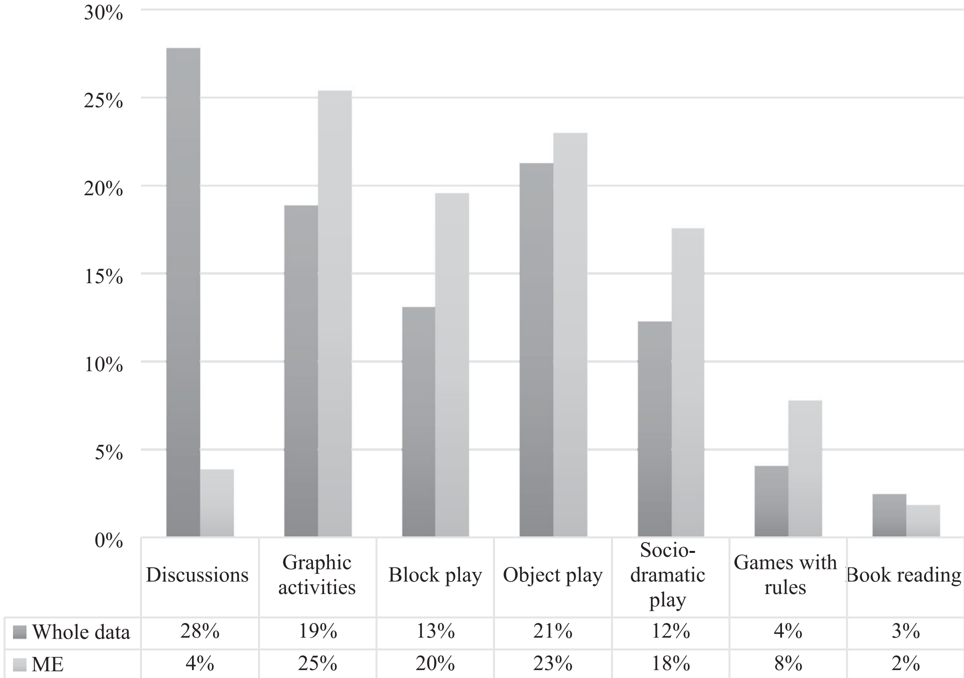

Figure 1. Display of children's mathematics in free-time activities across and within whole data and MEs.

factors related to their home and community environment 18 times in total. For instance, being occupied with an oven playset, Sofia pointed at a button and explained that 'this counts the hours, how much it needs to bake', while later she clarified that in her home oven she baked cookies with her mother. These data were strengthened and further enriched by examining both the information given by children's parents and the observed activities they had engaged in. This analysis resulted in the categories presented in Tables 4 and 5 which illustrate the factors that seemed to have an influence on children's mathematical knowledge as well as some representative examples.

Discussion

The amount and the breadth of mathematical topics that were found within the 122 observed play activities cover the areas included in mathematics curriculum (i.e. number and operations, measurement, space, geometry and algebraic areas) and confirm previous findings (Cook 1996; Ginsburg et al. 2003; Lee 2012; Worthington and van Oers 2016). What is more, the mathematical processes that children implemented during free time provided interesting insights into the way they used their mathematical resources.

Except for the mathematical representations expressed by children through drawing and writing, which are in line with previous findings (Papandreou 2019; Worthington and van Oers 2016), we also documented children's ease to create 3D and gestural representations of mathematical ideas. Children's keen interest for investigating math

Table 4. Family-based mathematical FofK.

Sources		Examples of family activities and mathematics expressed at school
Family activities	Parent's occupation	- Marilia often visits her parents' office, who are accountants, and observes them during work; she also likes collecting old receipts and uses them at her play. At school she often asked about and wrote big numbers, while she liked teaching her peers number writing - Nikiforos' father is an engineer. Nikiforos likes observing him during working on his PC and often asks questions about his job and the tools he uses, e.g. the measuring tape. He often used the measuring tape at school while he mentioned that his father used a similar measure in his job.
	Household activities	-During baking at home with his mother, Daniel often used the scale and measuring cups. They also used the measuring tape together for various tasks. At school, Daniel often pretended to bake or cook, measuring the quantities by filling a measuring cup with small pieces of playdough. He also liked measuring objects and distances with the measuring tape. - Apostolos goes shopping with his parents very often; he asks questions about the value of money and talks about money with them. During his play at school he often used money, made simple additions and referred to large amounts (e.g. fifty billion euros, a thousand euros)
	Intentional learning activities	Numerical activities such as, sharing and counting amounts tasks or other problem-solving games were reported by eight parents as well as by some children - Apostolos wrote various family telephone numbers at school, and explained that his grandmother had trained him memorise the numbers - Alexandros drew a house at school and while tutoring two other girls on how to do the same drawing, he explained that his grandfather had taught him how to draw those houses.
	Educational games & books	- At Alexandros' home, they regularly make puzzles all together as a family. At school one of his favourite games is to make puzzles and he used to teach his peers his strategy of checking the right pieces. - Sofia brought at school a number book from home and she pretended to teach her peers counting and number reading

Table 5. Community and school-based mathematical FofK.

Sources		Examples of community and school activities and mathematics expressed at school
Community- related activities	Pop culture	- After mentioning the size and capacity of the *Titanic* ship that he had seen on TV, Alexandros advised Apostolos on how to enlarge his boat, (made of blocks) so as it would look like the ship in the movie - Nikiforos and Daniel, motivated by the animated movie *Lightning McQueen,* talked about cars' speed
	Hobbies & leisure activities	- During reading an educational book at school, Kristallia, Eva, Katerina and Stella commented on a swimming pool drawing displayed in it. Making use of their previous experience of swimming lessons, they described specific swimming techniques by using directional words and body movements - During the construction of a castle, Eva described it by making evaluations and comparison and explaining that she wanted to make a tower like the White Tower of Thessaloniki. Her mother had mentioned that they take walks in the city and travel together
	Medical experience	- Marilia talked about her visit in the doctor's office and explained the way the doctor examined her and the tools he used - Marilia, Sofia and Katerina pretended to evaluate patients' fever with a thermometer, while they explained the sequence (using ordinal numerals) for taking the pills
School activities	Peer interactions	-Sofia taught Katerina how to draw a pattern of snowflakes and praised her - During block play, Apostolos explained to Stella and Christos what a symmetrical construction was
	School artefacts	-Nikiforos used the calculator as a phone by announcing of his home phone number one-by-one. Similarly, other children used various school devices such as laptop, sound recording, camera and measuring tools

symbols was shown in reading numerals in books and measuring tools, suggesting or asking for writing big numbers (e.g. 'Miss how much is one hundred?' 'Can I show you one thousand?'), picturing conventional representations (e.g. the layout of a numbered monthly calendar) or arithmetic operations (e.g. writing down '5 5 = 10' though without using the plus sign, Katerina explained 'five and more five … plus … plus, ten does five five … five and five'). Likewise, they explored patterns and symmetrical figures by creating block constructions and drawings, while they used gestures to describe geometrical and numerical properties.

Although less frequently, children also expressed reasoning and problem-solving processes related to different mathematical ideas (e.g. shapes, ages, volume, symmetry). For example, using manipulatives or other artefacts and discussing with each other, they described the way a pattern is made and investigated and/or explained strategies for making a puzzle or a symmetrical figure. Such evidence expands Sumpter and Hedefalk's (2015) findings of children's collective mathematical reasoning and problem-solving during outdoor play, as it additionally highlights the promising role of various indoor play activities for the enhancement of complex mathematical thinking in ECE. This argument is further supported by looking deeper into MEs, which provided us with a better understanding of children's shared involvement in sustained mathematical thinking. In these cases, apparently, not only did they use their mathematics for serving the demands of their play activity, but, more importantly, they played with and often elaborated further on specific mathematical meanings by making them the primary subject of their play. This evidence suggests that when similar events are noticed and valued within classrooms, they can constitute the starting points for promoting meaningful mathematical learning through play pedagogies (Dockett and Goff 2013; Wager and Parks 2016).

Examining the play activities that seemed to favour the sustained involvement of children with mathematical ideas can move the aforementioned discussion forward. This analysis (Figure 1) indicated that besides block (e.g. Ginsburg et al. 2003) and sociodramatic play (Worthington and van Oers 2016), which have already been associated with the emergence of mathematics, graphic activities and object play, had more or equally prominent role in prompting sustained elaboration of mathematical ideas (i.e. MEs). This means that these two types of play, overlooked in previous studies or included within pretend play, may have their own value for understanding children's mathematics, since we found that painting, drawing, writing and object play often occurred independently, out of other play activity (e.g. sociodramatic play). Another activity that has not been previously recorded and occurred quite frequently among children in this study was peer discussion. A common but essential activity, peer discussion allows children to share experience, negotiate views and expand knowledge by interchanging the discursive roles equally without being controlled by adults (e.g. Cekaite et al. 2014). In sum, we can assume that overlooking some types of activities or including them to other categories may result in concealing important information when looking for the breadth of children's mathematics. Therefore, extra attention should be drawn to exploring what mathematical meanings children elaborate on during a particular activity (e.g. discussions) and how these meanings may differ from the ideas they explore through another one (e.g. sociodramatic play).

Family proved as a primary source of influence on children's mathematical FofK. The children's familiarity with their parents' occupations, their engagement in educational games, book readings, domestic and intentional mathematical activities, all had a noteworthy impact on their expressed mathematics (Table 4). Apart from teaching activities, similar family resources have been reported in Hedges, Cullen, and Jordan's (2011) thorough study on young children's FofK, while Worthington and van Oers (2016) identified parents' occupation as a prominent source of children's mathematical knowledge. However, studies focusing on mathematical learning at home have also identified parents' teaching practices (Anderson and Anderson 2018; Aubrey, Bottle, and Godfrey 2003). Considering that in our study both parents and children referred to mathematical training at home, it is likely that such evidence reflects a common parental attitude of Greek families, which may also cover other subject areas. However, this speculation needs to be further studied.

As we know from previous research (Chesworth 2016; Hedges, Cullen, and Jordan 2011) pop-culture subjects, hobbies and leisure activities are frequently shared at school, often reflecting children's deeper interests and shaping the context for co-construction of new meanings during collaborative play. During the present study, various mathematical ideas emerged when children included similar subjects in their play narratives, a finding that renders such contexts favourable for meaningful mathematical learning at school. Furthermore, the evidence of mathematics in play incidents related to children's hobbies confirms the recently reported influence of swimming lessons on preschoolers' mathematics (Jorgensen 2018), and suggests that out-of-school activities (e.g. dancing, crafting), which are quite common in Greek families, when revisited by children and noticed by teachers may provide considerable opportunities for further mathematical learning.

Contrary to what we expected no impact of structured mathematical activities and teacher's authority was identified on children's mathematics; peer interactions and school artefacts, however, were found to associate with the emergence of mathematics, though in a different way. The sharing of mathematical ideas among peers was expressed either implicitly following each other's play interests (e.g. after observing how Apostolos constructed and described a 'symmetrical tree-house', Christos tried to make a similar one) or explicitly through peer-tutoring (Table 5). More importantly, some children tutored their peers by explicitly transferring mathematical understandings experienced in family teaching events and educational games (see Apostolos and Alexandros's examples in Table 4). Spontaneous mathematical tutoring among peers is a stimulating finding that needs further investigation and reflection on how to support this beneficial attitude within classrooms.

School artefacts functioned either as stimulus for uncovering and communicating FofK appropriated out of school (e.g. Daniel and Apostolos demonstrated an ongoing interest to measuring tools, often used within their family environment) or as original resources for investigating new learning (e.g. calculator and sound recording). By bringing in classroom artefacts mentioned by parents and examining children's involvement with these materials, we had the opportunity to understand as Chesworth (2016, 304–305) put it, 'how classroom resources acted as cultural artefacts, in which children drew upon their interests to imagine and ascribe new meanings to objects in their play'. However, attention must be paid to interpretations related to children's interests (Hedges, Cullen, and Jordan 2011), as, for example, Daniel's intensive engagement with playdough indicated. One day, as he was the leader of the play, he made small particles of playdough, represented the ingredients for 'baking a cake', and then measured the whole quantity with a measuring cup, while the next days he kept on filling with playdough the cup and taking measurements. Apparently, Daniel expressed his interest not in playdough per se, but in baking and measuring with a familiar tool, which eventually reflected family lived experiences, as his mother's account proved.

Conclusions

By conceptualising children's spontaneous mathematical activity through a wide-ranging perspective and working under the FofK approach, in this study we aimed to contribute to the discussion that foregrounds the links of play pedagogies and mathematical learning in ECE (e.g. van Oers 2010; Wager and Parks 2016). The results enhance our understanding of the emergence of children's cultural mathematical knowledge throughout the activities that take place within the routine of free time in ECE settings. The systematic participant observation of children in conjunction with their parents' information provided a breadth of evidence, which covers different aspects of their everyday mathematics (i.e. topics and processes, way of use, involved play activities, and influential factors). To the best of our knowledge, this is the first study that investigated children's mathematics during play encompassing all these aspects, which could constitute a framework for the inquiry of children's informal mathematics during naturalistic interactions.

In common with Worthington and van Oers (2016) findings, our results spotlight the meaningful mathematical play that emerges in school context when children draw upon and reconstruct their FofK. More importantly however, the present study provides new insights into the role of the different play activities for both the emergence and the elaboration of children's mathematical thinking, while it also depicts the contribution of various family, school and community activities in the appropriation of this knowledge. The amount of evidence related to children's FofK strengthens the critical role of observation during naturalistic play interactions for investigating aspects of their cultural ways of knowing, especially when the full access to family environment is not uncomplicated. Finally, our focus on children's various types of communication proved essential for uncovering mathematical understanding, otherwise unseen, expressed through gestures, material manipulation, constructions and drawings.

Although this research broadens our knowledge of young children's everyday mathematics, we have to keep in mind that the findings do not reflect the whole picture of the emergence of children's mathematics in ECE classrooms during free time. Apart from being a small-scale and cultural-specific research, the fact that a lot of play activities were occurring simultaneously during free time raises the question of what was lost in terms of the amount and the range of mathematical incidents.

Nevertheless, the present study provides informed recommendations for research and practice in ECE. It is argued here that the FofK construct, in conjunction with a wide-ranging conceptualisation of the emergence of children's everyday mathematics, offers an alternative framework for researchers and teachers who, valuing children's mathematical play as an important learning experience, seek to work out and advance mathematical learning in ECE through play pedagogies. More specifically, the mathematical topics and processes children may integrate within their play, the way they use, share and reconstruct them, the activities that favour their emergence, as well as the cultural foundation of this knowledge are all important aspects that should be taken into consideration for both investigating and extending children's mathematical resources. More importantly, noticing MEs as those documented during this study, educators can get involved with children's play and 'establish a mutual ground' in order to provide them with new challenges with the aim to expand their mathematical activity and thinking (Magnusson and Pramling 2018, 38). However, for having access on this corpus of information within the kindergarten class, teachers should make efforts and delve into children's narratives, constructions, discussions, embodied actions and material environment, and show genuine interest in all play activities without guiding, constraining or judging them.

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