Towards a Navajo Indian Geometry.

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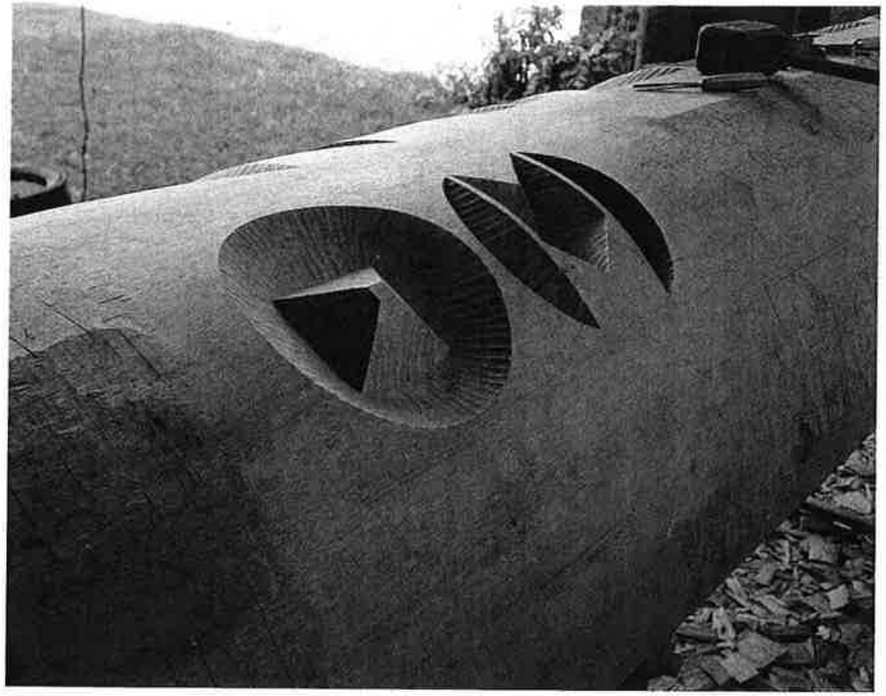
TOWARDS A NAVAJO INDIAN GEOMETRY



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Contents.

Introduction

This booklet gives an example of the way development can be set up, taking into account as much as possible the cultural heritage of the people. The example focuses on a specific culture and on a particular area of education, but the study should be seen as the expression of a general, multicultural perspective. That is to say, the philosophy of development we stand for is, mutatis mutandis, applicable to any culture and to any problem area. We propose to base all development on native concepts and (cognitive and cultural) procedures : the major addition by the developer should be his or her help in sophisticating native notions and strategies.

This study presents the analysis of the knowledge system of the Navajos (American Indians) and it focuses on the development of formal thought (i.c. geometry). We developed a curriculum for bicultural education. The curriculum was tried out on Navajo reservation for one year.

Part I

SPATIAL REPRESENTATIONS OF NAVAJOS

I. Navajo world view.

In a previous work (Pinxten et al., 1983) the world view of Navajo Indians was extensively discussed. Since the basic orientation of the Navajo view differs from the western one and since the teaching of geometry is bound to draw on spatial and general natural philosophical intuitions in the student's culture (cf. below Part II), it is deemed relevant to sum up the main points of the previous analysis.

In general, five main characteristics of the Navajo world view are esteemed as highly relevant in order to understand Navajo spatial representation. Other than these five aspects could be presented (cf. the literature in this area, more particularly Witherspoon, 1977), but they would not have the same importance for the spatial knower and agent. These five caracteristics are:

1. The structure of the world :

The Navajo world is conceived as a combination of two dishlike phenomena: earth is seen as a dishlike form stretched out in all four directions, and heaven is a dishlike form hanging some distance above the earth, upside down. In this way heaven is topping earth (without touching it) like a lid would top a can. The combination of both forms gives a bowlshaped construction, which is found also in the form of the hooghan or houses. Finally, the wedding basket (one of the most significant products of Navajo culture) has the same dishlike shape and refers again to the form of the earth.

The earth, like the hooghan and the wedding basket, is said to be closed on all sides, except to the east.

The earth, like everything else in the navajo universe, is changing continuously. In the first place, it is expanding in a gradual spiraling movement, which started from the exact center of earth (upon the moment of Creation). Furthermore, just like everything else, earth is in continuous and eternal change, which testifies for the fact that nothing is ever the same on the navajo world over different years, seasons and even days. The air is revolving in the sense of the directions, yielding a cyclic pattern of whiteness in the east in the morning, blueness in the south at midday, yellowness in the west of twilight and blackness in the north at night.

2. The dynamic nature of the universe.

Everything is inhabited by some sort of power or structuring principle (Navajos speak of winds: nilch'i), which keeps it standing throughout changes and against the forces that work on it (other winds, water, etc.). In fact, the substance that constitutes reality in the navajo conviction *is* events, changes, processes, much more than objects or states. This profoundly dynamic view on reality is guided, as it were, by the fact that navajo language consists overwhelmingly of verbs and verbal constructions (Young, R. & Morgan W., 1980).

While investigating the part/whole distinction in Navajo language Pinxten et al. (1983) found that this distinction was almost absent : Navajos express functional relationships rather than part/whole ones, which is of course more congenial with their dynamic view on the world.

To be fully clear on this characteristic it is advisable to present a few examples. When speaking about a mountain, the Navajo thinks about the way this mass is holding out against the process of erosion, about the constant force that keeps the phenomenal form the way we perceive it, about the dynamic interaction between valleys and peaks, and about the constant slow changes that keep going on within and at the outside of the mountain. When speaking about a human being, one thinks of the way the body is only an armature of appearance held in place by the same sort of force that inhabits the mountain (and is the mountain, in a sense), the *nilchi'i bii'gistiin*, and also of the genuine life force that came from the Holy Mountain and makes human beings walk, live and think and speak, the *nilchi'i bii'siziinii*. These vital elements or forces, all labeled after the wind (*nilch'i*) are the essence of things.

It is these aspects of reality that human beings should be very careful with : they can be influenced through careful and caring thought ('when you want something you should think hard about it and have constant care for it, and it will gradually happen'), and they are always focused upon and manipulated in the numerous ceremonies. The basic one of these, 'harmony, order', is the main point of focus of a Navajo's entire life (cf. sub. 5).

It is clear that the dynamic nature of everything in the Navajo universe is not only an epiphenomenal aspect of his understanding of the world, but has a very crucial, indeed essential place in the whole conception. Both in the structure of the language (the overwhelming amount of verbal forms), in the behavior (the constant reference to 'continuing in existence' and to constant change in everything) and in the cognitive representation the dynamic view is the most powerful and the 'natural' way to deal with things.

3. Boundedness of the world; center-periphery organization:

The Navajo space is bounded in each cardinal direction by a Sacred Mountain. Indeed, from any possible point on the reservation one can see at least one of these mountains, and each one is directed roughly according to the main four cardinal points. The world is thus bordered in a horizontal plane. Similarly, the world is vertically bounded by the shell of the sky and by the shell of the earth, thus realizing a neatly bounded bowl.

The same boundaries can be discussed in the hooghan. A similar circular construction obtains, bordered on all sides by the wall.

The world, just like the hooghan and the wedding basket are constructed around a center. This center goes from the nadir or center of the earth up into the zenith or center of heaven. People point to a particular star formation to locate the center of the sky and to a specific mountainous rock (Huerfano Peak) to identify the center of the earth. In the hooghan, the fireplace and its smokehole serve as similar references. Finally, the point where the spiraling movement of the basket begins is the central organization point of this artefact. Behavioral rules (sleep around the center of the hooghan in clockwise formation) and beliefs (ceremonies are performed in the center of the hooghan) all grant this crucial role to the center.

4. Closedness of the world; the notion of order:

According to mythology everything in the Navajo universe has been there since this universe was created. Put another way, this means that human beings cannot pretend to be able to add anything substantial to the world. Human beings can use their minds and their hearts to rearrange or reorder relationships between themselves and any aspect of nature, but they are unable to really enlarge the range of plants or animals, or to invent beings within their universe. This particularity of the Navajo world view is what is referred to here as the 'closedness' of the navajo world (somewhat in the sense of Koyré's understanding of open and closed world views, 1957).

All and everything in the navajo is fitting in a preconceived network of orderly relationships The order of this network is unknown to man. Navajos speak about the orderliness, harmony or beauty of nature (hozhoon) in this respect. The concept is a very basic one and it has been the subject of study for several major researchers on Navajo (e.g. Haile, 1947, Witherspoon, 1977, Farella, 1986). It is agreed upon nowadays that the single main formula of Navajo moral wisdom implies a central reference to this orderliness of nature : sa'ah naaghai bik'eh hozhoon (along the path of beauty/ orderliness/harmony (you should reach) longevity). It should be understood that order is thus projected into nature, or is presupposed to exist in nature without however its being known by the human beings. The latter should carefully observe everything in nature and try to grasp the orderliness in it. Such understanding can then form the basic information to yield action. Knowledge thus has a very important, but indeed risk-laden status in man's survival procedures. It should necessarily be sought, in order to guarantee a long and healthy life, but its acquisition is difficult and full of risks. Human beings are bound to make mistakes when trying to grasp the order in nature. The results of their actions (based on their understanding) will show whether their knowledge was adequate or not (cf. Pinxten, 1984). It is important to realize that knowledge has, for the Navajo, this very delicate position of an action that works between a presupposed order in nature and the unknowing human beings trying and groping to get insight about the former. Obviously, this way of seeing knowledge has its backdrops on any educational program.

5. The interrelatedness of everything in the Navajo universe:

This notion refers to some extent to the former one. Nothing in the Navajo world is on its own. Everything is imbedded in a complex network, by means of which it is linked to many other things and ultimately to the whole. The Navajo universe is homogeneous throughout and any disturbance on one particular place or time will have effects on everything else and for a considerable time. Human beings are as much imbedded in the englobing network of relationship as any other aspect of the world.

The network formed by the interrelationships between everthing has the peculiartity of being harmonious. Navajos say that everything 'was placed' in the proper way in the universe : sheep supply meat and wool to human beings and use the grass and herbs for feed, while eagles supply feathers and survey some of the lower animals, which in turn serve as food for them, etc.. In this way, the beings in the Navajo universe all have been placed in mutual functional relationships, except for human beings. They are ill-defined in the global harmonious structure and they constantly have to probe into the darkness of their ignorance to come to understand the order somewhat more. Since they are subject to 'placedness' themselves they will constantly have to take risks in their adventurous existence : they will disrupt or strengthen the interrelationships that tie them to the beings in nature, and the result of their actions will only make clear whether they did or did not understand the order they are investigating (i.e. they will fall ill, on the contrary profit from their action).

This short section ends here. The data presented thus far should suffice to have an elementary understanding of the basic perspective on nature in the Navajo tradition.

II. Navajo spatial knowledge.

The study of spatial knowledge in the Navajo culture concentrated mainly, though not exclusively, on linguistic material. Thus, the bulk (and hopefully all relevant material) of spatial differentiations in the Navajo language were semantically investigated to result in a model of Navajo space which will be used to work on for the educational program outlined. It is necessary to give a short overview of the Navajo model of space in order to understand the rest of this book.

1. The semantic model :

The study of Navajo space can be situated, from the point of view of methodology, in the branch of cognitive anthropology. However, the model proposed and the criteria of semantic analysis used are somewhat different from the standard ones within cognitive anthropology (cf. Pinxten, 1977, 1983): the description is constructed in such a way that no aprioristic elements of meanings should be 'smuggled' into the results. The criteria for semantic analysis allow for synthetic study only and grant as much room as possible to the native insights and the native remarks on every step of the work. Consequently, the result of semantic study, the model of Navajo space, has certain very particular characteristics. In the first place, it is agreed that a full definition of Navajo terms and categories is impossible, in view of the problem of translation or rather the unavoidable 'buffer zone' of intranslatability. Thus, we decided to define a minimal 'common ground' around which we start the ethnographic encounter. It is used to establish a first understanding by both parties (consultant and ethnographer) of the subject matter we are about to explore. Both ethnographer and consultant are constantly engaged in the processes of selection, recognition and identification of semantic categories. They correct one another's misunderstandings concerning concepts and representations and thus are involved in a continuous interaction process.

Furthermore, the final result (the identification of native categories and their representation in a cognitive model) is not understood to be definitive or full definition in the real sense. Both consultant and ethnographer are conscious of the gap of intranslatability and they will consequently strive for the ultimate or optimal representation or the least alienating cognitive category possible. It is therefore useful and careful to refrain from complete descriptions of native categories and to settle down for incomplete representations. In the actual model, the basic criterion was one of adequacy : the total spatial lexicon should be described in an unambiguous way, that is to say, such that all Navajo categories are mutually distinguishable. This minimal or incomplete identification of Navajo concepts and representations proved to be satisfying : the ethnographer was able to map out the total lexicon of spatial terms and the model enabled him to act and respond adequately in discourses. The control of consultants on any level of analysis guaranteed that less misinterpretation or uncontrolled presuppositions from the part of the researcher crept into the model.

Finally, the actual paraphrases of Navajo categories were used to construct the abstract representations, which resulted in the model. In practice, this meant that each and every category of Navajo space is composed of two subsets of 'features' or components: Const. I. and Const. H. The first formula (Is a constituent of) consists of a list of spatial (and some non-spatial) components, identified by Navajos, for which the category paraphrased is acting as a constituent or defining element. The second formula (Has as constituents) consists of the list of all components identified by Navajo which make up the meaning of the category paraphrased in an incomplete but sufficient way. Each category is an abstract construct which refers to and is practically 'filled in' by a set of terms in the language. (For example, a set of some twenty terms has to do with 'overlapping': each term gives a particular and exclusive aspect of overlapping, but the set of them defines the culturally relevant notion of 'overlapping' in Navajo, i.e. the category).

2. Spatial knowledge:

The graphic representation enclosed (fig. 1) gives an overview of the total cognitive domain of Navajo space. I will gradually develop the model by considering some of the basic distinctions first.

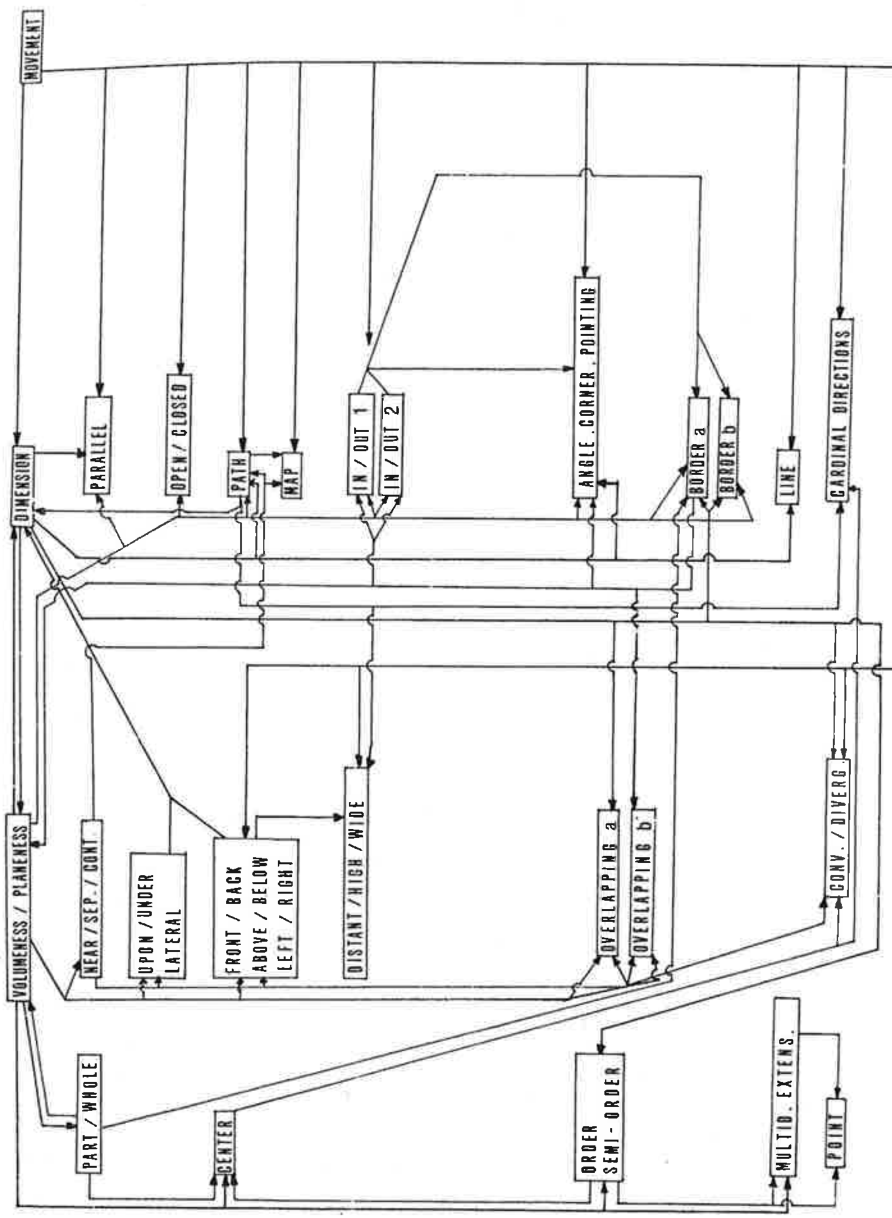
The three most dominant and thus most basic notions in Navajo spatial representation appeared to be 'volumeness/planeness', 'dimension' and

'movement'. These notions are more basic than others, since they are implied in the semantic identification of many other notions. Their semantic content (the Navajo understanding of each of them) is represented in the graphic model as well : the arrows pointing *toward* these notions indicate the semantic components that constitute the concepts. Each of these is distilled from a large set of terms (tens for each of them), which refer directly or indireclty to the notions themselves : there is not a single label or direct reference to 'dimension' in Navajo, but the notion is present in many composites (backward, forward, etc..), but there are large sets of verbstems and other verbal expressions denoting 'having expansion in all directions' (volumeness), 'having expansion in a planelike fashion, flatwise' (planeness) and 'movement' (displacement, rotation, translation, etc..).

My general intuition for the construction of a geometry curriculum is that it would be wise to start with these intuitive or broadly topological notions and to retrace the global field of Navajo space from these very broad notions out. I will stick with this intuition right from the beginning, that is, already while reporting on the data found in the previous study (Pinxten et al. 1983).

Volumeness/planeness :

in an oversimplification we can say that Navajo concepts of volumeness and planeness, in a way, the basic Navajo ontology. Indeed, a set of different phenomenal forms and events are introduced in the Navajo way of thinking about space. They are more often than not undefined and have little or none of the other spatial characteristics as their components. Thus the basic set of eleven classificatory verbstems (to be refined and multiplied at will) details eleven mutually exclusive and incomparable ways of manipulating or acting on phenomena : single multidimensional items with a solid appearance are delineated through the use of the si'á stem, while a mass of undifferentiated phenomena with considerable extension will be described in terms of sighí, and so on. 'Objects' in the western conceptualisation can travel from one to another category whether they are to be handled in a bundle, as a stack, in a container, and so on. Apart from these very general and pervading 'ways of being in place' (the classificatory verbstems), one finds a set of particular spatial formations like holes, bulks, bumps, and so on. It should be clear that the domain of what was called 'volumeness/planeness' here thus covers all non-metric and very rough or intuitive volumes and planes which are dis-



Graphic Scheme of the Semantics of Navajo Space.

Figure 1

tinguished in the cultural knowledge system of the Navajos. Some of the specific terms found here can be 'dismantled' in a small set of components, but most of them are basic or 'primitive' in the logical sense and seem to form the rock bottom on which all further specification and sophistication is to be defined within the tradition of the cultural knowledge system. The notions of 'volumeness/planeness' in the Navajo sense are components for the semantic fields of upto twenty other spatial notions (cf. arrows going from the node of 'volumeness/planeness 'into other nodes).

Movement :

Witherspoon (1977) can be quoted to have said that the verb 'to go' is for Navajo language and thought what the verb 'to be' would represent for westerners. One finds a practically endless series of forms and aspects of 'to go' and it is clear that any phenomenon in the Navajo world still at some point or other be subject to movement. The myriad of terms denoting movement in different senses (from actually moving back and forth, displacing and so on up to rotating and vibrating) can be seen as ever so many apsects of 'movement' in the Navajo view. The latter is not directly verbally expressed in Navajo. Some notion of 'movement' is acting as a component in the semantic constitution of eleven other spatial notions in the Navajo knowledge system. In the development of geometrical insights this fact should be represented adequately.

Dimension :

neither the abstract notion of 'dimension' nor any of the three spatial dimensions are labeled as such in the language. Rather, the three dimensions are formed as a composite of two parts : in front of + in the back of above + below, and so on. Otherwise, some aspect of movement or form expresses what comes closest to our understanding of dimension: e.g. something stretching out in a flatwise form as in a horizontal plane will be termed 'naanigo'. In this expression horizontality as such is not expressed, but the horizontal aspect of some way of existing through time is described. In consequence, the notion of 'dimension' used in this paper is an abstraction of what was recognized as an implicit concept in Navajo. It is a semantic constituent of eight other spatial notions in Navajo.

Primarily on the basic of these three irreducible spatial notions, and in particular combinations which are exclusively for each of the set of other spatial notions found in Navajo, a model of native spatial representation is built (see Fig. 1). It should be kept in mind that none of the notions represented below has been defined in a complete way. The aim of the study was to reach a model that would be satisfying for the Navajo consultants and that would allow for unambiguous differentiation of all relevant notions by means of this double constitutional relationship (being a constituent of, having as constituents).

Other spatial notions :

- NEAR/SEPARATE/CONTIGUOUS : things or events are near if they can be reached easily, have an easy accessibility, can be touched, and so on. This topological notion of 'small distance' is defined through reference to one constituent in the Navajo model, namely volumeness/planeness. The difficulty to reach something or the inaccessibility of something is rendered in the similarly intuitive notion of separateness. When two phenomena are actually touching one another then contiguity is the adequate description of the situation. All three notions are constituted by means of volumeness/ planeness.
- PART/WHOLE distinctions are rare in Navajo and they certainly do not play the same role that they do in the western spatial system. It was concluded (Pinxten et al., 1983) that functional relationships were constituents of the few part/whole distinctions found, and that functional relations were more often than not used in their stead. The constituents of notions of part and whole are volumeness/planeness and function.
- BOUNDARY : this spatial notion proved to be present in two, mutually irreducible forms in Navajo. In the first place obstructive boundaries were recognized, implying that agents could pass such borders by slightly modifying their behavior (e.g. climb a fence, jump over a wash, look for the pass in a hill, and so on). This notion of boundary is semantically constructed by means of the following Navajo notions : volumeness/planeness, movement, horizontal dimension or in/out.

In the second place Navajos distinguish ultimate boundaries, that is borders which cannot in any way be trespassed. One can think of the shore of an ocean, the borders of the world (in mythological thought) or the edge of a mesa. This notion of boundary is constituted by means of : volumeness/ planeness, part/whole, order or one of the three dimensions.

- IN/OUT AND CENTER/PERIPHERY : The first pair of distinctions again

has to be specified and divided into subcategories in order to comply with the navajo distinctions. Moving in or out of something or some space is an altogether different notion than being in or describing the inside or the outside of something. The former implies movement, border notions and volumeness/planeness distinctions, while the latter one is codefined by means of volumeness/planeness and border and has a connotation of defining specific instances of volumeness or planeness. For example, the latter will be used to describe types of containers or holes.

The **center/periphery** couple is a very important one in Navajo spatial orientation. Hooghans, the earth, baskets, and so on are all said to expand from the center out. Moreover, all phenomena are said to have a period of growth from their center out until they reach the ultimate periphery (for their nature), after which moment the shrinking movement from the periphery toward the center will be set in. This suffices to indicate that center and periphery have an important role in the actual spatial orientation and organisation. They are constituted by means of volumeness/planeness, part/ whole and order.

- OPEN/CLOSED : To the westerner and certainly the researcher who believes in the Gestaltist law of perception and cognition, the very general lack of recognition and processing of openness and closedness in pictures or situations comes as a surprise. It is in the action of opening or closing that the Navajo will recognize this notion, but its relevance will practically be confined to that. Again, for the geometry development program this culture specific emphasis is important. The notions of openess and closedness are semantically built up by means of volumeness/planeness and movement in Navajo.
- OVERLAPPING : Again, the emphasis on action or movement is striking when it comes to semantically characterizing 'overlapping'. It was considered preferable, in view of the data collected on this notion, to split the notion in two aspectival forms : clearly active overlapping and active/passive overlapping. The first notion refers to actions where something is hiding or otherwise covering something else (a shadow, an act of removing something from the sight of someone, and so one). The second notion expresses a more passive or constant situation (e.g. a mountain covers part of the panorama, and so on). This distinction is gradual, but it proved important enough to be made explicit. The constituents are volumeness/planeness, nearness/contiguity or border, and horizontality.

- CONVERGENCE/DIVERGENCE : These notions are closely linked with the Navajo notion of center. They express the movements from a center out or towards a center of a plurality of items. One can easily see that the explanation of the geometrical patterns of light or even water will benefit from the training of the present notions. Their semantic constitution in Navajo is fairly specific : center, near/separate, movement, horizontality and plurality of items.

- ORDER; SUCCESSION : Order is of course, a very basic notion in geometry. Navajos have a profound epistemological-ethical notion (hozhoon), which emphasizes the basic order and harmony of all things in the universe. Moreover, the particular emphasis is most adequately read in spatial terms : natural order is the way things have being placed and kept in place until the end of the world. (Pinxten, et al., 1983, Ch. 1; Witherspoon, 1977).

Navajos make a distinction between strict and unambiguous ordering of elements and nearly complete or strict ordering (semi-order), in a way very similar to the western distinction. These notions are constituted by means of the following Navajo notions : volumeness/planeness, dimension (one or more), and plurality of items.

- FRONT : BACK, UP/DOWN, AND LATERAL, NEXT TO : It is clear that the distinction between front and back, between up and down and between left and right (or lateral) are all made by means of reference to the human body in the Navajo mind. The body frame is offering to man a system of reference which permits to unambiguously and clearly organize space. Navajos, like many other peoples, use this implicit frame of reference. The notions expressing these distinctions are all built up in a rather simple and straightforward way : volumeness/planeness and movement (or direction) are the constituents in the Navajo world view.

Metric expressions of these very same distinctions are rendered in notions such as 'distance', 'height/deepness' and 'width' The basic differentiation remains the same, but their constitution makes use of one more specific semantic component, namely order. That is to say, Navajos base measurements along any of the three dimensions on the same procedure westerners would do : they apply a notion of order along the dimension selected.

- CARDINAL POINTS; COORDINATE SYSTEM : Instead of introducing a Cartesian system of coordination, Navajos refer to the system of two or three dimensions combined through their understanding of the movements of the sun vis-à-vis the earth. Therefore, different positions and movements of

the sun in its diurnal traject are named and used to build up the system of cardinal directions. The constituents of a spatial nature are : direction, path, center, movement.

- MULTIDIMENSIONAL EXTENSION : Surfaces or volumes which are measured in Navajo are handled in a more or less rough or imprecise way. The basic intuitive differentiation of instances of volumeness or planeness (roundish, bulky, pointing, flat, etc.. phenomena) are approached with comparative measures (so many times in a row this container, f.e.). The measurements are imprecise and the procedures to measure are, though intuitively satisfying, not carried out with enough zeal for exactitude and uniformity to allow for a more general and more abstract geometry. Nevertheless, the basic insights (volumeness/planeness combined with more or less uniform order) are there and geometry should be introduced by working on these rough foundations first and foremost.

3. Geometrical notions :

Navajos do have some geometrical notions. In a previous study some of these were analysed and their form and extent should be mentioned here (Pinxten et al., 1983, III and IV).

Point :

Navajos conceive of a point as the smallest possible extension of a surface or volume. This intuitive notion is hardly ever used, except for orientation in a wide open field. It was never used, according to my knowledge, for the construction or identification of other geometric notions in Navajo.
Its constituents are : multidimensional extension and order (smallest).

Line, straight line :

Again, line is not an abundantly used notion in Navajo. Rahter than lines are volumes and planes the basic material out of which the phenomenal world is built up. Lines are recognized in the forms of paths or as directions for movement. Straight lines are even less frequent and 'natural' in the Navajo experience. Lines are particular expressions of a notion that is built up semantically by means of direction and dimension.

Angle, pointedness :

Angles of less than 90 ° are clearly distinguished. Corners of things and constructions and shapes of body parts or formations of solid material are recognized as angles (cf. the importance of nazhaz forms, recognized in flowers, corals, the form of the vagina and so on). Whenever the form is clearly sharp or pointed a new series of terms is used, expressing the faculty to pierce or penetrate, rather than the formal aspect of being an angle. Angles and points are constructed by means of the following Navajo spatial notions : volumeness, inside, border, direction.

It remains to be investigated exactly what range of corners and angles is recognized as such by Navajos.

Parallel :

Parallellism is a notion which is developed by means of movement : two figures or two persons can run parallel to one another and thus identify the notion. There is a very rude idea of not losing distance or not coming together, but no sophistication of the sort we have been using in geometry proper. The components are : plurality, volumeness/planeness, dimensions, movement.

Geometric volumes, surfaces, figures :

No sophisticated notions or a geometric nature have been found. It is clear that several geometric figures are distinguished by Navajos and that their language allows for a delicate and very satisfying description of differences and identities. Nevertheless, all figures are more readily rendered in terms of actions or manipulations than in terms of exact forms. It is not clear to what extent this preschool knowledge can be used to come to construct geometric notions, except for the level of action-based and intuitive notions.

The following notions were found :
- ringlike, circular surface
- round, balllike volume
- curved, wavelike volume
- cylindric or barrellike volume.
- square (four times a corner)
- triangle (arrowhead or three corners).

To give an example of the construction of these notions, it suffices to mention one : square is understood in the language to be such a surface (or

volume with practically no thickness) that one has to trace lines four times, every time connecting in a corner. This very rough translation of a very rough term is expressing the basic line of thought and offers a hint as to the ancre to use for geometry proper. One could try to start from the movement-laden notion of square, triangle, etc. first, and grow towards abstraction from there on. However, it is clear the notions should be refined and made more exclusive to begin with.

This ends the short presentation of Navajo spatial terms. To be sure, in view of the construction of geometry there has been a sharp selection such that only notions which bear relevance to the intuitive geometry of Navajos in se, have been withheld. Some other notions (e.g. path, orientation) may be introduced as a supplementary entry, if they are needed in some specific part of the geometry program. Those interested in the complete model and the complete exposition of detailed ethnographic information on Navajo space are referred to the book cited before (Pinxten et al., 1983).

Part II

EDUCATIONAL AND TECHNICAL PROBLEMS.

It is impossible in the present study to extensively present and discuss the different approaches to geometry teaching and its psychological prerequisites, which have been followed in the West. Instead, I will only present a set of convictions I share with some of the scholars in this field and express my doubt or downright disbelief in the positions of some others. Against this general background it will become clear what option will be taken with regard to the Navajo geometry project.

I disagree with the 'Piagetian' approach to geometry (and mathematics) teaching : it states that spatial conceptualization follows a strict logical development in the child and that education should tread in these very same paths.(1)

It appears that a very rationalistic structure of eduction and geometry development emerges, as is apparent from the several criticisms (cf. below). In the cross-cultural studies of the Piagetian school this emphasis on the logical structure of the genetic development of concepts led to serious difficulties expressed in the rather generally agreed position there that children of other cultures are lagging some years behind (e.g. Berry & Dasen, 1974).

In contradistinction to the Piagetian 'program' I agree with the emphasis put forward by Hans Freudenthal : in order to construct abstractions one should start education from the richest and most diversily structured context and gradually have the child evolve into concepts that express more general and thus poorer and purer relations and entities. In practice, it is thought to be desirable to work with the richest possible examples and projects in the classroom to begin with (e.g. villages, islands, etc.. which are constructed at scale model) and to come to the understanding of gradually more general and more abstract notions by focusing first on the concepts that spring to mind in the given context for the child (e.g. Freudenthal, 1979; Five Years of IOWO, 1976; Freudenthal, personal communication).

In this approach, it is believed that, in order to reach abstraction, one should pass from the poorly differentiated rich context over specific visualizations into abstraction. Geometry, more than any other branch of mathematical thought, provides us with an adequate transitional discipline in this

(1) It should be mentioned that Piaget himself never claimed an educational application for his findings; rather mathematicians used his findings (e.g. Piaget & Inhelder, 1947) to find a psychological support for their rationalistic theories on geometry education.

regard: it feeds on spatial differentiations (and thus on a rich common sense context) and it works as an abstract system, just like other mathematical branches. Therefore, the training of visualization in the development of geometry and the training of the transition from rich contexts to abstractions of geometrical thought can lay out a foundation for the child on which to build further mathematical understanding. It is the pretention of this approach that abstractions, reached in terms of this solid foundation in visualization, will be better understood and will allow for easy understanding of other abstractions.

What is proposed in the present book can be summarized along the same lines of thought : Navajo children normally live in a context where the Navajo language is used to denote and analyse all aspects of daily life. The cultural as well as the natural context of the children will thus be framed and interpreted in terms of the Navajo tradition. One of the aspects of the tradition (and not a minor one, because of the all-pervasiveness of space in Navajo language, cf. Werner, 1982) is the system of spatial knowledge or spatial differentiations, which is found in the Navajo culture (cf. Part I of this book). Consequently, Navajo children come to school with a preschool context which is most of all Navajo. In order to reach the level of abstraction which would offer most guarantees for an adequate and proper understanding of abstract thought and its use, it would then be advisable to develop a program of visualization which feeds on and is to a great extent an expression of the differentiations of space and other aspects of the Navajo preschool world view. Therefore, in this study the emphasis is on the training of autochtonous differentiations in spatial talk and spatial perception and on the development of local cultural themes as means to master the road to abstraction. In the same thrust, it is understandable that one has to go one step further than any of the geometrical programs we have mentioned so far: in order to guarantee a good and proper understanding of abstraction and abstract thought when dealing with another culture (than the western one) I claim it is advisable and desirable to develop native abstractions in a native context and by means of native language. In other words, I think it is advisable to train the Navajo children in a Navajo context of education and by means of (properly agreed upon) Navajo terms in Navajo visualizations. This process will end in the construction, training and elaboration of abstractions that have a meaning primarily in the Navajo way of thinking (cf. the scope of the research on modern mathematics in Cole et al., 1971).

Ofcourse, this development will tend to change some of the perspectives of the traditional Navajo culture as well, but it will maximally guarantee a successful introduction of abstract thinking. A preferential means in this panoramic procedure is the development (to whatever degree will be feasible) of a Navajo geometry.

III. On the usefulness of visualization.

The study of space or geometry in other cultures is not very well developed. The study of curriculum programs on geometry in other cultures is practically nonexistent. On the one hand, this facilitates the job for a researcher in this area, since he can skip most of the inventory and comparisonwork that is typical of scientific analysis. On the other hand, this situation acts like a threat to the unsuspecting scientist : is this domain so desertlike because of the pitfals or is it just the case that nobody ever took the other subject quite serious ? I will take it that the latter is the good answer and try for a new development after the introduction of the little that has been done in this area.

Some scholars have been doing research on the potential role of geometry teaching in the western school curriculum, especially after the drastic removal of geometry out of the curricula of most schools in the western countries over a decade ago.

On the role of geometry in general, and of visualization in particular in the context of abstract thought, one can discern a variety of standpoints. Volrath (1976) distinguishes between not less than seven theoretical positions. In contemporary education (and most of all in the West) two alternative views seem to be dominating the competition : a) 'geometry as an inventory of concepts and theorems for constructing theories' (Volrath's number 2), and b) 'geometry as an inventory of theories of space' (his number 5). The former will put strong emphasis on axiomatization and will develop a curriculum with a strong logical structure (in this sense Piaget and Klein can be situated here, in as far as they focus on the rationalisitic nature of geometric thought). This latter will emphasize the 'mathematization' of the natural environment, the development of an intuitive geometry based on the experience of the physical world by the child, and so on (cf. Freudenthal, Minkowski, and others).

Different researchers have done work on the relationship between spatial ability or spatial knowledge on the one hand and mathematical ability on the other hand. The question treated here is clearly the following : does spatial ability have an influence on the capacity of a subject to master mathematical abstraction or not ? The positive answer to this question would naturally be a strong argument in favour of the intuitive geometry- or anti-rationalistic approach discussed above. The experimental studies set up thus far seem to yield contradicting or ambiguous results : Lean & Clements (1981) come to this conclusion, since their experiments in this area result in findings that partially contradict some of the work of others, partly cannot be interpreted in any strict sense (e.g. the Gestalt tests they performed do not fit in their general overview of results and seem to contradict their own work in this sense), and partly cannot be said to be confirming any of the approaches, since the tests are insufficient. The authors point out that they tested only a certain amount of variables, which are deemed relevant for the study of the relationship between spatial ability and mathematical performance. They were stuck, however, with the variables that were worked into the tests available, and they express their conviction that a whole set of non-mathematical variables are left out of the tests. From the contemporary point of view in these matters, however, it can validly be asked what non-mathematical variables play a role in the development of mathematical performance (cf. again the Freudenthal-critique on geometry curricula). The general idea one gets, then, is that of insufficient and biased testing in these researches, since tests are developed from a theoretical viewpoint on what mathematical performace is all about and this viewpoint is based on a rationalistic or a counter-rationalistic psychology of cognitive development to begin with. Until we have firm and convincing evidence on the role of spatial and of logical components in the development of mathematical (and more specifically geometrical) thought, that is until different educational strategies are agreed to produce clearly different results in this respect, it looks rather hazardous to develop strict tests in this domain. Indeed, one is not very clear on what one is testing. Further on it becomes clear how and to what extent cultural components in the development of geometrical thought can be differentiated and tested.

In the area of curriculum development for geometry some interesting results have been reported already. In his overview article, Kapadia draws some conclusions from this history of two decades. He claims that recent developments in curriculum studies, where geometry has been either eliminated from the school or adapted to be more rigid and more rigourous in order to

comply with the idea of geometry as a strictly deductive system of thought, have led to a decline in visual thinking and in abstract reasoning. He mentions experiments which showed that geometry is better suited to learn axiomatic thinking than e.g. algebra. His conclusion is that geometry proves to have a separate status vis-à-vis two subdomains of human thought : mathematics and language. His general analysis of the relationships between the three is as follows : language has a weak syntax and a very rich semantic and unprecise reference; algebra has a very weak semantics, a rich and strong syntax and high precision of expression. Geometry is positioned, on all three criteria, in the middle of the other two : it has a moderate semantics, a sober syntax and relatively high precision of expression. For example, the figure Δ can be linguistically interpreted as the mutual contingency of three straight lines. It will be defined as three straight lines of which no two will be parallell in geometry. On the other hand, it can be defined most precisely as the image of the affine transformation of the border of a 2-simplex, in algebra. From this analysis of the status of geometry Kapadia draws the conclusion that geometry can fulfill a unique role in the development of abstraction and axiomatic thinking, since it can gradually alter the vague notions of common language into the extremely rigid and precise notions of algebra. Geometry is the bridge discipline between both, because it feeds on visualization. Bender & Schreiber (1980) go into detailed analysis of the operative genesis of geometric concepts. Their work can be seen as a useful elaboration of some of the more general standpoint of researchers like Freudenthal and Kapadia, and their examples and concrete analyses will be used in the following chapters.

IV. Visualization and the study of other cultures.

Some researchers have been studying the problem of spatial ability, visualization and mathematical performance in the context of education in other cultures. Bishop and Lancy are among the most talented and outspoken among them. The former has provided us with general insights and farreaching conclusions.

In a general overview of the problem Bishop (1980) starts from the conviction that genetic psychology plays an important role in mathematical education these days. Soon it becomes clear that all is not well in this respect : the three types of differences in mathematical performance he discerns (mathematical skill, cultural differences and differences of an analytical nature between geometries) are scarcely dealt with in genetic psychology. Only the first one will get some attention, and scholars tend not to agree on the crucial component in it : Bruner is the only one to plead for a systematic training in visualization. In most of his work Bishop tries to identify and measure the component of 'cultural differences' in mathematical education and to draw educational conclusions from his researches.

An example of a 'Piagetian' approach will make clear what focuses and what type of modifications prevail in this area. Mangau (1978) analyzes why the Piagetian theory seems to yield little useful results in cross-cultural research. He identifies four components in Piaget's theory which are said to be determinants of development of psychological processes : biological factors, equilibrium factors, social factors and cultural and educational transmitters. The last component is practically believed to be reducible to language influences. Mangau reports that this theory omits the importance of values in cognitive development and tries to show that genuinely 'emic' analyses would yield important corrections on the theory. The proposal that follows from this critique is that culture specific operations and paradigms should be allowed to complete Piaget's model. The latter are framed in terms of values, rather than strict cognitive features : Mangau distinguishes between a 'mythicomagical' mode of cognition (typical for nonwestern cultures) and an 'empiricoscientific' mode of cognition (characteristic for westerners). The problem is not at all solved, I conjecture, but it is framed in such a way that some by-effect of the rationalistic 'Piagetian' approach (the fact that some performances can be seen, which cannot be coped with in the theory, and that some other performances seem to be ratable according to the analytic frame used) can be dealt with.

In other words, the primary emphasis on the determination of concepts and development from the logical structure of geometric thinking out is not denied or questioned at all, but it is modified to some by the elaboration of the 'cultural' influence that Piaget had vaguely provided for in his theory. The net result is that Mangau comes up with a hotchpotch conclusion, which is practically impossible to apply in education : apart from a scientific animal, man is also a religious, an economical and cultural animal. Since it is clear that any of these terms has been tens of definitions by scholars in the past and the present, it will be clear that this conclusion does not allow for a better understanding of the educational task which is thought to be mathematics here (Freudenthal, 1973).

It might be better to turn towards specific problems and their solutions in this field of study.

The representation of threedimensional (3 D) phenomena into twodimensional (2 D) forms through drawing has been amply studied. Mitchelmore (1980) reports that this representation meets with serious difficulty in non-literate cultures, but that once the means and procedures of representations of 3 D into 2 D are learned and trained the results get better. Not only that, but according to his results the learning of this representation results in a growth of spatial ability and spatial orientation. Bishop (1979) investigates the same problem in terms of visualization and its role in mathematical develoment. He comes to the conclusion that part of the problem lies in the fact that the convention of pictoral representation was not known before in the culture under study. Consequently, primary to training in precise representation of 3D by 2D drawings, the teacher should implement the use and meaning of picture drawing to begin with. Another aspect of the problem is the heavy dependence on cultural biases in the western use of drawing and pictoral representations : Papuans performed very adequately when they had to draw maps (which is more 'natural' and 'purposeful' in their culture than drawing rectangles for example), than in the case when parallel lines, photographer's perspectives and the like were

implied. Bishop concludes : 'We don't realize how much it e.g. photographer's view] conditions us in our drawing.' (1979, p. 139).

A still further going remark of Bishop's concerns the implicit role and meaning of visualization in a particular culture : he found that some tests based on visualization procedures in the western context (e.g. Gestalt completion tests, filling out a drawing by connecting a series of dots, etc.) yielded very poor results, while others (e.g. copy the drawing of a cube under some particular angle) yielded very good results. His first conclusion is that 'even if the 'objects' were known to them /the Papuans/ the representations were not.' (1979, p. 141). Going further into the investigation of the logic in his results he found that Papuans were led in their attempt to make graphic representations and copy drawings by features like colour, shape, texture and size, but not at all by name. In a specific study in depth on the linguistic influences in representation he was able to construct a list of words which matched with a series of concepts (e.g. beneath, far, close, long, inside, etc..), and a list of concepts which had no terms in the language (e.g. line, forard, round, form, picture, pattern, etc..). Finally, in trying to use these insights in an attempt to consciously develop spatial abilities, he worked out a series of tasks where dual relationships between levels of representational abstraction are systematically elaborated into an integrated program of training in spatial abilities : object to object relation (reduplicating, manipulating objects), object to picture relation (from 3D to 2D), object to word relation (description of objects), picture to object relationship (constructing an object from a diagram), picture to picture relation (copy pictures, etc.), picture to word relation (read maps, describe pictures, etc.), word to object relation (describe object on the basis of linguistic data), word to picture relation (recongize maps, etc. on the basic of linguistic data) and word to word relation (describe phenomena on the basis of other descriptions).

All in all, Bishop found some interesting problems in his material and managed to prove their importance by systematically searching the relative impact of a set of cultural variables. One variable which is rather scanticy treated in his approach is the role of social influence or social pressure or collaboration. Lowenthal (eg. Lowenthal & Marcq, 1980, and Lowenthal & Severs, 1980) did several experiments about this particular variable and found that group identitiy and mutual cooperation tended to improve results considerably in mathematical education.

Summing up the results of reserach in this area, in as far as it is relevant for the purposes of this book, I can depend heavily on Clements & Lean's (1980) résumé concerning Bishop's work. I will add a few conclusions which are not mentioned there :

1. Pictorial representational conventions are teachable' (1980, p.3)
2. Training students in drawing and sketching techniques improves their ability to read and interpret other people's diagrams' (idem).
3. Manipulation of concrete apparatus develops spatial abilities.
4. Languages differ in their capacity to express spatial differentiations.
5. Differences in the quality of lexicon will yield differences in visualization skills.
6. Orientation and map drawing seem to be more readily developed than other spatial skills.
7. Memory problems do not need verbal interference.
8. Variability can be more important within a group than between groups.
9. Less acculturated groups seem to be better developed visually than more acculturated groups.
10. Social cooperation is favorable to mathematical education.
11. Mathematical performance has strong dependence on cultural components.

I think it can be defended that these conclusions should be taken as the starting point for further research. First of all, they may be applied to whatever extent is possible in education and thus serve as a basis for further research 'in the field'. This is what will be done in the present research : the proposal in part III of this book will be directed according to these (provisional) results and the observations of the use and development of this curriculum proposal will offer further insight in the working of the mathematization-via-visualization perspective.

V. Learning strategies and teaching devices.

In previous chapters of this part of the monograph, different arguments in favour of a so-called intuitive geometry and against a more rationalistic approach to geometry development and teaching (i.c. the 'Piaget-approach') were brought forward. I think we thus have been able to convince the readers sufficiently of the feasability of an intuitive geometry approach. Its implimentation and test in the field of Indian schools will further prove its validity and powerfulness. The question which remains largely to be dealt with is that of the actual teaching strategies that are possible or preferable for this endeavour.

1. General strategies.

Very generally I will follow the guidelines laid down by Freudenthal (e.g. 1979), stating that one should work from the richest possible context of the child out toward a more selective and more formal representation of aspects of this context in geometrical formula. It is useful and indeed unavoidable in this approach to work through visualization exercises and tests. In practice, this rationale is best followed by working in educational tasks or projects, each concentrating on some set of geometrical concepts.

Bender & Schreiber (1980) translated this emphasis in their notion of 'principle of operative concept formation' in geometry teaching. They reject the classical, rationalistic approach of teaching geometry by means of gradually constructing definitions and develop a series of concrete strategies in everyday context to come to geometrical thinking : the fitting of forms is an important characteristic for geometry and should be trained in the child's natural environment, the displacability along certain patterns is a similar feature, as well as the application of every geometrical concept on the real space in order to become conscious of its utility. They present a set of six educational goals for geometry teaching :

- look through geometric forms and visualize them,
- detect and describe the goal and efficiency of geometric forms (e.g. a

sphere can hold the pressure of gas),
- make and model geometric forms by means of anything: 3 D artefacts, design, language, etc.,
- solve both geometric and non-geometric problems in a geometric way (e.g. football is a polyhedron),
- build a system of geometrical notions, and
- recognize the aesthetic aspects of geometry.

Bishop (mimeo) gives another, even more practice-oriented list of goals. The acceptance of these goals by necessity implied the acceptance of certain strategies of teaching : the student should be able to understand the words (a), the diagrams (b), to understand and represent the situation (c), to understand the changes of situation (d), and to represent the situation on paper (e). These points should be kept in mind and should be reflected in any teaching strategy of geometry.

Both foregoing guidelines are echoed in a lot of the contributions in this field. It is impossible within the scope of this paper to give a complete overview, and I will rather point to particular suggestions on this common base in some other sources of interest. In the same scope, Egsgard (1970) proposed to subdivide the general procedure of geometry teaching in three stages : the teacher should first focus on form recognition and labeling, then on size bigness, and only in the last instance on measure.

Fielker (1979) develops a few examples in the same general perspective : he focuses on the fact that form recognition is feeding heavily on manipulating and displacement of forms. For example, the exploration of the realtionships between a circle and a straight line will learn (by displacing one or the other to reach all possible variants) that they can be either separate, contiguous or intersecting each other; the exploration of the possible relations between two circles will learn that they can form one out of five different forms (separate, contiguous on the outside, intersecting, contiguous on the inside, enclosed). And so on. The detection of these possible relationships teaches the child about the peculiarity of the figure, with respect to its form. In the last example, size played a role as well. These aspects, bounded by and founded on form and size, should be mastered first, before a reasonably successful attempt can be made to teach metric features and operations.

A useful and highly practical means of exploring the form and size of things in a context is highlightened by Freudenthal : he points to the fact

that contexts often are represented by the child in terms of polarities, and that these context-bound polarities can thus be used as keystones in order to explore the geometrical concepts exemplified through the objects and their context : above/below, head and tail, left/right, backward/forwards, inside/outside, inward/outward, from here to there, this/that, and so on. This idea of space being structured to some degree in terms of polar pairs can be seen to have cross cultural value in the results I summarized in Part I (cf. also Pinxten et al., 1983). A specific emphasis which completes Freudenthal's perspective is the focus on manipulation of objects in the child's everyday environment. For example, the concept of 'line' will gradually emerge out of the folding experiences of the child, while symmetry will be understood from folding, cutting or watching in a mirror, and so on. These ideas are developed in a series of projects for geometry teaching (Five years of IOWO, 1976):
- the 'Island of geometry' is a scale model of an island, which is then used to become aware of perspective, of the overlapping of one phenomenon by another one in a certain viewpoint, and so on. The reseachers state : 'there was mathematics in it : orientation, reasoning and geometric forms' (idem, p. 194-195).
- a 'viewbox' is made by the children, and different drawings and stand-in figures are created to fill it up. Thus, the children learn the notions of size, relative size, up and down, and so on.
- 'our earth' is a series of exercices concerning the spherical shape of the earth (an earth model is constructed by the children), and the different astronomical elements which can be seen to have different positions with regard to the earth (e.g. the sun can be depected at different heights, with rays coming down to the center of the classroom, from which the different lengths of the rays and the different angles of sunlight can be learned).

With these and similar examples Freudenthal's group managed to map out the basis of intuitive geometry teaching. They thus gave a lot of suggestions concerning the way a set of intuitive concepts can be grouped in a meaningful project, and the way the abstract notions will be reached through visualization and manipulation of objects and situations of everyday life.

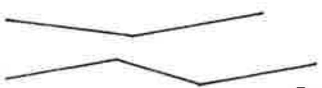
2. Some geometric concepts and the corresponding learning strategies.

Dienes (1969) gives some examples of topological notions and the way they can easily be learned in a school setting. First of all he advises to construct a labyrinth and have the child find out by means of walking or drawing how the labyrinth can be traversed without occupying any place twice. The topological notion of *'path'*, but also that of *'overlapping'* can thus become consciously known. Furthermore, Dienes conjectures that the best way to come to geometry proper (in a metric sense) is through the systematic exploration of the three important transformations that enable one to define a position : *translation, rotation and symmetry.* A systematic training of all possible effects of these transformations will give the child an exceptionnally useful entry into the system of geometric thought. It is obvious they can be trained very easily and without sophisticated means : the body can be used as an object, things can be folded, turned around, and so on.

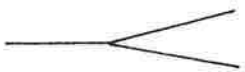
A somewhat more technical device is needed for the teaching of topological notions in the paper by Wallrabenstein (1973). The author works from a drawing where five domains, with two completely enclosed by the three other one, are given. The question is who can visit whom in this setting, where one element per domain is to be visited. The restriction is that one should not 'cross' (intersect with) an existing path. It becomes clear from the drawing attempts of the child that this exercise is impossible to resolve with five domains, unless a 'bridge' (a special access) to the fifth domain is built. Networks of this and other types are then explored and the child gets an idea of *connection, path, neighbourhoud, contiguity,* and the like by actually manipulating the spatial objects or domains concerned.

Fielker (1979) explores the concept of *'line'* and the geometric orders made possible by different amounts of lines and *'angles'*. He suggests to use Mecano, Construct-o-Straws, geostrips or anything of that kind. Immediately, these devices will make clear that the connection of two elements will yield one order (the two pieces are linked and form an angle), while three elements will allow for two orders, and so on :

2 elements :
3 elements :



Or :



From that point on, one can introduce the notions of open and closed figures : from the above configurations, one can make a closed figure with three elements by *displacing* one element (which then yield a triangle). Building along these lines, the multiple forms of four elements can be conceived : a square, a diamond, a convergence of three elements in one point, and so on). The Tangram play can be used to learn to construct figures according to order : e.g. not less than four different figures are possible, which all have eight straight angles, whereas only one (a cross) can have twelve straight angles. By manipulation and visualization the child will become familiar with these figures, because it learns to know its manipulative constraints and features.

In a particularly inspired paper Bishop (1978) gives a long list of devices he collected or developed himself in order to test the spatial abilities of Papuans. His special interest is the difference in competence between three- and twodimensional recongition and interpretation by the Papuans. Some of the devices Bishop listed can easily be turned around and used as teaching devices, I conjecture. They would then be used to develop the twodimensional representation (in drawing and in visual imagery) of the concepts concerned. Bishop mentions the following ones :

- reading and drawing of maps, indicating the roads, houses, and so on of the known environments;
- recognition of pictures at different angles and taken from different distances;
- copying a design;
- spatial talk : translating or paraphrasing some seventy terms from Papuan to English, or the other way around;
- modeling : constructing a model of something by means of sticks, plasticine, etc;
- sorting maps;
- imagery : imagining a situation and talk about the particularly geometric aspects of it (how many angles, etc.);
- location cards : cards show circles and lines in specific configurations ; their place is determined orally;
- square labyrinths : a series of gradually more difficult labyrinths is presented and tested;
- camera : a series of pictures are shown and the child must determine the angle of the photographer;

- visual memory : cards are spread and taken away; the child is then asked which card held what place;
- drawing blocks : uniform blocks make up a specific figure; the child is asked to make a drawing copying the figure before him;
- orientation of the bottle : a bottle with some water in it is shown in an upright and a tilted position, and the child is asked to draw the surface of the water in it;
- drawing of perspective;
- embedded figures : four figures are shown of which one is embedded in the others; which one ?
- match box : a child is requested to draw a dot on the place indicated by the teacher;
- word completion tests;
- picture completion tests.

Bishop used all these for testing spatial ability. It is clear they can be used as well, eventually varied or elaborated to be used to learn the concepts aimed at. The manipulation which is implicit in most of these may add considerably to the success of the teaching procedures.

With this example of a broadly set up observation test this chapter will end. Given the general perspective developed in the beginning section of the chapter, and the often very particular advices and devices mentioned in the second part, I think it will be clear that a lot of material is already available to start building a more intuitively aimed geometry teaching approach. The overview will end here for that reason. The main part of this book, namely the development of a curriculum and a set of strategies and techniques for the Navajo geometry classes, will be the subject of the rest of this report.

Part III

NAVAJOS, TEACHING OF GEOMETRY, RODEO AND OTHER ISSUES.

In the previous parts of this paper it was mentioned several times that education should start from the natural or culture specific knowledge and context of the child. We made explicit that we want to follow Freudenthal in this respect (and Piaget to a lesser extent) when he advocates such a strategy for the teaching of geometry as well. In the case of Navajo children we have the particularity that we should not only start from the world of experience of a child, but from the world of experience of a child in another culture. Therefore, we aim at developing a geometry course by means of terms and expressions which are Navajo, and in a cultural context of representations and choices which are, again, Navajo. Practically, this means we select five rich contexts in the Navajo world, which we present in the building of geometric concepts. It is within this rich context of experience in the Navajo world that gradually more and more precise and semantically poorer concepts will be detected by the child, while exploring these wellknown contexts in the classroom.

VI. Preliminary work.

The present work was used in a Navajo class at the primary level : children would be 8 to 10 years of age. This means they have passed the 'topological level' of spatial representation of which Piaget is speaking (Piaget & Inhelder, 1947, Mangau, 1978), but did not yet have the time or the mental capacity to work out a systematic alternative (projective or Euclidian geometry, for example). Moreover, in the research on spatial representation (Pinxten et al., 1983) we found that a considerable amount of the adult Navajo spatial differentiations tend to be very much akin to intuitive topological notions, somewhat in the way the Piagetians talk about the latter. Therefore, the following strategy is proposed :

a) One should first search for the actual set of forms that are differentiated by the children, both in their language and in their behavior. The Topological Test (a set of drawings) can be used to check on visual differentiations, while the material of the previous research (Pinxten, o.c.) will allow to delineate a set of relevant terms and expressions. Finally, the teachers should be alert in their contact with the children to come to a full understanding of the behavioral and manipulative space of the latter.

b) The Navajo teachers should then decide, preferably through discussion with Navajos specializing in the study of their own language, what terminological conventions should be respected or engaged in on the spot. That is, with Freudenthal (e.g. 1973) we believe that the development of representation goes hand in hand with the development of language, and,more in particular, we think that it would be in vain to isolate concepts or more standardized representations in the geometry class without at the same time introducing conventional labels for them. The actual abundance of detailed terms for spatial differentiations in Navajo language should be reduced to a managable set of more abstract terms in the course of the development of more abstract concepts. E.g. the notions of line, point, volume, surface, and so on should get standard labels in Navajo, notwithstanding the particular features of the spatial forms each label will and can refer to. This is a task of primary importance, it appears, and Navajo native linguistics will come to face such tasks more and more the greater their interaction with the white world.

c) Finally, we can get to the development of spatial and geometric exploration in the cultural scope of the Navajo tradition. The child will have to explore by itself in the classroom setting or in the open air. We propose to work by mean of five projects for geometry teaching, of which the first will be elaborated immediately : 1. rodeo
2. hooghan
3. school compound
4. herding sheep
5. rug weaving.

Each of these projects deals with activities and phenomena the children know personally. Each one is rich in spatial meaning and can be explored, reconstructed, visualized, described and acted out by children of this age. Therefore, these cultural contexts seem to offer the ideal means to have the children explore the inherent geometry in their understanding of their environment.

In order to explore these contexts in a way that is close to the child's own experience, we think it is essential to allow for manipulation, active explorations, remaking and the like as much as possible (on this point Freudenthal and Piaget agree totally, for once). Therefore, we aim to maximalize the possibilities to do, rather than just watch and repeat. We propose the following simple auxiliary means for this line of didactics : material of all sorts (carton, boxes, paper, wood, iron wire, and so on);
- mecano or geo-strips, pipe cleaners, and so on;
- drawing material; plasticine;
- rope, strings, and so on;
- nailboard and rubbers;
- any small item from the natural environment, depending on the need for each specific project.

It should be clear that teachers will use the natural environment as much as possible. They might work part of the time outside in the open or reconstruct scale models of environments in the classroom (e.g. the school compound). It is essential that the child's body should be seen as the first and foremost instrument of the exploration of the environment. Therefore, manipulation and actual bodily exploration, such as displacing things or

'walking' a figure, will be favored as much as possible and will be the first steps in the detection of geometric forms and relationships.

TEACHER'S NOTE :

The construction and use of a *"nailboard"*:

The teacher and /or the children can easily construct a nailboard :
- take a rectangular plank of 2 inches thick, and 14 by 10 inches;
- divide the surface into a grid of 140 square inches by drawing 13 straight parallel lines in width and 9 straight parallel lines in height;
- at each crossing of lines a small nail is driven in to some extent, making sure it sticks out such that a small piece of rope or a rubber string can be attached to it. We get the following board (part of the real one):

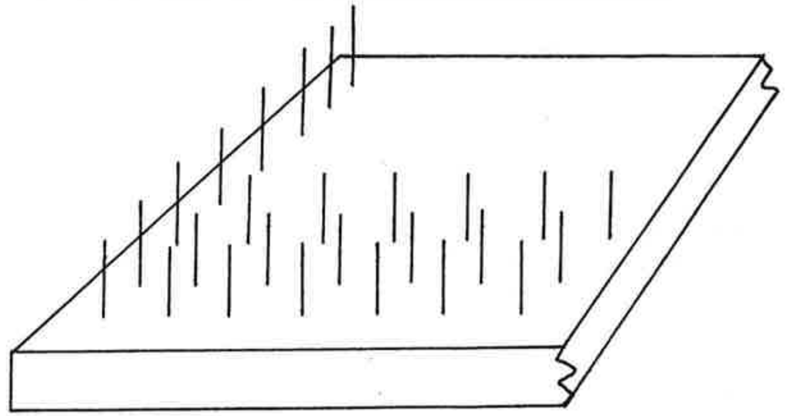


Figure 2

The board can be used to try out *by doing* (i.e. using the rubber or the rope) the mental image of a figure a child has in mind. For example, the straight angles of a square or rectangle can be detected by means of the board (i.e. counting so much units-nails-sideways and so much up, then so much sideways again, etc.), different triangles can be explored by picturing them on the board by means of the strings or rubbers. Correction is always possible and proves to be very easy; the child can compare a drawing or a real situation with his imitation on the board, and so on.

The board has proved to be very useful in the development of visualization, and it allows the refinement of collaboration between hands, eyes and mind.

LINGUISTIC CONVENTIONS :

The following terms will be introduced / considered in Navajo to serve as technical terms for geometric thought; their meaning will be determined in the course of the geometry teaching experiment, since this meaning is to grow and to be identified in the gradual evolution towards Navajo abstractions :

- point : dah alzhin
- line : k'ee (footsteps)
- straight line : k'esdon
- spatial forms : all classificatory verbs
- near, separate, touching :
- border : -dzoh
  linelike border : anit'i
- (being) inside : ghone'e
- (being) outside : t&'oo'di
- center : alnii'
- open : ą́ą́'
- close : kał
- overlapping, covering : —
- succession, order : iit'i'
- horizontal/vertical : —
- far (metric) : —
- big, small : neez, yazhi
- thick (metric) : ditą́
- wide, broad (metric) : hoteel, ts'oozi
- angle : tł'ah
- surface : circle : hobas
  square : —
  rectangle : —
- volume : ball : homas — cube : —
  cilinder : — — piramid : —

VII. The Projects.

Rodeo has grown to be a major point of interest for Navajo children. Especially boys are eager to assist the rodeo events and not seldom the dream of a boy or young adult is to become a rodeo rider himself. It should be mentioned that, although the atmosphere of an Indian rodeo is different from a classical cowboy rodeo (most of the time Indians are silent, for example), the importance of this sort of amusement for modern Navajos cannot be underestimated : it is possible to find rodeo poems in some houses, and everybody wears the wellknown attire these days. Moreover, it often happens now that upon the birth of a boy one of the parents buries a string of horsetail under the hooghan So that he may become a good rodeoman' (Frank Harvey, personal communication).

Rodeo is a family event. Gradually, rodeos were organized on the reservation, which prescribed that only Indians can compete for the prices (cf. the All Indian Cowboy Rodeo, Ganado, 1977). Navajos are growing to be specialists in cattle roping, wild bull riding, and so on, and the sport is catching on as a major form of amusement for the summer period. In the light of all this, it seems appropriate to use the rodeo scene as a project in the geometry curriculum. Moreover, the 'western' forms and units which are displayed in the rodeo setting will be studied in this case through Navajo eyes (cf. the 'rethinking' of a car in the booklet of Rock Point, MS). Motivation is guaranteed, and the specific way of integrating this phenomenon in the Navajo cultural scope offers many perspectives on the problem of form identification and representation in the Navajo knowledge system. Therefore, this phenomenon can be a good point of departure to have the children explore their environment inview of geometric thinking.

1. Strategy :

Following the principles of Freudenthal and Bishop in this respect, the primary way of exploring this context is to imitate or reconstruct it (at scale model).

The children will be asked to make a rodeo ground in the classroom or on the school compound. Anything can be used here as material for construction : old boxes, pieces of wire or fence, etc. The main thing is to get an idea of the rodeo ground as a unit, and details will be left to the appreciation of the children. Children will have to work at scale model size, since otherwise the task would be beyond them. In general we will get the following sort of structure :

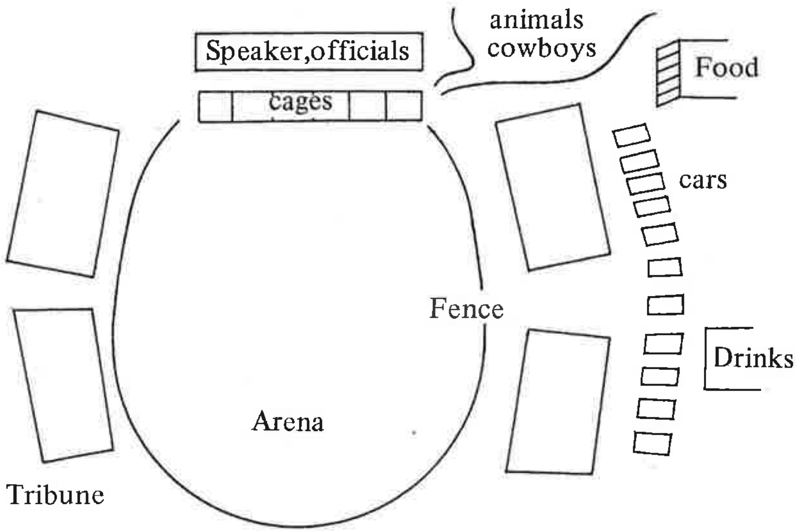


Figure 3

The following steps should be 'found' and followed by the children :
1. everybody should work together to realize the project,
2. one first has to draw a rough map of the site : indicate flat surface, place of arena, of cars, and so on;
3. one should respect proportions in order to make it look 'real' : the arena should be big enough to hold several horses and people, in comparison with the cages (one for each horse or bull) or the stage of the spectators; the tribune of spectators should be bigger than the cars which contains people, and so on. The exploration and discussion of proportion is one of the basic aspects of map drawing and should take up considerable time. It also allows for the child's imagination and spatial representation (in the mind) to develop and be subjected to criticism or support from others. As such, it is a very important means to develop visualization (representing the situation in the child's mind) and is a main avenue for the development of abstraction in geometric thought.
4. by dividing work so that everybody in the class has some task the gradual realization of the whole project, i.e. the making of a small rodeo ground, can be reached. It is important to stop at regular intervals and discuss the progress as well as the difficulties of all involved.

Once the rodeo ground has been constructed the exploration of its features can begin.

2. Exploration :

At this stage the teacher should assist and eventually guide the children in their role of explorers of this construction. Many different approaches are possible here, and a combination of a multitude of perspectives might be advisable. These are some possibilities, although they certainly do not exhaust the range.

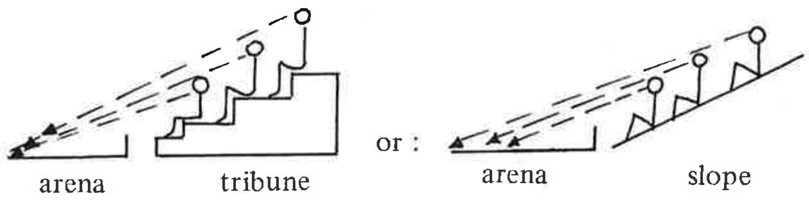
- detection of the usefulness of regular (geometric) forms :

The *arena* is a polygon or a circular construction. Why is that ? Because it allows for easy construction and easy action (one can never be 'caught' in a corner with the animal that way). Thus, a certain form allows for a certain range of actions. In fact, the form of the arena can be defined in terms of action: it is corresponding to the set of all actions of animals and men in this space.

The *speaker's lodge* is up front and well above the level of the arena. Why is that ? Children can try out different positions and it will result that they have a better overview of the total area (arena + public) when they stand on top of a chair or ladder in the speaker's corner.

The *spectators* are sitting or standing on the outside of the arena. They are sitting in rows on a special construction. The use of straight angles and the construction of stairs to sit on can be demonstrated by using a few benches one on top of the other or simply using a slope to sit on : (Fig. 4 A and B).

It will be clear to the children that the meaning of the slope or staircase can be deduced from its capacity to have more people have a better view on the scene all at once. This will make clear the interdependence of height and overview, of obstructing or allowing a view depending on the position of the viewer. The principle can be traced in other contexts as well.



Figuur 4 A and B

The *fence* between the spectators and the arena is a border : it indicates how far the spectators can go without risk, and to what limit the animals will be able to progress. Thus, the fence divides the total space in two very distinct units : the place of action (arena) and the place of watching (outside). It furthermore defines an inside and outside position of the units : the arena is on the inside or encircled by the spectators' place.

The *cages* are up front. They open and close. The latter means they regulate the supply of animals and cowboys by letting them into the arena or refrain them from doing so. Again, this opening and closing principle can be found in many other contexts (door, reservation, etc.).

The *total context* can be grasped in its form and function by approaching the model from different angles : standing far away from it, it will offer a different view from standing inside it, and so on. Children will grasp the meaning of front and back, inside and outside, englobing and other relevant concepts better if they can explore the total context of the rodeo ground from different distances and from different angles. They can get an overview of the context if they can have a bird eye's view (from some distance and from some higher point) and they will be able to see the structure of englobing and of background-rodeoground distinction better that way : the total scenery can be seen as three concentric rings or circles with the unit 'arena-cages-speaker's lodge 'in the center', separated from the encircling

ring of tribunes by a fence, separated in turn from the cars and food stands behind. The whole complex in its turn is perceived as one unit in the landscape that surrounds it and that is gone through when the family comes to a rodeo from some place else.

3. Details :

Several of the particular units and forms on the rodeo ground can be studied as particular phenomena in themselves. The following examples may suffice, although they are certainly not exhaustive :

- The *cages* : cages can be built by the children by means of matchboxes or strings and pieces of wood. It is important to explore the different dimensions which are apparent in the use of the cages : the animals are introduced in the back, after which the cage is closed. The cowboy climbs in on the top (which must be open) and lowers himself on the back of the animal. He can barely fit in the cage together with the animal. One then opens the front in order to allow the rider and animal to go in the arena. This box must thus have a complex and highly functional form, although its construction proved fairly easy
- The *lasso* : cowboys use lassos to catch and to rope the animals in some exercices in the program. Children can be trained to understand the knots which are important here : the specific ways of overlapping, crossing and gliding of the knots offer important means of understanding the basic geometric notions (topology).

How is the knot? Why is it that way? How does it work? What form does the rope have (line + circle)? How can these be formed out of just one piece of rope?

- The *movements inside the arena* : the rodeo ground is first and foremost a place of action. The different actions of cowboys (storm out straightforward, trying to get close to an animal, or sitting 'glued' on a wild animal for some time) are important elements to explore the form of the arena, the spatial relationships between animal and rider, the relevance of folding and bending to get a better grip and view of the animal, and so on.
- The *tribune* is constructed as a staircase : why is that so? In this way the notions of overlapping, covering, hiding, and looking over can be explored in some detail. What angles are necessary and sufficient ? (e.g. a shallow angle offers a lot of possibilities already, while a wide angle may

be uncomfortable). What surfaces are needed at each level?

4. Conclusion and guidelines :

It is clear the forgoing paragraphs offer only a sketchy outline of the ways and means to use the rodeo theme in the classroom. The different and highly detailed actions and statements of children were not mentioned and should 'give life' to the picture drawn. The present paper can only indicate the lines along which the actual procedure of learning can develop in the classroom. The material presented should then be used in a broad and well-bred way, including for all steps and for all detailed parts of it at least the following stages of teaching :

- children should first of all detect and identify the spatial forms talked about by manipulation. That is the reason why they should construct a model of the rodeo ground.
- once they have become familiar with the forms and situation through manipulation, one should take care that they will use the proper words (in Navajo) for each element and for the spatial features displayed at whatever level of this cultural context. Sometimes, the teacher should introduce and exercise some new terms (e.g. a standard word for border, or for the steps of tribunes).
- in a third stage parts (and the whole) of the built models should be drawn on paper, exercising the graphical representation of the spatial relationships. In this instance the nailboard can be used as well.
- finally, the situation constructed and depicted on paper can be alterred in the child's mind. The alternative should be drawn on paper. E.g. the teacher can ask : what would it look like if the arena were square ? Draw it. This stage will train the visual representation (through imagery) and the graphical representation of the children.

It should be clear that, already after and through the teaching of the 'rodeo ground' context, a tremendous amount of geometric material will have become more conscious in the child's mind, laying the ground for further and more systematic concept formation in the future. It is good to keep in mind that the concepts and labels developed at this stage of teaching geometry should all be Navajo and that no attempt whatever should be made to alter the child's conception toward a 'western' or 'Euclidean' understanding of the notions discussed. If this evolution will take place (and this is un-

certain for the time being), it will be at a much later stage, when abstraction and the role of visualisation in the acquisition of native abstract notions will be fully established.

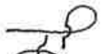
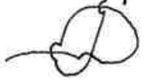
The following notions (at least) have been touched upon and are taught at a first level in the rodeo context : map, drawing, box, arena, flat, surface, proportion, polygon, circle, up, front, above, inside, outside, rows, angles, slope, stair, border, dividing space, englobing (containing), opening, closing, distance perspective, knot, fold/bend, and so on. All of these are actually present in the child's natural environment or in his wellknown world. All of them have their particular meaning in the child's world view. It is this level we are aiming at first and foremost and from there we will gradually evolve towards a more conscious and a more logical way of dealing with these meanings, in the child's world.

Appendix :

It is obvious that the making and understanding of knots is a crucial and powerful means to come to the understanding of topological (elementary geometric) notions.

We know that a lot of Navajos are able to play the so called string game (*tł'óółts'ósí*), making all kinds of figures by folding, turning, having overlap, and so on a piece of string held in both hands. Moreover, the different sorts of knots and wrappings which are displayed in the rodeo games can be understood as ever so many ways of producing different knots. Since knots are important to come to understand elementary transformations in geometry (turning, folding, overlapping, stringing through,...) we advice to teach and make explicit these aspects of the games. Concretely, one can explain and have the children do themselves (by imitation or better still by finding out themselves) how a single rope or string can be used and handled to come to the different sorts of knots and figures that one knows. The use of the nailboard can substitute that of strings or ropes.

The importance of these practices cannot be stressed too much : we suggest children can play with knots and strings that way at regular intervals, for example whenever there is a little bit of time left in another course. By having this regular practice of knotting they will readily learn the concepts of geometric transformation and the teacher can help this learning by labeling these concepts in the appropriate standard Navajo way, thus introducing an important part of the geometric lexicon. We should reach an agreement on the standard expressions to be used :

- folding = 
- overlapping = 
- turning completely around (to make a knot) = 
- turning + overlapping = 
- stringing through = 

VIII. The hooghan.

The hooghan is the single most important construction of the Navajo tradition. Although more and more Navajos go to live in new prefabricated houses or trailers nowadays, the hooghan is not only still in use by many, but it remains the necessary ceremonial construction for all. Thus, it is not rare to see people live in a house or trailer and build a hooghan on the side for ceremonial purposes. Again, although the traditional rather primitive and barely furnished form of the hooghan might be vanishing rapidly now, the modern and often well-equipped hooghans still have the same basic form as the traditional one. (cf. e.g. David McAllester & Susan McAllester, 1981).

All in all, it is to be expected that all children know hooghans from their personal experience, either because they assisted a ceremony or because they have lived in one for a longer or shorter period (with grandparents, for example). Not only that, but because of the fundamental structural caracteristics of the building, all children will have some knowledge of the meaning and the form of the hooghan (cf. Haile's introduction to Blessingway, Wyman, 1975).

1. *Constructing a hooghan scale model :*

Children will go and visit a hooghan in the neighboorhoud. They will ask about the way it was constructed, eventually assist at the construction of a new hooghan in the vicinity. They will note down what they learn and explain everything in the classroom.

Starting from a set of this material, one proceeds to construct a scale model hooghan in the classroom, again using simple materials (and mud) most of all.

Apart from this, we advice to simulate a hooghanspace in the classroom : e.g. by sitting together in the proper way, surrounded by 'wall' of carton boxes with only one opening (to the east).

With this preparation, the exploration period can start. In the construction of a scale model hooghan or of a 'hooghan space' to sit in, in the classroom, the following aspects of the hooghan should be kept in mind (whatever variation of form one might reach) : the hooghan has a center; it has one dooropening (to the east) and it is more or less cicular.

A first, rough approximation of the hooghan can be simulated; during the lessons the faults and difficulties of this construction will be detected, after which a more accurate version can be tried out.

2. exploration.

a. "Playing hooghan" :

the children are visiting their grandparents who live in a hooghan. What should they do ? When is eating time and where will it be cooked ? When is sleeping time and where do they sleep ? From examples and talks of the children, it will appear that the center is the main structuring element of the internal space of the hooghan : people sleep around the center, walk around it, etc. The center is where the fire usually is and where the sand-painting of a ceremony will be at. Around the center one can detect the walls and the poles in all cardinal directions. The notion of center can now be explored more systematically :

- try to point to the center of the 'sit-in hooghan'in the classroom;
- walk from any "wall" towards the center and count your steps;
- adjust the center of the hooghan until it is more accurately in the middle of the hooghan space;
- how can we know the distance between center and wall? by stepping : but the length of steps differ with the length of one's legs (explore with teacher and student stepping). How can we get to a uniform and unique measure ? By using rope or stick. One chooses a rope or stick this way and reconstructs the distance between center and "walls" in a more accurate way. To exercise this notion of 'measure', smaller and bigger hooghans can be constructed in the classroom in order to grasp the idea of interdependence between center and distance (width), by using different lengths of sticks or ropes. One should always conserve each of the ropes or sticks used, in order to aid visualization.

b. The structure of the hooghan :

What do you need to have a hooghan : the bowlshaped form, consisting of

a shell-like top and a shell-like bottom. These represent the heaven and the earth in Navajo tradition. How are those forms constructed ? By defining a center and four cardinal directions (east, west, south, north). These can be detected in the classroom construction as well and oriented the proper way.

Around the four main poles (one in each direction), sticking straight up in the sky, legs or planks are attached, one on top on the other. In this instance the difference between vertical and horizontal extension is clear. The horizontal logs reach a certain height (man-high); on top of this construction the roof is constructed. So what you have in different stages of construction is :

- a *flat* surface, with *center* and peripheral walls;
- a determination of the *border* of the hooghan space by pointing out the place of the standing up *poles* and the horizontal walls;
- this gives a sort of open *cilinder* (more or less), which is then topped of by a roof, making a top — border in a way;

Children will try to represent this process in drawing, and in actual manufacture.

Once the principles of this construction procedure are more or less grapped, one can go on and represent the model of the hooghan at a more abstract level by using *folding* figures (in the sense of the UNICEF Educoll-system) :

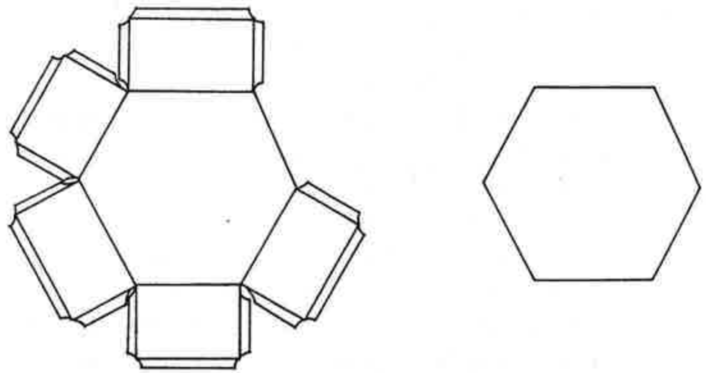


Figure 5

In the above example, the hooghan has six sides, one of them being the dooropening. The figure should be prepared by the teacher and can be cut and coloured by the children. The lid or roof is somewhat problematic : it might be best to use a flat hexagon and fill it up (if need be) with some waste material to have it bulb a little bit.

One can now explore the hooghan shape in a systematic way. While turning around the hooghan model, it looks approximately the same from all possible directions. This corresponds with the real hooghan : its geometric shape (hexagon or octogon) makes it appear more or less identical from any angle. The following exercises in visualization can now be performed :

- looking at it from one direction, one sees a rib extending toward one's eyes in front, whereas the back ribs cannot be seen. Is this rib bigger than the other ones? Why do we see it bigger and not see the other ones?
- turning around the paper model, we always have a similar look (except for the door), however we turn the construction; how is this possible ? In this way, the uniformity of the geometric shape is perceived and consciously understood.
- looking on top of the hooghan (like from a rim of canyon down on top of the hooghan in the canyon), we perceive the polygon as a whole. When looking right from above, we cannot see the entrance, and the polygon appears to be regular on all sides, structured by a hole in the center (the smoke hole).

Once this point is grasped, children will try to draw the hooghan from different perspectives. These drawings will be imprecise, because the children cannot master the use of the drawing instruments of geometry. Nevertheless, they will freely try to approximate the form in a graphic representation. They will gradually learn that they should start from the centerposition and 'measure' a uniform rod distance from there. Eventually, a stick or a piece of rope can be used at this instance, bringing us to the following way of constructing the polygon. (Fig. 6)

By means of these drawing exercises, it will become clear that the polygons fit nicely into a circle and that many polygons can be fitted into the same circle. This may lead to the drawing of different maps of hooghans : hooghans with six, eight, ten walls and perfectly round hooghans (like the mud hooghan that can be seen at some places still).

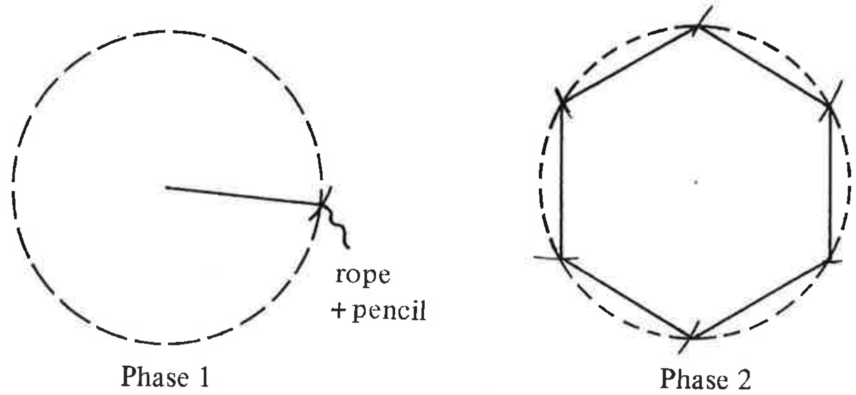


Figure 6

c) Details :

- The logs or planks which are used to erect the walls of the hooghan have a particular shape. By means of inteviews the children will become aware of this shape and the functionality of it in the construction : what shape do they have ? (straight, smooth, just long enough for one side). Why would that be so ? (it is easier to build with flat and straight logs or planks than with irregular or bulky ones; they fit nicely on top of each other).

By imitating the construction of hooghan walls by means of building block games, it will become clear that straightness along the length dimension and regular straight angles for the thickness dimension makes it easier to put one on the other. By making corners (to reach the hexagon figure), the construction gets more stability : each wall supports the other ones. This can be easily tested by the children : construct a wall of light material and try to push it. The wall easily gives way. Now do the same to make a cube or a hexagon : the construction is much more stable and cannot be pushed so easily. Even human beings making such forms by their intertwined bodies can illustrate this principle (children in classroom).

This exercise learns about the use of flat surfaces (walls) to make a stable space (hooghans). We take up notions like straight, angle, surface, hexagon, bottom/top and center in the process.

- the above mentioned aspects of space can be handled abundantly in drawing lessions : make a plan of a hooghan with eight sides; draw the walls one next to the other as flat surfaces.

- develop different perspectives on the hooghan and represent these sketchily in drawings :
  - looking from the top down (from th rim of a canyon), we see the polygon structure with center and walls.
  - looking at it in a horizontal plane (like when coming up to a hooghan when sheep herding, looking at it up front) we see a rectangle with a bulbing roof (arch).

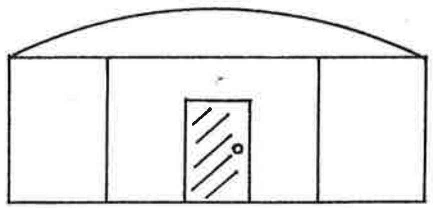


Figure 7

- looking at the dirt hooghan from aside, we see a half sphere. This shape can be studied some more. First it should be drawn :

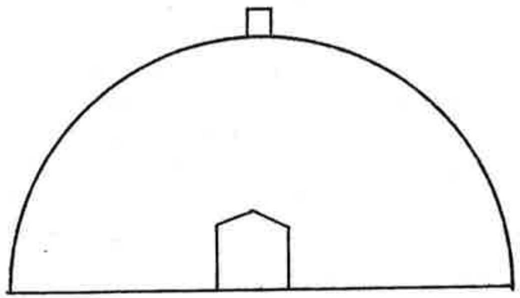


Fig. 8

What are the caracteristics of this shape ? Use the rope to detect these caracteristics (from center out the whole circumference can be drawn, ending in a straight line on the surface). Suppose we would put two dirt hooghans on top of each other, one upside down and touching each other's base, what would we have? A sphere. What can you do with a sphere? (roll on every side), when is it standing up, when is it lying sideways?

Children will then look for other spheres in their environments (like balls, tumble weeds, etc.). They will explore the peculiarities of the sphere and try to reconstruct one in plasticine or other material.

General remarks :

The same points hold here as in the rodeo example : children should gradually grasp the terminology (standardized for the purpose), the graphical representation, the situational and the processual aspects of all geometric figures in this context. We think it is clear that one should always start from the natural context, then visualize the features in it and turn to graphical representation.

IX. Herding sheep.

Navajo children tend to have some experience with sheep herding, most of all those living in the poorer areas of the reservation. Indeed, it is not uncommon to see children herd the sheep in different parts of the reservation. From very early age, they accompany older ones who are roaming the vast desert planes with the sheep herd. Therefore, the spatial experiences which go with the sheep herding activities can be used profitably in the classroom in order to build geometric notions. We have the intuition that, the area and the landscape being represented in the child's mind by means of the movements and needs of herder and sheep, this praxiological basis can be abstracted in a gradual process : the movements can be imitated at scale level and gradually the graphic representation and the mental operation will become substitutes of them. The needs of herder and sheep (water, reachability of lands,safety of wide open spaces, and so on) can become the more abstract criteria of spatial organization in geometry.

1. Storytelling :

Teachers may start by having some of the children tell the story of their experience with sheep herding. Eventually, these stories can be supplemented by one told by the teacher (from her or his own experience or from a story book).

Following the exposition of the stories, the class will now detect the spatial aspects of the events of sheep herding by actively asking questions to the story tellers and investigating in each case what particular answers are yielded. All children note, draw or represent in some way or other the answers.

The questions can be of a diverse nature :

- How do you herd sheep ? Do you have to move on or hold the herd together ? How do you realize their sticking together ? (these questions will yield answers with notions like 'progressing', 'covering a big surface', 'encircle the sheep by dogs', and so on).

- How big is your herd ? Is it bigger or smaller than x's herd ? How can you tell ? (these questions may lead to notions such big, small, estimate by size, and so on).
- Where do you head with the herd ? What is you point of departure ? What distance do you cover in one day ? Where do you stop ? Why ? Tell us, as detailed as you can, the exact path you follow. (These questions will allow for several very important notions to come to the fore : beginning and end (border), distance in terms of walking, points of interest in the landscape, orientation, accessibility of sources of food and drink, and so on; the detailed presentation of the route will allow for a better understanding of the criteria of spatial orientation and of concept formation in geometry).
- Do you always follow the same route(s) ? Can you arbitrarily point out a route for the herd or are there standard routes ? (answers to these questions will lead to the identification of the notion of 'path': standardized routes of walking between two places).
- What places do you avoid ? Why ? (here a notion of 'separation' is built up : keeping some unprecise distance from some places or things).
- Suppose you are lost, how can you find your way back home ? Undoubtedly, notions like cardinal directions in terms of the movements of the sun and in terms of the place of the hooghandoor will be yielded here. Moreover, the recognition of the form of mesas, mountains and valleys will play a role. All these spatial caracteristics should be recognized, their function in spatial thinking should be trained and their contextuality should gradually be de-learned in order to use them in genuine geometric thought.
- When do you decide to go back or turn around ? (answers can point to temporal or spatial limits, we conjecture ; either the day or week or season is coming to an end or the border of the territory is reached).

It is clear that different paths and different forms of a path can be detected in the source of this sort of search. We think of the following examples : a path is closed if it seems to turn at some point and return to the place of departure, whereas a path can be essentially a connection between two separate places if it does not lead back to the initial place. By means of examples in the stories told, this distinction between at least two different sorts of paths is reached. Conventional terms should be agreed upon for each of these types and for the general notion of 'path'. Finally, children should learn to draw the paths on paper, thus representing graphically the difference between openness and closedness of a path, or between

linear and circular paths. We think it would be advisable to have these notions trained thouroughly, e.g. by having the children construct graphical representation on the basis of mere oral instruction, or, preferably, by having the children direct each other orally. We see the following possibilities by way of example :
- H. Dingler (1933) introduces a geometric system on the basis of actions by the human being : a line is something one constructs by following a surface in a certain way : in the present case a path can be drawn by the child by 'enacting' a search through an imaginary environment (of rocks, mesas, etc.), where he or she follows the surface of the valleys and mesas as much as possible. Someone directs another person who is drawing, by telling him or her where he or she has to turn in what direction and for how far, and so on.
- in modern attempts at 'turtle geometry' (Abelson & deSessa, 1981) a similar emphasis on exploitation through ordered action can be seen : the turtle robot (eventually to be acted out by a child) draws a figure on the basis of very simple commands by the child. One should take care here to use as little different orders as possible, and preferably (like in the book by Abelson & deSessa) stick with geometric or quasi-geometric notions only to construct all other figures, i.c. all possible paths.

2. Map drawing :

A second avenue to treat the context of sheep herding in its geometric potentialities is to focus on map drawing. The following approaches seem to be valuable in this regard :
- the school possesses a scale model of the environment, and a set of photographic pictures of the environment; the children can now start to identify the mesas, mountains, valleys and washes systematically on the pictures and find them on the scale model. Picture, label and place on the scale model are then joined. This happens for the total environment.
- the storyteller (cf. sub 1) or the other children in the class can train themselves by means of the scale model and pictures : they can retrace the path of the sheep herder on the scale model, and eventually seek other paths which would be equally possible for the herder. Several alternative situations can be modelled in this way, from text or from oral testimonies by children or teacher.

- finally, the foregoing procedures should lead to a supplementary step toward abstraction: children will now try to represent the total situation and the paths of the sheep herder in a graphical way, by drawing a map of the landscape and of the sheep herder's path through it. This step may have to be taken gradually : the teacher will guide the children's exploration of this shift from three- to twodimensional representation by pointed questions and invitations to 'act out' the ideas they propose. First, a group trial can be set up where the total class or two or three groups cooperate to construct a map on the basis of the scale model :
  - all important figures (mountain, mesas, etc.) should be depicted on the map, at approximately the size and place they have on the scale model.
  - different colors should be used to indicate the mesas, mountains, and valleys. The color ascribed to all of them in Navjo knowledge (e.g. Black Mountain near Rough Rock) may be useful here.
  - finally, a step by step mapping of the movements of the herder and his sheep will be aimed at. A continuous referring back and forth from the model and the visualized places to the map is necessary.

Once this shift from scale model to map is realized one time, it has to be trained thoroughly : different paths can be copied from the scale model. And, finally, the construction of a path can be trained solely and completely by means of orders to move and draw, given by the teacher or by fellow pupils. The child then becomes more and more the reasoner-designer he is growing into by geometry.

3. Details :

Black Mountain :

This is the dominant mountain in the immediate vicinity of the school. Its form is very particular (elongated, log-like with a little nub on top of it towards one end). The children can be asked to copy the form of the mountain, to rebuild it in plasticine, to cut out in paper its typical shape, and so on. Since the mountain is geometrically rather regular, it offers a nice opportunity to explore the possibilities of geometric figures in reality. (cf. Freudenthal, 1979).

Movement and geometric figures :

- The stories, the representation on the scale model and the graphic representation all made clear that the movements of the herd can be characterized by means of different quasi-geometric notions : we distinguished between an open and closed path, where the herd gets away from the original point or finally gets back to the point of departure. The second notion is, of course, the topological equivalent for a circular path. It should be trained thourougly in order to have the children understand the concept of closedness :
- The nailboard can be used to represent some of the possible closed paths in sheepherding : their formal correspondance will become clear from their comparison. That is, it be borne in mind for the child that all the different paths have in common that they are closed, i.e. have the same point of start and finish, and are completely continuous (do have a cut or jump along the way). This distinguishes them from the open paths. Some examples :

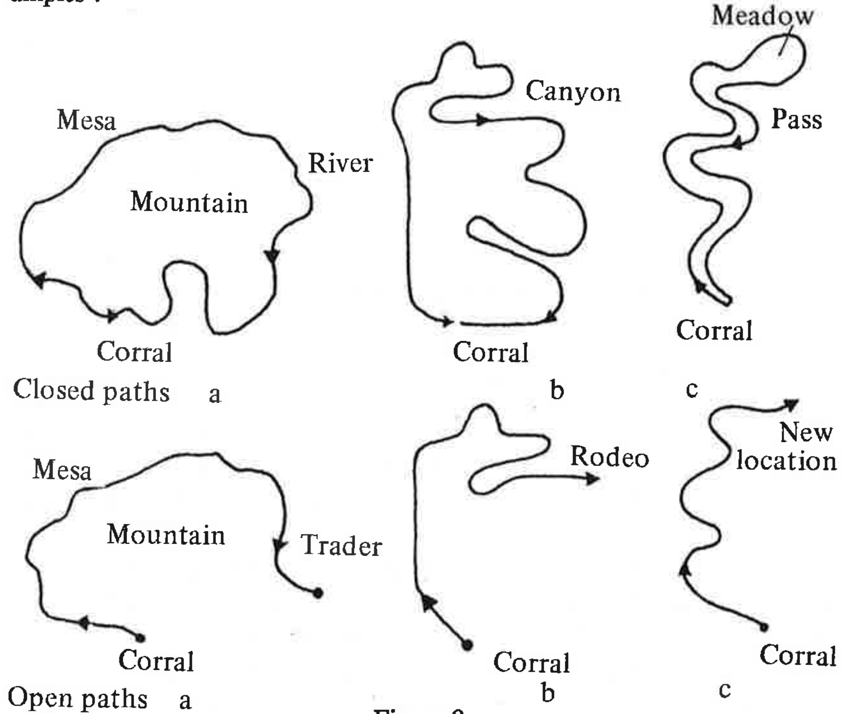


Figure 9

- Ad example 'closed path a': the herd starts from the corral and wanders around a mountain to come back after some time (e.g. weeks) into the corral. In example 'open path a' the same path is followed during some time, but finally the sheep are sold to the trader at the other side of the mountain, and do not come back to the corral. Although the path is the same for some part, the difference between both is striking : in the first place the circle is closed and the sheep 'come home', in the second example the path is broken down at some point ('trader').

Ad example 'closed path b': the sheep start out from the corral and wander through a canyon where they can graze, after which they are led back to the original corral. In 'open path b' the sheep follow the same canyon to some point where they are used as meat for a rodeo event. They do not come back to the corral anymore, and thus their path is broken off at that point.

Ad example 'closed path c': the sheep are led to the meadow on the other side of the mountain by crossing a pass over the mountain. After some time they will cross the same pass in the reverse direction and come back to the corral. In example 'open path c' the sheep are led over the pass to a new location on the other side of the mountain : this is the case of a family which is 'relocated'. They will never come back to the original place and the path between 'corral' and new location is broken, will not be closed again.

These and similar examples can be trained. They are important for the better understanding of geometric figures later on. It is important to vary situations, imaginative or real ones, as much as possible here, in order to assure a thourough grasp of the concepts :

- where do we find similar sorts of movement ? The sun has a broken path as far as we can see : comes up in the east, moves over the south to the west, where it vanishes to come up again in the east. Or, the sun makes a full circle (going partly under the earth : Pinxten, et al., 1983). Draw these representations on paper.
- when we visit town, we can go by different ways or use different paths to go there and come back. Represent these possibilities graphically.
- an exception to the foregoing paths : Sometimes, when herding sheep, we will have a situation where a small part of the path will have to be used twice, i.e. where the initial path is crossed for a short part. We have the

following examples of a sheep herd setting off along the road to land in the valley x first, then make a loop, cross the road again and land in a loop y.

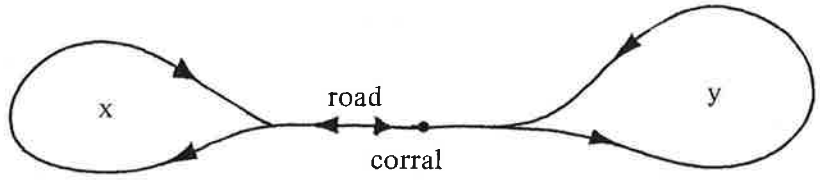


Figure 10

The case is interesting : we only have this situation when a path overlaps with itself for some part. We then have real 'loops' at both ends of the common path, that is, we have two close paths which finally make up together one closed path. There is no cut or break in the path, but part of it is run through twice. This situation should again be varied to yield two or more loops (cf. Freudenthal, MS : try it out by yourself).

- the herd itself has a particular spatial character : all sheep stick together, and when one or two separate from the herd, they are gathered and driven back into the group. This unity of all sheep constitutes an entity, called a *neighbourhoud* and is distinct from the herder and the landscape since it acts as a unity. However although the herd varies in form (it will become elongated to go through a pass, or roundish in the wide open field, or come to a triangular form when entering the corral), it will always be recognized as the same entity. Here again, the topological character of the spatial unity prevails over its many particular forms. This topological features are highlighted in the Navajo language : the herd will be referred to by means of the *sikaad*-stem (mainly *nanishkaad*).

Rocks :

Different forms and shapes of rocks can be identified. It is essential that a set of more or less standard formations can be selected and identified : e.g. some rocks are identified by stressing the horizontal dimension (Rock Point : *Tsé nitsaa deez'áhí*), or the vertical dimension (Standing Rock : *tse* *li'áhí*), or the particular form of the rock (Round rock : the table aspect of it is stressed : *tsé nikání*).

In this sense all different and wellknown rocks in the vicinity of the school can be dealt with. Their names can be identified and studied. Their geometric structure can be explored, for example by means of the Tantram game (mapping out the forms by means of the pieces of that Chinese puzzle).

It should be borne in mind that this way the apparently incomparable forms of rocks can prove to be composed of (or to be describable in terms of) similar or identical geometric forms in particular combinations. Eventually one can thus come to a typology of rockformations in terms of their component of a geometric nature. Drawing exercises will have an important part in this exploration.

Telling the time :

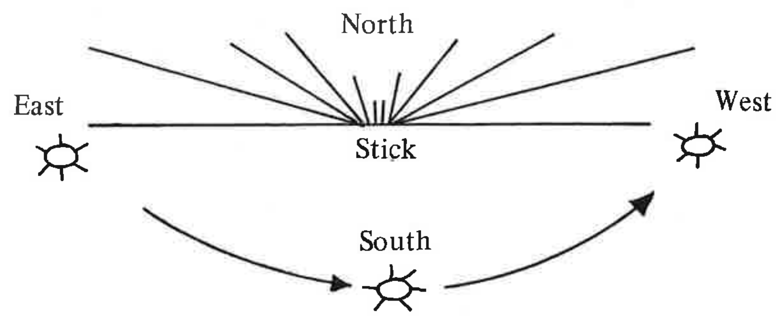
When in the wide open field with the sheep herd, one can learn to tell the time of the day without having a watch. The play of sunrays and shadows will help us.

Children use their body to stand as an indicator of the sun : standing erect in the middle of a sunny space, one throws a shadow in the direction opposite the sunrays. This situation is observed and then represented graphically in the classroom. Further exploration can be directed by using another rod in the role of the child : a stick is planted in the middle of the sunny spot and the shadow of the stick is drawn in the sand. Every hour or so the displaced shadow is drawn in the sand again. (The story of Spider rock can be used in this instance : the rock in Canyon de Chelly throws a long shadow on the bottom of the canyon, and throughout the day a 'web' of shadows covers the canyonfloor). After a day's work in this sense, the results are fixated in drawing : we get the following complicated picture : (Fig. 11).

The morning sun (in the east) throws a long shadow to the west, shortening during the morning until it is practically vanished at noon (sun high above us, no shadow) and gradually growing to the west with shadows toward the east. Now, depending on the length and the direction of the shadow, we can determine the stand of the sun and thus the time of the day.

In the classroom a sunray-meter can be designed. Eventually, in a

higher class this reasoning can be taken up to come to measure the angles formed by the sunrays when they touch the earth (cf. IOWO, 1976).



Figuur 11

distances and agents :

Different notions of distance will be used throughout the exploration of the sheep herding context. It might be worthwhile to note them down and compare them in some detail. We will present some ideas in this regard : distance might be the ideal way to come to understand the operations of measurement.

When herding sheep, it will happen that you can touch or grasp some of the animals of the herd : they are within grasping distance, very easily reachable. Some other animals will be outside of this range, further away : you have to walk over to them to grasp them. The distance between yourself and the animal is expressed by means of one unit of length : the length of your arm or of a stick, whereby you can touch them.

When you are walking through a pass or through a wide open space some of the sheep may be lost out of your sight (they are ahead in the pass, past a curb in the road, or they are covered by a rock so you cannot see them). When they have bells around their neck or when they bleat, you can always hear them and you can hear how far away from you or from the main group they wander. The distance here is different : they are within hearing distance. You know you can reach them or the dog can catch them easily when they are within hearing distance. The unit of measure here is the distance your hearing can carry, or the distance their noise will carry.

When you spot an animal that has drifted apart on top of a mesa you may not be able to touch or even to hear it, but you can still see it. You can then climb the mesa and get it. The distance here is one of sight : as far as you can see the sheep clearly. The unit is the distance covered by the eye.

When one tries this out in the field, one can have very intriguing experiences : children are brought in the wide open field. They first determine and note down the 'touching distance', then experiment to determine the hearing distance, and finally note down the seeing distance of each other (one separates from the group who keep talking, or separates until it can hardly be seen anymore). After that, the distances are measured where the touching or hearing distance are subsequently taken as the unit for measuring, and the length of the greater distance is gradually determined as 'x times' that of the smaller distance.

This notion of measure and unit of measuring is to be trained thoroughly. It will become clear that , although the actual and particular notions of distance and unit of measure differ, the operation is identical.

These are some examples of how the context of 'sheep herding' can be used in order to detect and develop geometric notions. It will become clear that, here again, the gradual 'detection' by the child and the systematic and gradual progression towards more abstract use of particularities and regularities in the context are essential for the success of the enterprise. Again, the labeling, pictorial and graphical representation follow the thorough manipulative and visual exploration of the context. Visualisation is only meaningful here, and its use will be highlighted in every instance possible.

X. The school compound

On the reservation all schools are built in such a way that they form a small village in the desert. Since Navajos live in more or less isolated hooghans, the schools form a particular figure in the landscape : in some places they are the only concentration of buildings in a large area. Often, schools are built next to a mission or close to a dispensarium or hospital (or the other way around) ; they always have a supply of water (most often a watertower) and electricity. Finally, the schools consist of some educational and administrative buildings and a large amount of houses for the teaching staff. The totality is most often surrounded by a continuous fence and/or a ditch in order to keep the animals out. Schools are always connected with the outside world by means of a road (in contradistinction to some hooghans).

Since children are brought and returned to school and hooghan by buses, all children know the school and the system of school compounds to a certain degree. Therefore, the context of the school is absolutely relevant and important in the world view of the Navajo child. Schools are, of course, creations from the west in this area. That is to say, their form and function are specific for the western way of seeing and representing space, among other things. Concretely, the buildings on the school compound have straight walls, right angles, and so on. Their functionality is first and foremost computed in western traditions, although some modifications have been introduced in the total system to meet the specific demands of the reservation and the Indian traditions. It is important to note all this from the beginning : we are not introducing a context which is Navajo here, but a context which is very well known to all Navajo children, although it is foreign (i.e. 'Anglo'). Children will have developed their view and their representations of this alien institution, of its forms and functions. It is these representations, which are interesting to us now. Is it such that the Navajo represent the school and everything to it in a way which is specific for them (as they did with the car), and what does that look like ? How does the Navajo child picture this extremely geometric construction in his own world view? In other words : how is a particular geometric tradition of forms modified in the spatial representation system of a particular culture ? How do they perceive these same forms, and how do they conceptualize them ? In this respect, the geometric regularities of the school compound offer a unique possibility to come to know and understand the way Navajo children cope with these rather unfamiliar forms of the 'carpentered world' of the west (cf. Campbell, 1964)

1. Making an overview and map of the school compound :

It is clear that children should first and foremost explore the compound, in order to have a clear overview of what is there and where each part of the school is situated. We propose that the class should be split up, giving the different groups precise tasks as to their exploration of a particular part of the school compound : some will go around and determine the place and function of all classrooms, some will locate all inhabitants on the compound, and so on. They may be proceeding by asking questions to as many people they meet, note down the answers and eventually make a little map of the places they visit. Eventually, they may already use the map of the school ground here, in order to determine the precise tasks of each and every group. (They may have learned what a map is in one of the forego-ing projects).

The teacher should have a more or less correct ground plan of the school compound ready[1], when the children come back from their tour in the school. The plan is then introduced in the following way :

- the teacher asks what this ground plan of the school expresses, how it works (reference can be made to the ground-plan of the hooghan).
- the plan is oriented according to the cardinal directions; entry of the compound is designated.
- the places are determined on the plan where the different buildings should be located, according to reality (the children may have to go out and look for themselves several times in order to be able to do this identification task).
- from this identification of places and building a first and tentative exploration of the function of the total form and structure can be set in : why is the building of the main office placed here, and the houses are located on the outskirts? What is the function of all these educational buildings and why are they situated the way they are ? In this respect, a first understanding of the usefulness of the geometric designs is reached.

(1) We think it is advisable to make a big ground-plan, such that all buildings of the school can figure in it in the size of shoeboxes, for instance (cf. further on). Children will be more at ease since they can then more or less move around in it.

The following buildings can now be imitated (e.g. by means of carton boxes or shoe-boxes) and applied to the groundplan :

- houses, with tags indicating who is living there,
- classroom buildings and administrative building(s), with tags indicating who is working there, also the dormitory, etc,
- the water tower(s) can be constructed out of sticks and a ball (e.g. mecano sticks),
- roads can be drawn on the ground-plan, connecting the different buildings.

It is important to note that the above procedure may take a considerable amount of time, since the representation on scale model of such a complex phenomenon as a whole compound will confront the children with a myriad of problems of visualization and abstraction : detection of directions, detection of orientation of buildings according to the cardinal directions, detection of the proportion (size and form) of buildings with respect to each other and to the total ground to be covered,etc.. When the location and orientation problems have been solved, it may be advisable to color the whole setup, in order to aid imagination during the exploration procedures.

Finally, in the following stages of the exploration of this context, children will gradually come to the graphic representation of the groundplan, of particular buildings and of parts of the total setup. The teacher should provide, again, for pictures of the school compound, taken from different perspectives and under many different angles.

2. Exploration.

Children identify on the plan where they are situated now. The questions that should be put from there on are : how can you reach the office of the director of the school from this spot out ? What different paths can you take, and which is the shortest one ? Try to reach the director by using as few doors as possible; try to get to him by using two doors and by coming out in the open air only twice, etc:. For each of these tasks the visualization should be central : Children should try to imagine the road on the basis of the plan, speak out and then indicate on the plan how they would go about in practice. Finally, a few children (or the whole class) can try out in reality and check whether the proposed solution works or not.

Similar examples of exercices can be found with other instances : x lives in building A and wants to pay a visit to the teacher w. The teacher is either in the classroom (F) or at home (in building C). How should x go in order to check F and C, using as little distance as possible? Again, visualization and map reading will go hand in hand for the solution of this problem.

After some of these examples, children can draw or be handed out preprinted maps of the school compound, on which similar exercices can be done by each child individually or by small groups of cooperating children. The roads or paths can be traced on the maps in different colors.

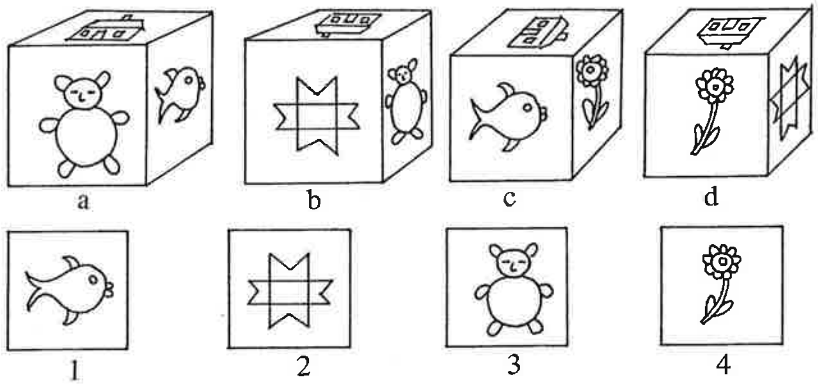
Orientation and perspective can be trained by making use of the same devices, complemented by the photographs of the school : different pictures will be shown, representing the compound from different angles. The children are asked to imagine, on the basis of the groundplan, where each of these pictures were taken. Furthermore, they are asked to locate and model on or next to the groundplan the constructions or environmental features which are relevant for the pictures shown : e.g.

- a picture showing the school from the top of Black Mountain implies the identification of that mountain, as well as its entering into the scale model some way or other ;
- a picture showing a road coming to the school is taken from the east, since the only paved road is heading from east to the school (continuing in a dirt road towards the west) ;
- a picture showing a flat plane coming up to the school is taken from the north, with Black Mountain showing in the south ;
- a picture where the watertower is partially hiding behind a set of school buildings is taken from the ... ;
- a picture showing a bird's eye view of the school without the water tower can only be taken from the top of the latter, and so on.

These and similar exercices should be maximalized, since they offer the

children the opportunity to check their capability to visualize correctly.

Following such a set of exercices the abstract corresponding puzzles can be used in order to train the step towards more context-free visualizations. We propose the following set, which can be enlarged and elaborated at will :



Figuur 12

Children should try to find the answer to the following puzzle, without making or drawing anything, but using more visualization : which one of the squares 1, 2, 3 and 4 should be placed in the cubes a, b, c or d. Fill in the back side or the missing parts in each of the four cubes.

The exercise is meant to train the mental moving around (turning) of the structure, such that the imagination can take the place of actual manipulation and fill in the exact square at the back side of the cube. To reach a good result the whole series must be visualized, just like the whole school compound must be 'kept in mind' in order to recognize and locate correctly the building by just using pictures.

Similarly, pictures can be shown of the school compound, asking the child to locate the place and the angle of the picture vis-à-vis the school.

3. details :

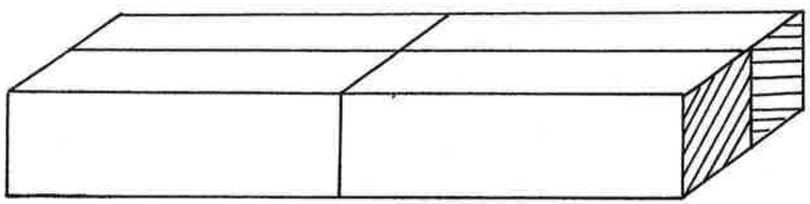
Building a *scale model* of the school compound in the classroom is done primarily by using waste material (shoe boxes, etc.). The *proportion* of the different buildings vis-à-vis each other and with respect to the total surface covered by the compound was very roughly recognized and respected in this regard. However, a more precise and more realistic attempt should be made now.

Children are given a whole stack of identical little blocks (either wooden blocks or Lego blocks or something similar can be used, as long as they have a standard size and shape). The children will now try to reconstruct a scale model of the school, respecting the proportion between the buildings and between a building and the total surface of the compound as much as possible. At this point, it will become clear that some sort of measure instrument (a ruler, a conventional stick indicating one unit of measure, etc.) is necessary.

One can gradually move up to more exact proportions by starting first with comparisons between units of the environment :

- the school compound is > or < than a house.
- the school compound is > or < than Black Mountain.
- the school compound is > or < than an educational building.

From there, we can make rough estimates of the proportions : the schoolbuildings are x times as high and y times as long as the houses; the water tower is z times as high as the educational building. Let us now take the lowest and smallest buildings as a unit (e.g. houses) and construct them with our Lego material. For example, an individual house :



Figuur 13

Then, the educational building should be x times as long and y times as high as any house. We can go on and construct the educational building in Lego blocks. We can now make all houses and other buildings needed.

By using different colors, we can easily distinguish between different sorts of buildings, just like we did in the drawings and in the construction of the first scale model.

It is important to note that these excercises will train the children's awareness of the use and meaning of measures. We are convinced it is useful to start from comparisons first, since after all all acts of measuring can be understood as more complicated acts of comparing (e.g. Reichenbach, 1958).

- Roads and paths :

On the compound, as wel as outside of it, roads and paths are first and foremost means of connecting places and means to allow human beings to get in touch with one another. This can be most readily seen by having children solve problems as to how to get from one building (e.g. home) to another one (e.g. to the classroom). When barring a road (because of an accident, because of a fence, etc.), one has to seek for another one in order to reach someone's place.

One can now ask children whether they would know similar phenomena, outside of the compound : how do you get to your aunt living on top of the mesa ? (follow the dirt road) how do you get to Many Farms, to the city of Cortez, etc.? It will gradually emerge that roads are, notwithstanding their physical appearance of a line, means to reach someone, to get from one place to the next one.

From this level on we can then proceed towards the more abstract notion of 'line' : how do you recognize a road ? How do you draw a road ? What do you get if the road is barred at some point (by a mountain, a flooding wash, etc..) ? How can we remedy this situation ? Represent graphically what can be done.

- Size :

Houses and the educational and administrative building have a certain form and a certain size. How big are they, as compared to the height or length of the children ? And to the adults ? What do they look like inside ? Why is that so ?

It is important here to have the children explore and detect the connection between their own size (and that of the teachers) and the form and size of the building they either live in or take classes in. Imagine smaller sizes of the building. Draw this situation.

Finally, we have to stress again that in this project, as in the foregoing ones, we emphasize heavily the manipulation and visualization of space and spatial forms by the children. Furthermore, the following steps should be present in the course, based on the context :
- recognition through manipulation and exploration;
- labeling of the units and relationships explored in Navajo language; becoming conscious of the Navajo words and their particular meaning ;
- graphic representation of the situations and of units in the situation;
- imaginary and graphic representation of changes in the situation.

XI. Rug weaving.

The weaving of rugs is, without doubt, one of the most extensively developed and one of the most profitable cultural activities of Navajo women. Already in 1884 Washington Matthews emphasized that Navajo weaving reached a level of artistic perfection, which was not matched by any other Indian tradition in the Americas.

Children will know what weaving is all about, from their mother or grandmother, although they will not be able to do it themselves until after their puberty rites. Through a small amount of instruction in the techniques of weaving, one can progress to the exploration of any geometric aspect of this context.

1. *Visiting a weaver :*

The teacher will take out the class to go and visit a weaver in their neighbourhood. Most likely somebody will be found on the school compound, but it may be more instructive to visit a traditional weaver, i.e. one who speaks only Navajo. After looking at the setup and the activities of the weaver, children will ask questions about the setting up of the loom, the making of a warp and the activities of weaving itself. All instruments and materials used can be extensively looked at.

When back in the classroom, the knowledge gathered this way will be discussed : the words used will be trained to be remembered and the activities will become more prominent in the following sections of this context.

2.*Constructing a loom.*

In the classroom children will construct an elementary loom. The idea is to get a better understanding of the parts and workings of the loom, such that the loom's form and function is reinvented by the children. The teacher should be careful not to 'instruct' the children, but rather have them reinvent the thing. For example, they will find that one way or another a square or rectangular form is to be constructed in order to produce square or rectangular rugs. Also, they should find that weaving can only proceed by intertwining a frame of threads (the warp) with a second, orthogonally positioned thread.

The following aspects and concepts will emerge :

- two horizontal poles at a certain distance from one another, more or less parallel,
- two vertical poles in similar position to each other and being fixed tightly to the horizontal ones to make a square or rectangle. The angles are approximately straight.
- the warp has a set of approximately parallel strings or threads in order to make a tight and neatly woven rug,
- the warp is hung up in front of the (sitting) weaver : it is attached by knots on the upper and lower beam. It is stretched as much as possible in-order to have a smooth (flat) surface of the rug ,
- the batten is used in a double way : it is used to tighten the rug after each new horizontal thread, and it is used to open the threads of the rugs to such a degree that a thread can be introduced in the warp to intertwine with it. Why cannot a stick be used as a batten ? (wrong form, which would not serve the function of opening the warp).

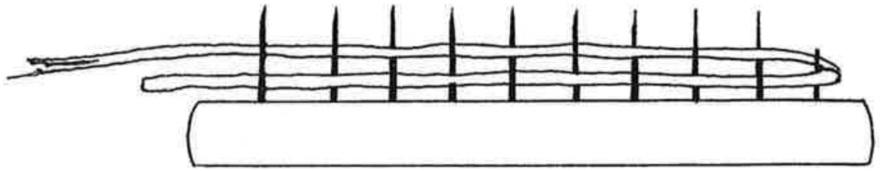
Again, terminology and concepts of function and form should be rehearsed to some extent to get a clear view for their implications.

3. *Use nailboard to understand weaving :*

The children will now search on their own how weaving is functioning : they can use the nailboard in the first place and find out by means of pieces of thread or strips of cotton how the intertwining should be reached.

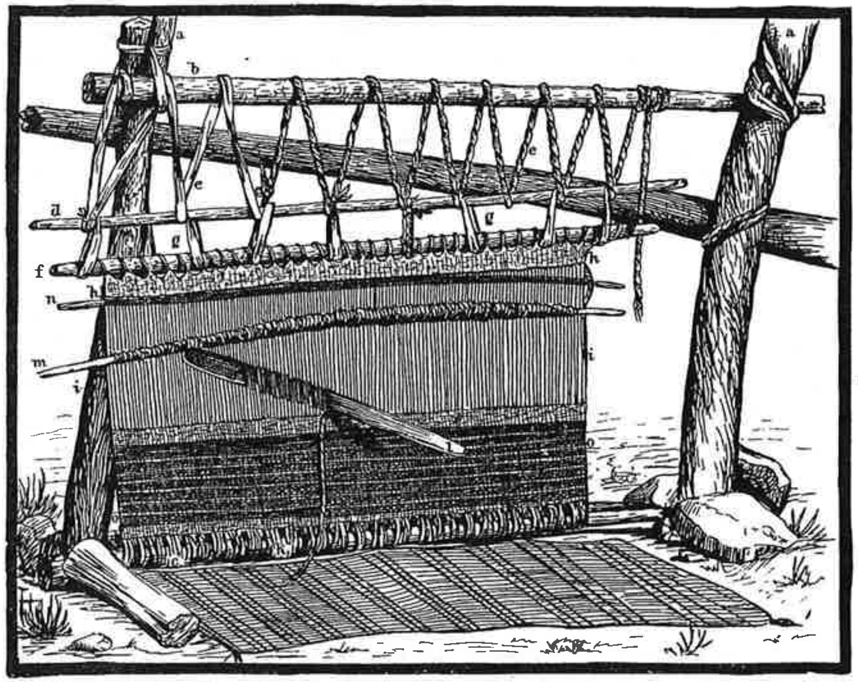
It is useful to advise them to use the bottom row of nails as representation of the warp and to try and find a way of weaving threads on this 'warp'. This will give the following structure : (Fig. 14).
This drawing below (Fig. 15) from W. Matthews (1884) gives an overview of different constituent parts of the loom : horizontal and vertical beams, the rug which is tightened between the upper and the lower beam by means of a frame (the warp) and ropes, the warp hung in front of and facing the weaver, the batten and the thread which is introduced to 'fill in' the

warp.



Figuur 14

In other words, the tread has to pass over the first nail and behind the second one, over the third one and behind the fourth one and so on. This order is essential to have the intertwining effect.



Figuur 15

Children can, if they are able to use the string games which are known to Navajos, to develop other forms and figures with basically the same sort of knowledge (Kluckhohn, et al., 1970).

4. *The weaving wheel :*

Children come to master the idea of weaving, in terms of overlapping and knotting, even further by producing a simple rug themselves. We propose the following device in this instance :

- cut out a circle in carton (used box, etc.); the circle should be big enough to handle easily (e.g. 10 inches in diameter), but should not be precisely circular.
- mark the circumference of the circle an uneven amount of times (e.g. 15) by making small incisions with the scissors at regular intervals (about every three to four centimeters).
- now introduce a long piece of thread in one incision and stretch the thread to the opposite incision, where it is fixed in the incision again. Then chose the 'longest' distance on the circumference (one half of the circle has now six and the other seven incisions left). Go to the next incision on the carton (on the back side) and introduce the thread in this incision, then cross the circle over its center and introduce the thread again in the opposite incision. Proceed this way until all incisions have been come across. We get the following result :(Fig. 16 a and b).
- next the center should be tightened and the 'rays' on the front side should be fastened so as to function like a warp. This can be done by 'weaving' the loose end of the finish through all the threads, starting on the right side : first under the first thread to the right, then over the next one and so on. Once a round is finished the thread is tightened : it forms a last ray to complete the structure. The children should now keep on weaving with the last loose end for several rounds (about ten is a minimum) in order to stabilize the center and to have strong warp to work on. After the last round (e.g. the tenth, or when the thread comes to an end), the last bit is simply hided under the center, which is by now a mini-rug.
- pieces of worn cotton of different colors are cut in long strips (not wider than an inch). The children can now weave their own rug by applying the pieces to the warp like a weaver does. By fastening the different rows towards the center by means of a comb or simply by hand, the structure will get a good shape. When the whole warp is filled with subsequent rows in this way, the piece is finished. It can either be used as a bonnet or as a decoration in the classroom.

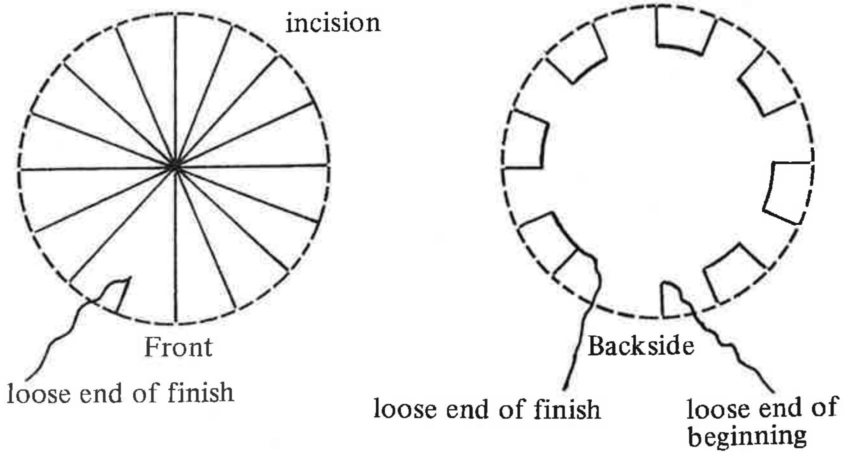


Figure 16 a and Figure 16 b

The teacher should now rehearse the different concepts used in the weaving technique :

- circle (by cutting), center, circumference;
- incision,
- the line constructing the surface : the thread making out the flat figure of the woven circle,
- the spiral realized through the systematic addition of a line to a former one in a circular progression (cf. spiral of the wedding basket),
- overlapping, crossing of the threads,
- succession in two dimensions,
- continuity : the thread realizes a continuous line.

5. *Details :*

When going to the trader to sell a rug, Navajo women will have their work checked and controlled by the trader. What does he look for ? How can he determine whether it is good or bad ? Children may go to the local trader and ask him about it. The following elements will certainly result :

- The trader will see whether the rug is rectangular and regular : he will fold two ends together and check whether the rug can be neatly folded in two to overlap completely. This aspect is important to get to an understanding of the geometric properties of the rectangle : straight angles everywhere, sides of equal length (two by two), parallel sides.
- this finding can now be trained in the classroom : cut out a figure that is square or rectangular without measuring anything : how can you proceed ? By folding paper, cloth, etc..
- the trader will see whether the figures on the rug are regularly distributed all over the surface of the rug : a certain harmony and balance is wanted. How to check this ? Same procedures within the borders of the rug.

Exercise : draw figures on a piece of a paper such that a perfect equilibrium between two sides is realized : Fold the piece in two and determine where the outside-most figures will come in both cases ; Fold in two again and do the same thing for this surface, and so on. The same can be done in all directions when a perfect balance in all directions is needed. Exercise this with squares, rectangles, triangles and circles.

Metric notions : the weaver will make out how large and how long a rug will be before starting the actual weaving. She will measure the size by means of the *woshigishii* (caterpillar) movement : stretch and fold the hand between two fingers (forefinger and thumb) alongside the rug, on all four sides :

For example :

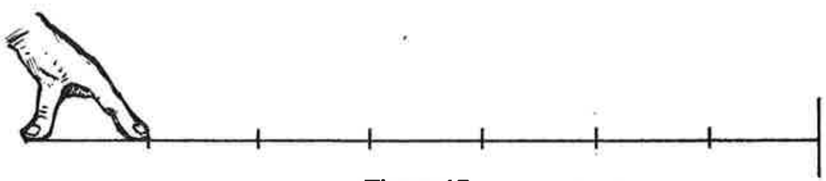


Figure 17

When analysing this practice it will become clear that measurement is involved : equal distance (between forefinger and thumb), as many times as needed to cover the whole distance.

Where did we find something similar ? In making the plan of the rodeo ground and the school compound, we used the width of a step to measure distance. What is similar and what is different here ? By this search children will come to the understanding of the measurement mechanism : a unit is chosen (e.g. width of step or width between thumb and forefinger) and applied along a line as much as needed, that is to say, such that the whole distance or length of the line is covered once.

How can we agree on the unit to be used ? Convention : e.g. *tsin sita* (a mile, i.e. the distance between two mile posts), a meter, etc.

6. *Concluding remarks :*

Again, just like in all previous contexts, the different and consecutive steps mentioned by Bishop should be followed here : children should first learn to manipulate things in the cultural context they know (build, imitate, etc.), then name the structures they detect themselves, then visualize and represent graphically, then visualize situations and finally draw situations and changes in situations on paper from mere instructions.

XII. Going into abstractions by means of detection.

The final step of the whole geometry curriculum for this level of schooling consists in the detection or building of genuine abstractions by the children. The general principle here is : *decontextualize by cross-contextualizing.* From comparison between concepts used in different contexts, we will finally get to the construction of theoretical or abstract geometric concepts. This construction again has to be founded in action.

1. *Decontextualize by cross-contextualizing :*

By briefly taking up all conventional words and concepts which were detected and learned throughout the total process of five contexts, the following lists will be found :[1]

Rodeo :
- map, proportion
- (flat) surface
- polygon and circle
- front/back, sideways
- border (fence)
- open/closed
- up/down (top)
- knot (line, circle, overlapping, lasso)
- proximity, distance
- folding
- stair

Hooghan :
- circle, center
- opening

(1) Children can make these lists themselves when running through the written notes they have of each context. It may be advisable to work in groups here.

- border (wall)
- distance
- cardinal points
- horizontal, vertical
- flat, straight
- cilinder form
- hexagon
- folding
- proportion
- perspective
- angle
- ball, sphere

Herding sheep : - big/small
- orientation
- distance
- border
- path, road
- separation, neighborhood
- mesas, mountain, valley
- forward/backward
- turning
- open/closed
- surface
- map, scale model
- loop
- length
- measuring

School compound : - map
- surface
- orientation
- path
- distance (reachability)
- cube, square
- back/front
- scale; proportion

- size
- unit

Rug weaving : - distance
- square, rectangle, circle, triangle
- horizontal, vertical
- parallel
- line, surface
- straight line
- straight angle
- front/back
- knots
- width, length
- folding, overlapping
- succession
- continuous
- measuring.

By going back and forth between the concepts which are identical or very similar in different contexts (e.g. length, horizontal, etc.) children will detect the similarities and distinctions at each instance. From there on, they will gradually come to see the invariants, the geometric abstractions in the figures and concepts used. At this point, the definition of each of the relevant geometric concepts can take place (cf. sub 2 below). It is important that the similarities and differences detected between concepts by children should be thouroughly trained in any possible way. Again, stress should again be on visualization and graphic representation.

One can easily devise educational means to provide for this training: e.g. distance in the case of rodeo is different from, but similar to distance in the sheep herding context, and again to distance in the weaving context. What is the difference ? (size, measure, units, tools to measure). What is similar ? (the measuring procedure). What other uses or instances can you imagine ? (e.g. flying by plane, me asuring the Navajo country, etc.). How would we have to proceed there to determine the distance ?

For each and every concept which was detected by the children in the course of the previous months, a similar search for similarities and differences can be investigated. Similarities between the concepts in the different contexts will be found, and these similarities will hold good in many more different contexts as well. This systematic exploration by imagination can take up considerable time, that is until everybody is convinced by his own search of the regularity and generality of the notions found and of the need and usefulness of these general concepts. In order to get clarity and better mutual understanding one can then go in search of the ways to define the concepts found.

2. *The definition of Navajo geometric concepts and the building of a Navajo geometry :*

It is clear that the definitions of the concepts and the construction of a genuine geometry can only begin to be undertaken at this point. That is, instead of installing preconceived definitions and relationships (like one does in the usual Euclidean geometry course), one should first 'detect' the differentiations and become clear on the demarcations one needs within one's mental representation of space. This is what has been done by the children so far. Since Navajo language and the Navajo contexts were never abandoned throughout this exploration period, the teachers, the children and the researchers will now have a clear overview of the 'materials' out of which a geometry can be built. Let us conclude with some speculative remarks.

H. Dingler (1933) already made a start with a very different and experience-based reinterpretation of geometry in the European context. It has been developed a little bit since, but it has been largely unknown to the Anglo-Saxon world because most of it has been written in German (Dingler, Lorenzen, Janich). Freudenthal (1973 especially) refers to this work in intuitive geometry, but a systematic exposition in English is not available. We will not bring a systematic report on it here, since the details of the tradition may not be relevant for Navajo geometry at all. However, the principles laid down by representatives of this tradition may be of importance. Dingler states that the basics of geometry should be understood and recognized in as far as they mean something in action, and, furthermore, their meaning should be expressed and defined in terms of action for the particular member of a culture. For example : a line is not in the first place a connection between two points, since neither points nor line have any meaning in the experience of the subject. Rather line can be first and foremost seen as the result of a movement of gliding something on a surface

(e.g. when it is possible to glide a stone without interruption on a piece of wood or ground, in any direction whatever, you have the notion of line). When two such results of action meet or overlap at some point, you have the line intersection in a plane. And so on. From these elementary prerequisites for the meaning of geometric lines, one can go on and refine or sophisticate the demands yielding other and connected or more detailed notions (straight line, etc.). Again, introducing direction in action, one can bring in commands to alter the directions of the movement and thus come to grasp the notions of angle and geometric figure (rectangle, square, etc.): follow the surface so many steps, then turn right and do the same amount of steps, turn right again, etc.. producing a square or another regular geometric polygon.

K. Lewin stresses the differences between physical and psychological geometries. He follows a similar reasoning as Dingler and tries to identify different geometric characteristics of psychological experience. On these genuine abstract and even formalized systems of representation, i.e. a genuine geometry of the world of experience can be based, as was proved by him in his Topological Psychology (1936). Without into a detailed presentation of any of these works, they have been mentioned to show that such attempts and even elaborated theories exist. Moreover, in the present crisis in geometry education more and more prominent thinkers in this discipline turn towards such works to get clear on the possible solutions for the crisis. (the most prominent thinker in this respect is certainly Freudenthal, with Bishop and Lancy following at a small distance).

What has to be done now is to implement the educational layout presented in the previous chapters for a year or more in one or several of the bicultural schools (e.g. on the reservation). The results of the teaching along these lines of geometry detection and exploration will then yield the 'materials' we need in order to build a more formal and theoretically consistent Navajo geometry. At that point,we will be able again to continue the present research, keeping in tune with the representations and the emphases which are Navajo (and not aprioristic and maybe partly European, as is the case with Euclidean geometry). However, once a proposal will be made to formalize the Navajo geometric material, each and every generation of children will have to begin the exploration of geometric features again in the way outlined above, since the abstractions found at the end of this exploration period will only be grasped and mastered, provided the children will have 'detected' them themselves. In other words children will always have to reinvent geometry in order to learn it. It remains to be seen now in what sense Navajo children will produce a different sort of geometry than the western one, and where Euclidean geometry will hold a place as a particular and rather idiosyncratic example of this fascinating area. For partial results of empirical research along these lines see Pinxten & Vandenbogaert, 1987; Pinxten/ in press).

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