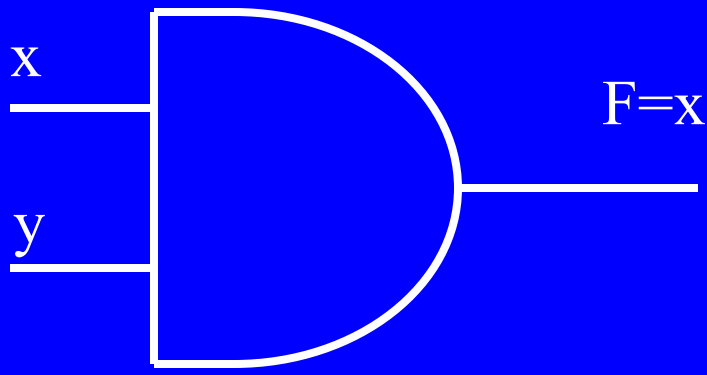
Binary logic

- AND
- OR
- NOT

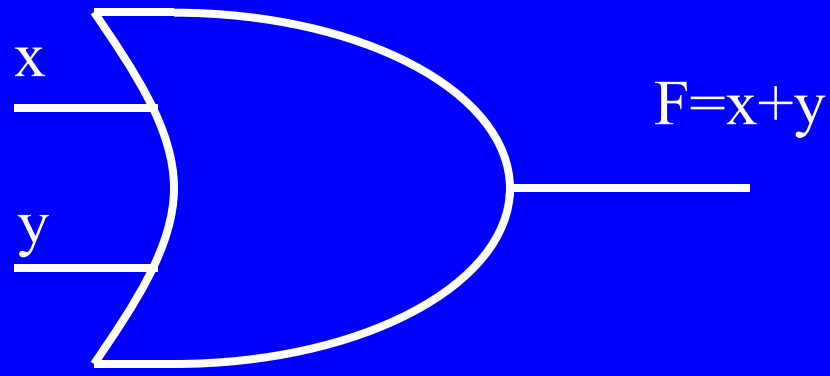
x	y	AND (x·y)	OR (x+y)	NOT (x')
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

AND gate



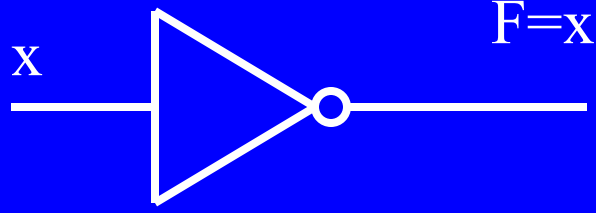
x	y	F
0	0	0
0	1	0
1	0	0
1	1	1

OR gate



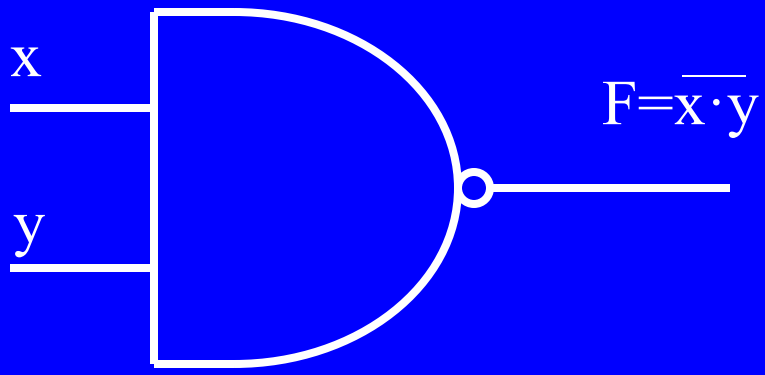
x	y	F
0	0	0
0	1	1
1	0	1
1	1	1

NOT gate



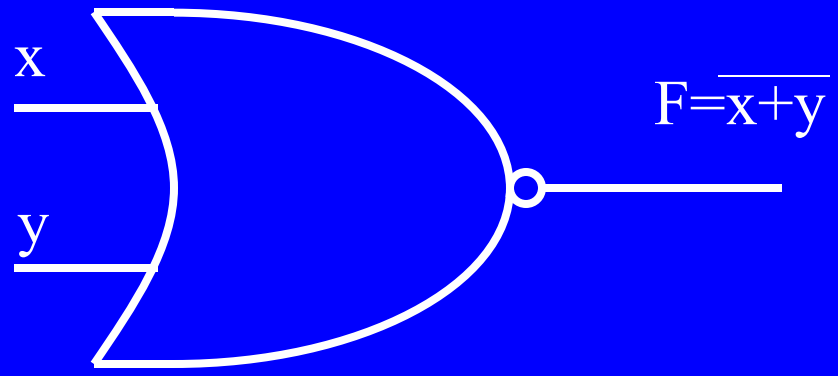
x	F
0	1
1	0

NAND gate



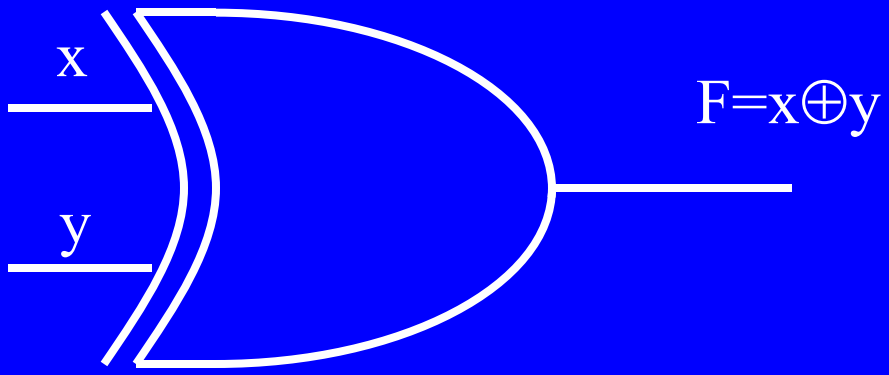
x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

NOR gate



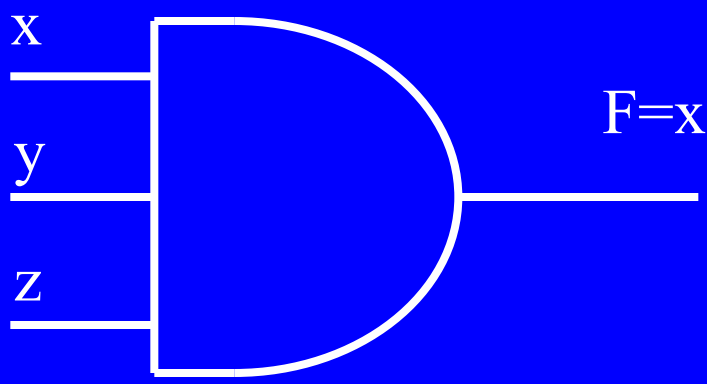
x	y	F
0	0	1
0	1	0
1	0	0
1	1	0

EXCLUSIVE-OR (XOR) gate

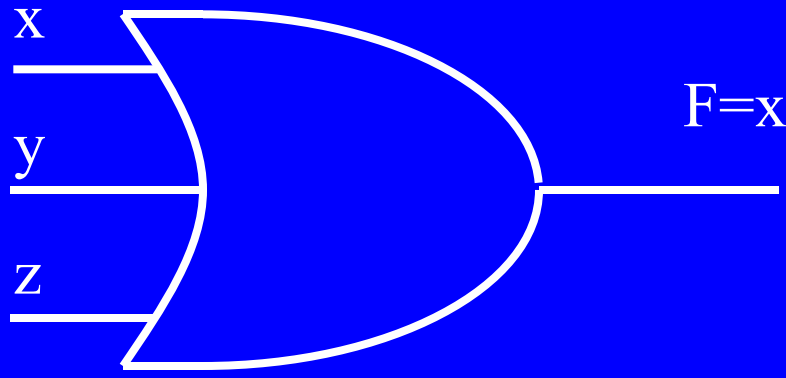


x	y	F
0	0	0
0	1	1
1	0	1
1	1	0

Other gates



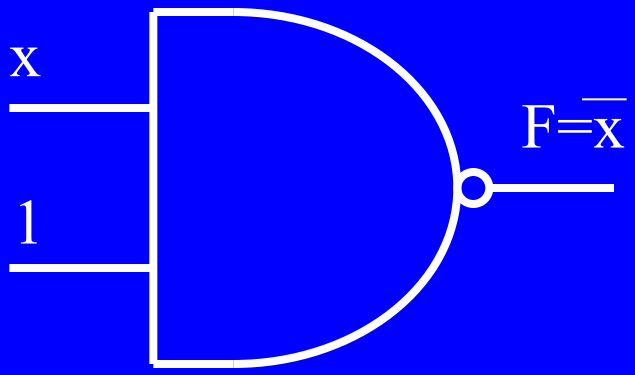
$F = x \cdot y \cdot z$

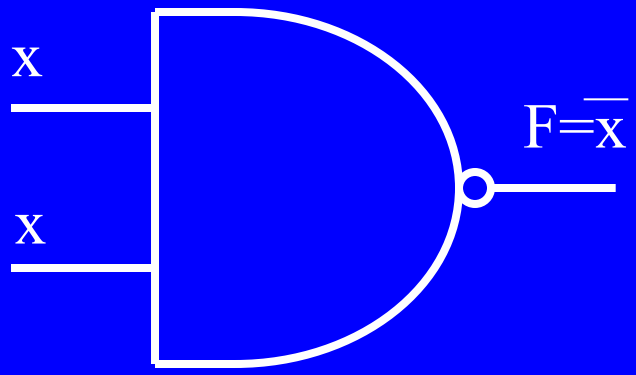


$F = x + y + z$

Universal gate

With two-input NAND or NOR gates we can implement any other gate





Boolean Algebra

Key definitions

- Closure x*y in S
- Associative law (x*y)*z=x*(y*z)
- Commutative law x*y=y*x
- Identity element e*x=x*e=x
- Inverse x*y=e
- Distributive law x*(y&z)=(x*y)&(x*z)

Axiomatic definition (Huntington)

- Closure with respect to +, •
- Identity element
  - with respect to +: x+0=x
  - with respect to •: x•1=x
- Commutative
  - with respect to +: x+y=y+x
  - with respect to •: x•y=y•x

Axiomatic definition(Huntington)

- Distributive
  - with respect to +: $x+(y \cdot z)=(x+y) \cdot (x+z)$
  - with respect to •: $x \cdot (y+z)=(x \cdot y)+(x \cdot z)$
- Complement
  - with respect to +: $x+x'=1$
  - with respect to •: $x \cdot x'=0$
- There exist at least two elements x,y in the set S.

Axiom 1: Commutative Laws

a. Commutativity with respect to +

$$A + B = B + A$$

b. Commutativity with respect to •

$$A \bullet B = B \bullet A$$

Axiom 2: Distributive Laws

a. Operator + is distributive with respect to •

$$A + (B \cdot C) = (A + B) \cdot (A + C)$$

b. Operator • is distributive with respect to +

$$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$$

Axiom 3: Identity element

a. The identity element of operator + is 0

$$A + 0 = 0 + A = A$$

b. The identity element of operator • is 1

$$A \cdot 1 = 1 \cdot A = A$$

Axiom 4: Complements

a. With respect to the operator +

$$A + A' = A' + A = 1$$

b. With respect to the operator •

$$A \bullet A' = A' \bullet A = 0$$

Theorem 1

A + 1 = 1

A + 1 = 1 • (A + 1)	Axiom 3
= (A + A') • (A + 1)	Axiom 4
= A + (A' • 1)	Axiom 2
= A + A'	Axiom 3
= 1	Axiom 4

Theorem 2

The complement of 0 is 1

The complement of 1 is 0

Theorem 3

$A + A = A$

$A + A = (A + A) \cdot 1$		Axiom 3
$= (A + A) \cdot (A + A')$		Axiom 4
$= A + (A \cdot A')$		Axiom 2
$= A + 0$		Axiom 4
$= A$		Axiom 3

Theorem 4

$A'' = A$	
$A'' = A'' + 0$	Axiom 3
$= A'' + (A \bullet A')$	Axiom 4
$= (A'' + A) \bullet (A'' + A')$	Axiom 2
$= (A'' + A) \bullet 1$	Axiom 4
$= (A'' + A) \bullet (A + A')$	Axiom 4
$= (A + A'') \bullet (A + A')$	Axiom 1
$= A \bullet (A'' + A')$	Axiom 2
$= A \bullet 1$	Axiom 4
$= A$	Axiom 3

Theorem 5

$A + (A \cdot B) = A \quad (A \cdot B \neq 0)$

$A + (A \cdot B)$	$= (A \cdot 1) + (A \cdot B)$	Axiom 3
	$= A \cdot (1 + B)$	Axiom 2
	$= A \cdot (B + 1)$	Axiom 1
	$= A \cdot 1$	Theorem 1
	$= A$	Axiom 3

Theorem 6

$A + (A' \cdot B) = A + B$

$A + (A' \cdot B) = (A + A') \cdot (A + B)$ Axiom 2

$= 1 \cdot (A + B)$ Axiom 4

$= A + B$ Axiom 3

Theorem 7

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

We set $X = (A \cdot B) \cdot C$ and $Y = A \cdot (B \cdot C)$

$A + X = A + [(A \cdot B) \cdot C]$	
$= [A + (A \cdot B)] \cdot (A + C)$	Axiom 2
$= A \cdot (A + C)$	Theorem 5
$= A$	Theorem 5
$A + Y = A + [A \cdot (B \cdot C)]$	
$= (A + A) \cdot [A + (B \cdot C)]$	Axiom 2
$= A \cdot [A + (B \cdot C)]$	Theorem 3
$= A$	Theorem 5

Άρα $A + X = A + Y$

Theorem 7 (continued)

$A' + X = A' + [(A \cdot B) \cdot C]$

$= [A' + (A \cdot B)] \cdot (A' + C)$ Axiom 2

$= [(A' + A) \cdot (A' + B)] \cdot (A' + C)$ Axiom 2

$= [1 \cdot (A' + B)] \cdot (A' + C)$ Axiom 4

$= (A' + B) \cdot (A' + C)$ Axiom 3

$= A' + (B \cdot C)$ Axiom 2

$A' + Y = A' + [A \cdot (B \cdot C)]$

$= (A' + A) \cdot [A' + (B \cdot C)]$ Axiom 2

$= 1 \cdot [A' + (B \cdot C)]$ Axiom 4

$= A' + (B \cdot C)$ Axiom 3

Therefore $A' + X = A' + Y$

Theorem 7 (continued)

$(A + X) \cdot (A' + X) = (A + Y) \cdot (A' + Y)$

$[(A + X) \cdot A'] + [(A + X) \cdot X]$

$= [(A + Y) \cdot A'] + [(A + Y) \cdot Y]$ Axiom 2

$(A \cdot A') + (X \cdot A') + [(A + X) \cdot X]$

$= (A \cdot A') + (Y \cdot A') + [(A + Y) \cdot Y]$ Axiom 2

$(A \cdot A') + (X \cdot A') + X$

$= (A \cdot A') + (Y \cdot A') + Y$ Theorem 5

$(X \cdot A') + X = (Y \cdot A') + Y$ Axioms 4+3

$X = Y$ Theorem 5

Theorem 8 (De Morgan)

$(A + B)' = A' \cdot B'$

$(A + B) + A' \cdot B' = [(A + B) + A'] \cdot [(A + B) + B']$	Axiom 2
$= [(B + A) + A'] \cdot [(A + B) + B']$	Axiom 1
$= [B + (A + A')] \cdot [A + (B + B')]$	Theorem 7
$= (B + 1) \cdot (A + 1)$	Axiom 4
$= 1 \cdot 1$	Theorem 1
$= 1$	Axiom 3
$(A + B) \cdot (A' \cdot B') = (A' \cdot B') \cdot (A + B)$	Axiom 1
$= [(A' \cdot B') \cdot A] + [(A' \cdot B') \cdot B]$	Axiom 2
$= [(B' \cdot A') \cdot A] + [(A' \cdot B') \cdot B]$	Axiom 1
$= [B' \cdot (A' \cdot A)] + [A' \cdot (B' \cdot B)]$	Theorem 7
$= (B' \cdot 0) + (A' \cdot 0)$	Axiom 4
$= 0 + 0$	Theorem 1
$= 0$	Theorem 3

What we keep

Theorem 1: A+1=1	A•0=0
Theorem 2: The complement of 0 is 1 and vice versa	
Theorem 3: A+A=A	A•A=A
Theorem 4: A''=A	
Theorem 5: A+(A•B)=A, (A•B)≠0	A•(A+B)=A, (A+B)≠1
Theorem 6: A+(A'•B)=A+B	A•(A'+B) = A•B
Theorem 7: (A•B)•C=A•(B•C)	(A+B)+C=A+(B+C)
Theorem 8: (A+B)'=A'•B'	(A•B)'=A'+B'

XOR operator

Commutativity

$$A \oplus B = B \oplus A$$

Associativity

$$(A \oplus B) \oplus C = A \oplus (B \oplus C) = A \oplus B \oplus C$$

Distributivity

$$(AB) \oplus (AC) = A(B \oplus C)$$

Boolean expressions

- Venn diagrams
- Boolean funstions
- Truth tables
- Canonical and standard forms
  - Minterms
  - Maxterms
  - Conjunctive Normal Form (CNF)

Boolean functions

Combinations of boolean variables produce logical functions. These functions take the form of:

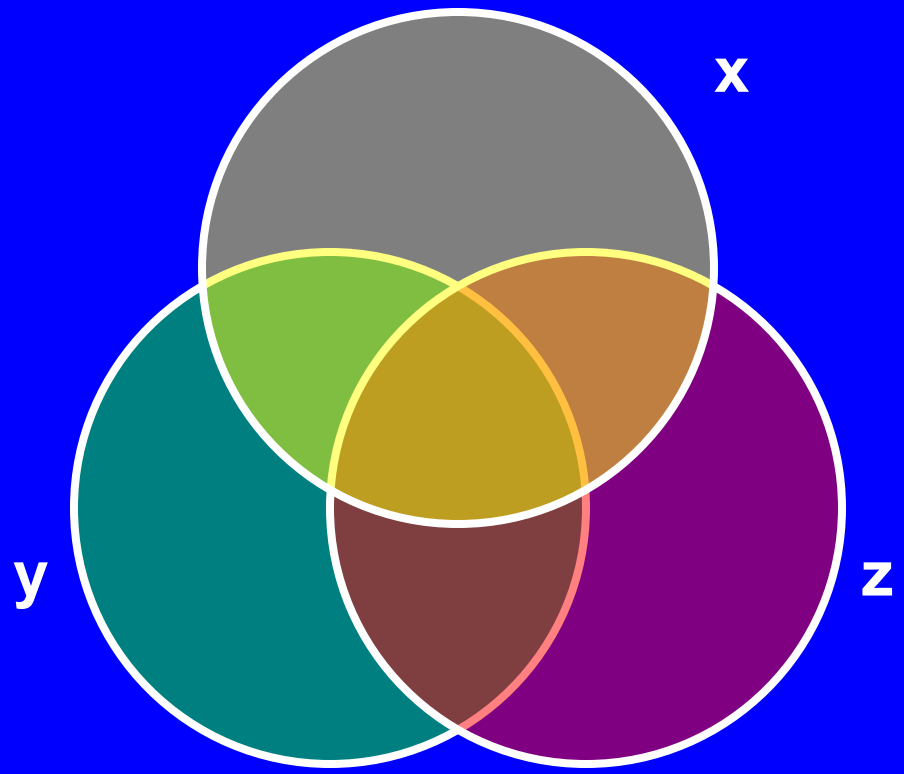
$$F_1 = x + yz$$

$$F_2 = xy + yz + zx$$

$$F_3 = xyz + x'y'z + x'yz' + xy'z'$$

$$F_4 = xy + z'$$

Venn Diagrams



Truth Tables

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

Minterms or standard products

As minterm or standard product of a boolean function $F = F(x_1, x_2, ..., x_n)$ we define an expression (AND) of all variables x_i either in their regular or complementary form.

Maxterms or standard sums

As maxterm or standard sum of a logical function $F = F(x_1, x_2, ..., x_n)$ we define an expression (OR) of all variables x_i either in their regular or complementary form.

Minterms and Maxtems

x	y	z	term	desc	term	desc
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$$

$$F_1 = x'yz + xy'z' + xy'z + xyz' + xyz$$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$$

$$F_1 = x'yz + xy'z' + xy'z + xyz' + xyz$$

$$F_2 = m_3 + m_5 + m_6 + m_7$$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$$

$$F_1 = x'yz + xy'z' + xy'z + xyz' + xyz$$

$$F_2 = m_3 + m_5 + m_6 + m_7$$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$$

$$F_1 = x'yz + xy'z' + xy'z + xyz' + xyz$$

$$F_2 = m_3 + m_5 + m_6 + m_7$$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$$

$$F_1 = x'yz + xy'z' + xy'z + xyz' + xyz$$

$$F_2 = m_3 + m_5 + m_6 + m_7$$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$

$F_1 = x'yz + xy'z' + xy'z + xyz' + xyz$

$F_2 = m_3 + m_5 + m_6 + m_7$

$F_2 = x'yz + xy'z + xyz' + xyz$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$F_1=m_3+m_4+m_5+m_6+m_7$

$F_1=x'yz+xy'z'+xy'z+xyz'+xyz$

$F_2=m_3+m_5+m_6+m_7$

$F_2=x'yz+xy'z+xyz'+xyz$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

$F_3=m_1+m_2+m_4+m_7$

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$

$F_1 = x'yz + xy'z' + xy'z + xyz' + xyz$

$F_2 = m_3 + m_5 + m_6 + m_7$

$F_2 = x'yz + xy'z + xyz' + xyz$

$F_3 = m_1 + m_2 + m_4 + m_7$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$

$F_1 = x'yz + xy'z' + xy'z + xyz' + xyz$

$F_2 = m_3 + m_5 + m_6 + m_7$

$F_2 = x'yz + xy'z + xyz' + xyz$

$F_3 = m_1 + m_2 + m_4 + m_7$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$

$F_1 = x'yz + xy'z' + xy'z + xyz' + xyz$

$F_2 = m_3 + m_5 + m_6 + m_7$

$F_2 = x'yz + xy'z + xyz' + xyz$

$F_3 = m_1 + m_2 + m_4 + m_7$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$F_1 = m_3 + m_4 + m_5 + m_6 + m_7$

$F_1 = x'yz + xy'z' + xy'z + xyz' + xyz$

$F_2 = m_3 + m_5 + m_6 + m_7$

$F_2 = x'yz + xy'z + xyz' + xyz$

$F_3 = m_1 + m_2 + m_4 + m_7$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

$F_1=m_3+m_4+m_5+m_6+m_7$

$F_1=x'yz+xy'z'+xy'z+xyz'+xyz$

$F_2=m_3+m_5+m_6+m_7$

$F_2=x'yz+xy'z+xyz'+xyz$

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

$F_3=m_1+m_2+m_4+m_7$

$F_3=x'y'z+x'yz'+xy'z'+xyz$

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

$$F_1=m_3+m_4+m_5+m_6+m_7$$

$$F_1=x'yz+xy'z'+xy'z+xyz'+xyz$$

$$F_2=m_3+m_5+m_6+m_7$$

$$F_2=x'yz+xy'z+xyz'+xyz$$

$$F_3=m_1+m_2+m_4+m_7$$

$$F_3=x'y'z+x'yz'+xy'z'+xyz$$

$$F_4=m_0+m_2+m_4+m_6+m_7$$

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

$$F_1=m_3+m_4+m_5+m_6+m_7$$

$$F_1=x'yz+xy'z'+xy'z+xyz'+xyz$$

$$F_2=m_3+m_5+m_6+m_7$$

$$F_2=x'yz+xy'z+xyz'+xyz$$

$$F_3=m_1+m_2+m_4+m_7$$

$$F_3=x'y'z+x'yz'+xy'z'+xyz$$

$$F_4=m_0+m_2+m_4+m_6+m_7$$

$$F_4=x'y'z'+x'yz'+xy'z'+xyz'+xyz$$

Truth table

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	1
0	0	1	0	0	1	0
0	1	0	0	0	1	1
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	0	1	1	1	0	0
1	1	0	1	1	0	1
1	1	1	1	1	1	1

Minterms and Maxterms

x	y	z				
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

2-36

$$F_1=M_0M_1M_2$$

$$F_1=(x+y+z)(x+y+z')(x+y'+z)$$

$$F_2= M_0M_1M_2M_4$$

$$F_2=(x+y+z)(x+y+z')(x+y'+z)(x'+y+z)$$

$$F_3= M_0M_3M_5M_6$$

$$F_3=(x+y+z)(x+y'+z')(x'+y+z')(x'+y'+z)$$

$$F_4= M_1M_3M_5$$

$$F_4=(x+y+z')(x+y'+z')(x'+y+z')$$

Shannon's Theorem

$$F(x_1, x_2, \ldots, x_n) = x_1 \bullet F(1, x_2, \ldots, x_n) + x_1' \bullet F(0, x_2, \ldots, x_n)$$

$$F(x_1, x_2, \ldots, x_n) = [x_1 + F(0, x_2, \ldots, x_n)] \bullet [x_1' + F(1, x_2, \ldots, x_n)]$$

Convert a function to a sum of minterms

- We apply Shannon's theorem until we have a sum of minterms
- We bring the function in the form of a sum of products
  - If a product is a minterm, we keep it
  - For each variable x_i that does not exist in the product we multiply it by $(x_i + x_i')$
  - We perform the operations and delete the terms that appear more than once

Convert a function to a product of maxterms

- We apply Shannon's theorem until we have a product of maxterms
- We bring the function to a product of sums
  - If a sum is a maxterm, we keep it
  - For each variable x_i which does not exist in the sum we add $x_i x_i'$
  - We perform the operations and delete the terms that appear more than once

Example

$F_1 = x+yz$

$F(x_1, x_2, \ldots, x_n) = x_1 \bullet F(1, x_2, \ldots, x_n) + x_1' \bullet F(0, x_2, \ldots, x_n)$

Example

$F_1 = x+yz$

$$F(x_1, x_2, \ldots, x_n) = x_1 \bullet F(1, x_2, \ldots, x_n) + x_1' \bullet F(0, x_2, \ldots, x_n)$$

Example

$F_1 = x+yz$

$= x(1+yz)+x'(0+yz)$

$F(x_1, x_2, \ldots, x_n) = x_1 \bullet F(1, x_2, \ldots, x_n) + x_1' \bullet F(0, x_2, \ldots, x_n)$

Example

$F_1 = x+yz$

$= x(1+yz)+x'(0+yz)$

$= x+x'yz$

$F(x_1, x_2, \ldots, x_n) = x_1 \bullet F(1, x_2, \ldots, x_n) + x_1' \bullet F(0, x_2, \ldots, x_n)$

Example

$F_1 = x+yz$

$= x(1+yz)+x'(0+yz)$

$= x+x'yz$

$= y(x+x'1z)+y'(x+x'0z)$

$F(x_1, x_2, \ldots, x_n) = x_1 \bullet F(1, x_2, \ldots, x_n) + x_1' \bullet F(0, x_2, \ldots, x_n)$

Example

$F_1 = x+yz$

$= x(1+yz)+x'(0+yz)$

$= x+x'yz$

$= y(x+x'1z)+y'(x+x'0z)$

$= xy + x'yz + xy'$

$F(x_1, x_2, \ldots, x_n) = x_1 \bullet F(1, x_2, \ldots, x_n) + x_1' \bullet F(0, x_2, \ldots, x_n)$

Example

$F_1 = x+yz$

$= x(1+yz)+x'(0+yz)$

$= x+x'yz$

$= y(x+x'1z)+y'(x+x'0z)$

$= xy + x'yz + xy'$

$= z(xy+x'y1+xy') + z'(xy+x'y0+xy')$

$F(x_1, x_2, \ldots, x_n) = x_1 \bullet F(1, x_2, \ldots, x_n) + x_1' \bullet F(0, x_2, \ldots, x_n)$

Example

$F_1 = x+yz$

$= x(1+yz)+x'(0+yz)$

$= x+x'yz$

$= y(x+x'1z)+y'(x+x'0z)$

$= xy + x'yz + xy'$

$= z(xy+x'y1+xy') + z'(xy+x'y0+xy')$

$= xyz+x'yz+xy'z+xyz'+xy'z'$

$F(x_1, x_2, \ldots, x_n) = x_1 \bullet F(1, x_2, \ldots, x_n) + x_1' \bullet F(0, x_2, \ldots, x_n)$

Example

$F_1 = x+yz$

$= x(1+yz)+x'(0+yz)$

$= x+x'yz$

$= y(x+x'1z)+y'(x+x'0z)$

$= xy + x'yz + xy'$

$= z(xy+x'y1+xy') + z'(xy+x'y0+xy')$

$= xyz+x'yz+xy'z+xyz'+xy'z'$

$= x'yz+xy'z'+xy'z+xyz'+xyz$

$F(x_1, x_2, \ldots, x_n) = x_1 \bullet F(1, x_2, \ldots, x_n) + x_1' \bullet F(0, x_2, \ldots, x_n)$

Example

$F_1 = x+yz$

$= x(1+yz)+x'(0+yz)$

$= x+x'yz$

$= y(x+x'1z)+y'(x+x'0z)$

$= xy + x'yz + xy'$

$= z(xy+x'y1+xy') + z'(xy+x'y0+xy')$

$= xyz+x'yz+xy'z+xyz'+xy'z'$

$= x'yz+xy'z'+xy'z+xyz'+xyz$

$= m3+m4+m5+m6+m7$

Example

$F_1 = x+yz$

$= x(y+y')(z+z') + (x+x')yz$

$= xyz+xyz'+xy'z+xy'z'+xyz+x'yz$

$= x'yz+xy'z'+xy'z+xyz'+xyz$

$= m3+m4+m5+m6+m7$

Example

$F_1 = x+yz$

$= x(y+y')(z+z') + (x+x')yz$

Παράδειγμα

$F_1 = x+yz$

$= x(y+y')(z+z') + (x+x')yz$

$= xyz+xyz'+xy'z+xy'z'+xyz+x'yz$

Example

$F_1 = x+yz$

$= x(y+y')(z+z') + (x+x')yz$

$= xyz+xyz'+xy'z+xy'z'+xyz+x'yz$

Example

$F_1 = x+yz$

$= x(y+y')(z+z') + (x+x')yz$

$= xyz+xyz'+xy'z+xy'z'+xyz+x'yz$

$= x'yz+xy'z'+xy'z+xyz'+xyz$

Example

$F_1 = x+yz$

$= x(y+y')(z+z') + (x+x')yz$

$= xyz+xyz'+xy'z+xy'z'+xyz+x'yz$

$= x'yz+xy'z'+xy'z+xyz'+xyz$

$= m3+m4+m5+m6+m7$

Conversions between standard forms

Relationship Between maxterms and minterms for a boolean function

$$F(x,y,z) = \Sigma(1,3,5,7) = \Pi(0,2,4,6)$$

For the complement of the function

$$F'(x,y,z) = \Pi(1,3,5,7) = \Sigma(0,2,4,6)$$

Complement a function

F = A + BC

F’ = (A + BC)’ = A’ (BC)’ = A’ (B’ + C’)

F = A(B + CD)

F’ = A’ + (B’(C’+D’))

Complement a function

F = A + BC

F' = (A + BC)' = A' (BC)' = A' (B' + C')

F = A(B + CD)

F' = A' + (B'(C'+D'))

Theorem 8 (A+B)'=A'•B' (A•B)'=A'+B'

Conversion to product of sums

1. We bring the function to a sum of products
2. We calculate the complement
3. We bring the complement to a sum of products
4. We find the complement of the complement

Example

$F_1 = x+yz$

Example

$F_1 = x+yz$

$F_1' = x'(y'+z')$

Example

$F_1 = x+yz$

$F_1' = x'(y'+z')$

$= x'y'+x'z'$

Example

$F_1 = x+yz$

$F_1' = x'(y'+z')$

$= x'y'+x'z'$

$F_1'' = (x+y)(x+z)$

Example

$F_1 = x+yz = (x+y)(x+z)$

Example

$F_1 = x+yz = (x+y)(x+z)$

$= (x+y+zz')(x+yy'+z)$

Example

$F_1 = x+yz = (x+y)(x+z)$

$= (x+y+zz')(x+yy'+z)$

$= (x+y+z)(x+y+z')(x+y+z)(x+y'+z)$

Example

$F_1 = x+yz = (x+y)(x+z)$

$= (x+y+zz')(x+yy'+z)$

$= (x+y+z)(x+y+z')(x+y+z)(x+y'+z)$

$= (x+y+z)(x+y+z')(x+y'+z)$

Example

$F_1 = x+yz = (x+y)(x+z)$

$= (x+y+zz')(x+yy'+z)$

$= (x+y+z)(x+y+z')(x+y+z)(x+y'+z)$

$= (x+y+z)(x+y+z')(x+y'+z)$

$= M_0M_1M_2$

$F(x_1, x_2, \ldots, x_n) = [x_1 + F(0, x_2, \ldots, x_n)] \cdot [x_1' + F(1, x_2, \ldots, x_n)]$

Example

$F_1 = x+yz$

$F(x_1, x_2, \ldots, x_n) = [x_1 + F(0, x_2, \ldots, x_n)] \cdot [x_1' + F(1, x_2, \ldots, x_n)]$

Example

$F_1 = x+yz$

$= [x+(0+yz)][x'+(1+yz)]$

$F(x_1, x_2, \ldots, x_n) = [x_1 + F(0, x_2, \ldots, x_n)] \cdot [x_1' + F(1, x_2, \ldots, x_n)]$

Example

$F_1 = x+yz$

$= [x+(0+yz)][x'+(1+yz)]$

$= x+yz$

$F(x_1, x_2, \ldots, x_n) = [x_1 + F(0, x_2, \ldots, x_n)] \cdot [x_1' + F(1, x_2, \ldots, x_n)]$

Example

$F_1 = x+yz$

$= [x+(0+yz)][x'+(1+yz)]$

$= x+yz$

$= [y+(x+0z)][y'+(x+1z)]$

$F(x_1, x_2, \ldots, x_n) = [x_1 + F(0, x_2, \ldots, x_n)] \cdot [x_1' + F(1, x_2, \ldots, x_n)]$

Example

$F_1 = x+yz$

$= [x+(0+yz)][x'+(1+yz)]$

$= x+yz$

$= [y+(x+0z)][y'+(x+1z)]$

$= (x+y)(x+y'+z)$

$F(x_1, x_2, \ldots, x_n) = [x_1 + F(0, x_2, \ldots, x_n)] \cdot [x_1' + F(1, x_2, \ldots, x_n)]$

Example

$F_1 = x+yz$

$= [x+(0+yz)][x'+(1+yz)]$

$= x+yz$

$= [y+(x+0z)][y'+(x+1z)]$

$= (x+y)(x+y'+z)$

$= [z+(x+y)(x+y'+0)][z'+(x+y)(x+y'+1)]$

$F(x_1, x_2, \ldots, x_n) = [x_1 + F(0, x_2, \ldots, x_n)] \cdot [x_1' + F(1, x_2, \ldots, x_n)]$

Example

$F_1 = x+yz$

$= [x+(0+yz)][x'+(1+yz)]$

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$= (x+y)(x+y'+z)$

$= [z+(x+y)(x+y'+0)][z'+(x+y)(x+y'+1)]$

$= (x+y+z)(x+y+z')(x+y'+z)$

$F(x_1, x_2, \ldots, x_n) = [x_1 + F(0, x_2, \ldots, x_n)] \cdot [x_1' + F(1, x_2, \ldots, x_n)]$

Example

$F_1 = x+yz$

$= [x+(0+yz)][x'+(1+yz)]$

$= x+yz$

$= [y+(x+0z)][y'+(x+1z)]$

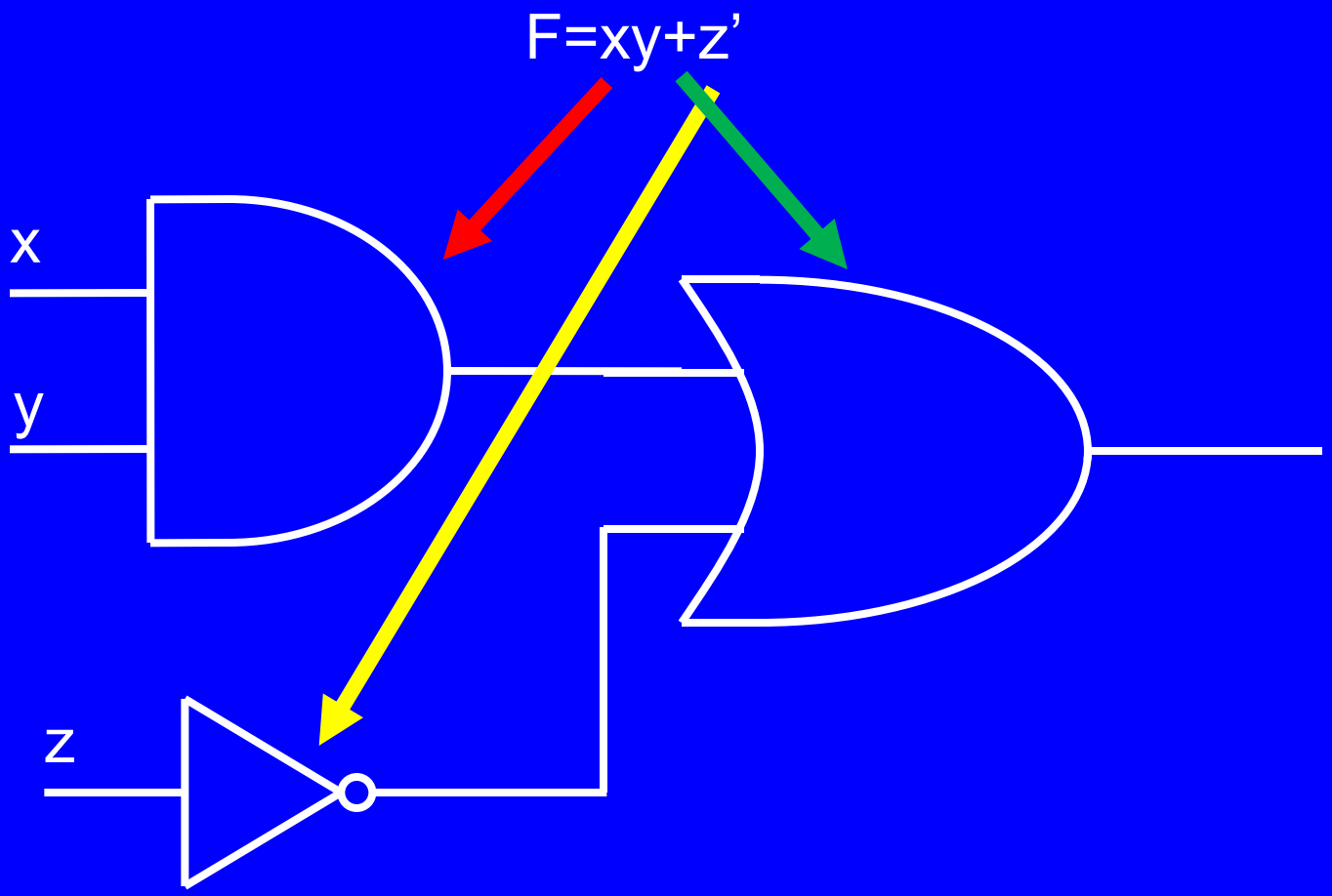
$= (x+y)(x+y'+z)$

$= [z+(x+y)(x+y'+0)][z'+(x+y)(x+y'+1)]$

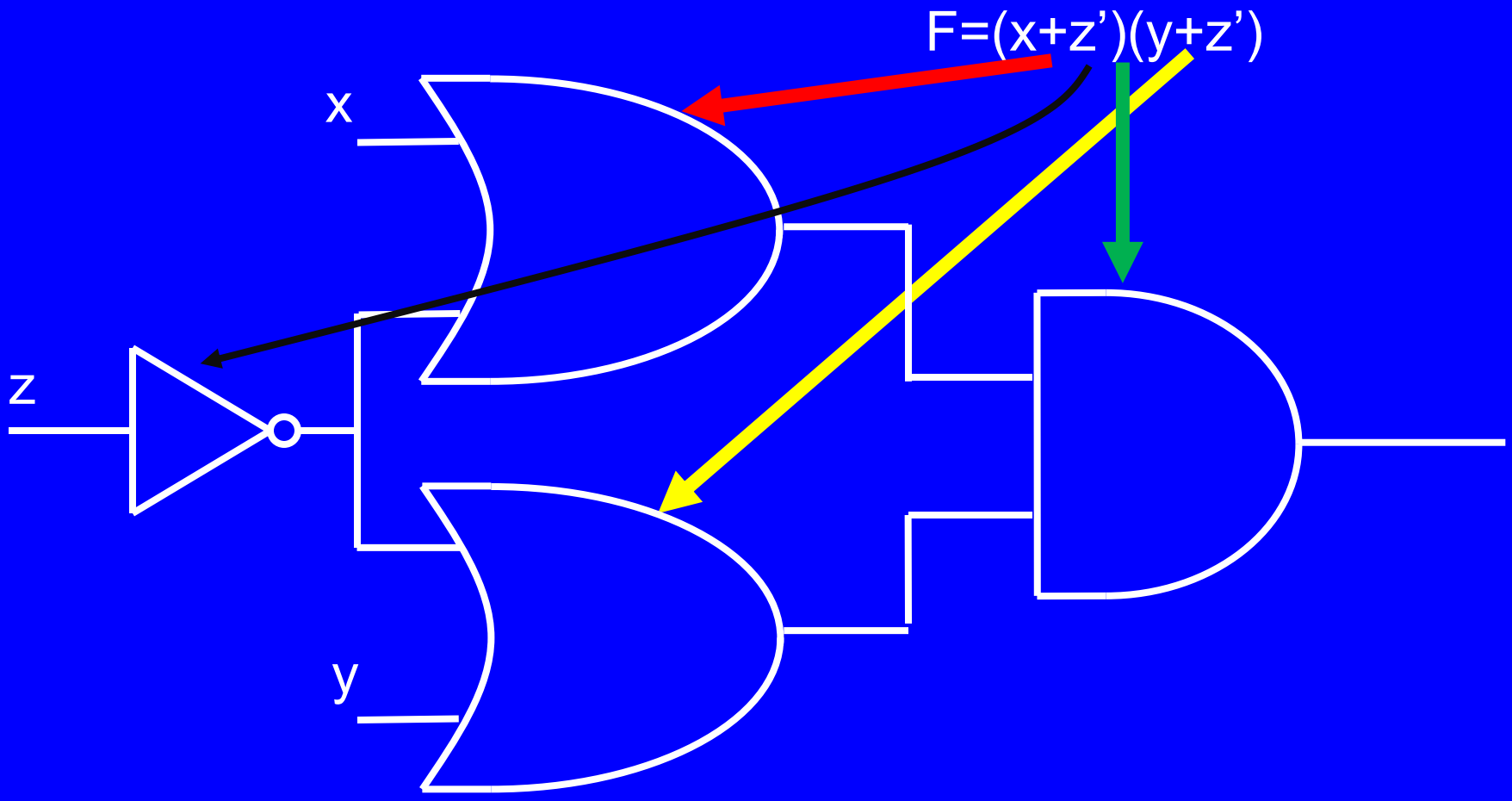
$= (x+y+z)(x+y+z')(x+y'+z)$

$= M_0M_1M_2$

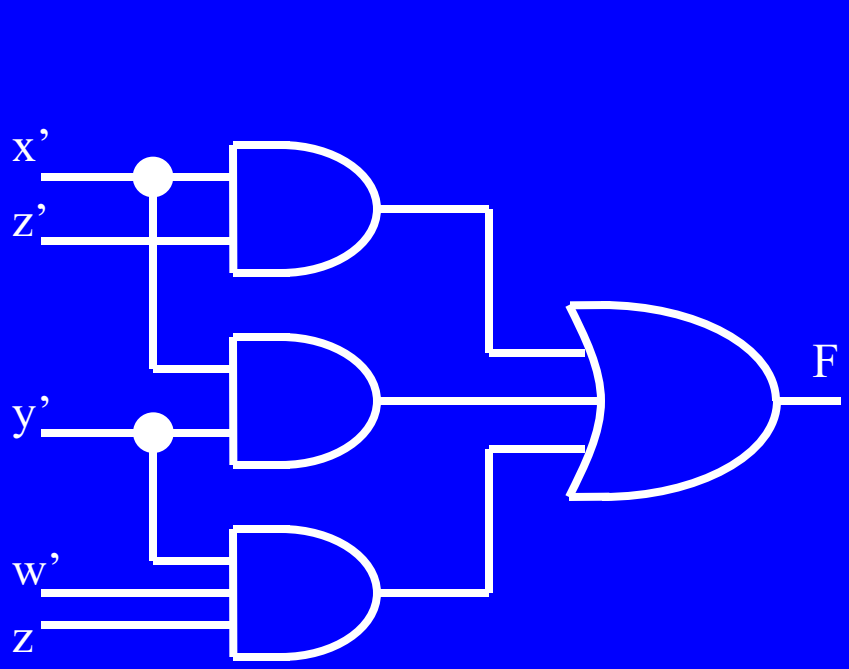
Implementation of boolean functions

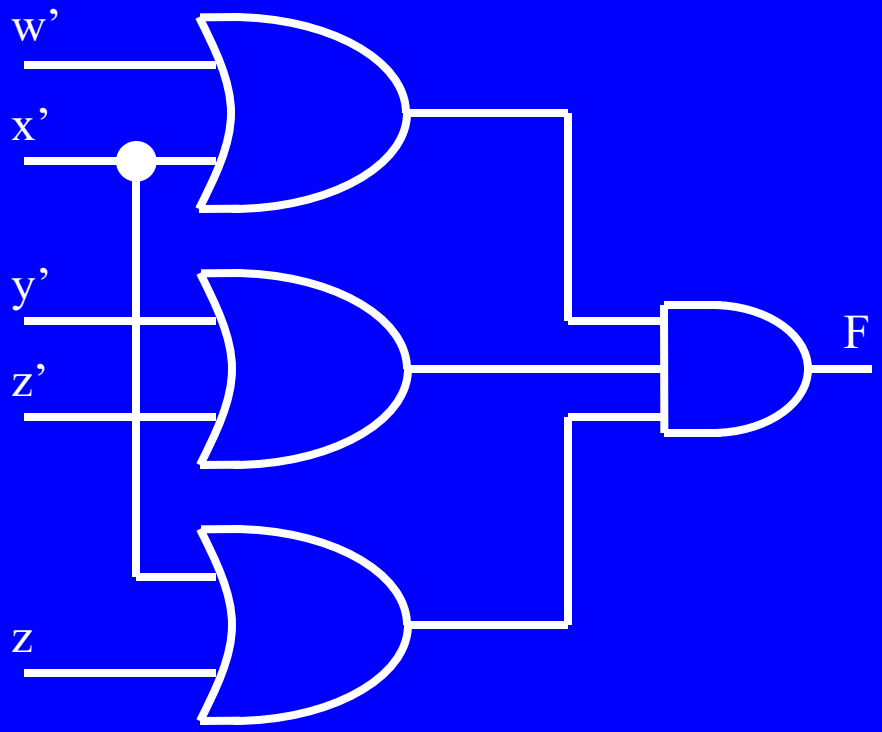


Implementation of boolean functions

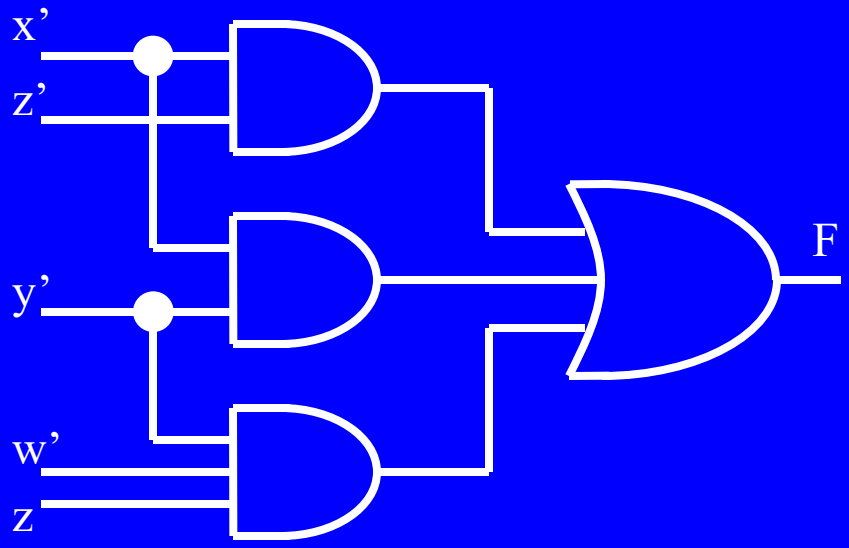


Implementation with AND and OR gates

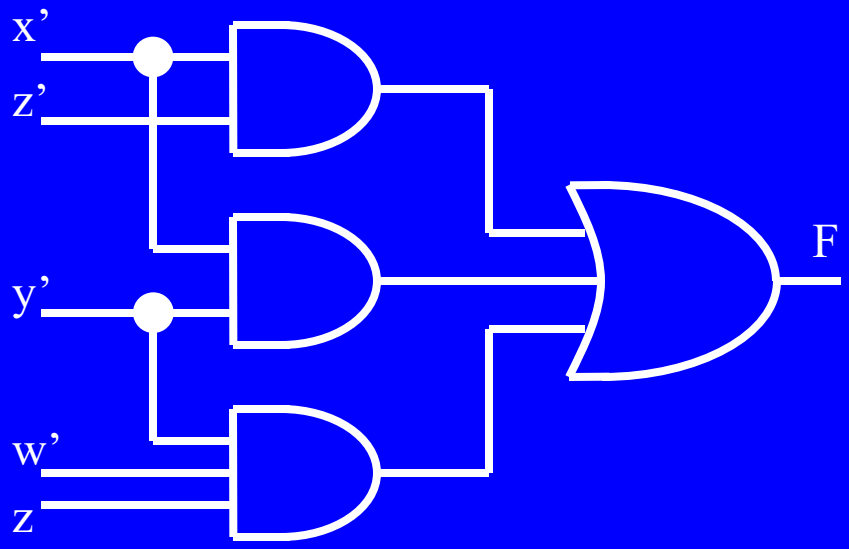


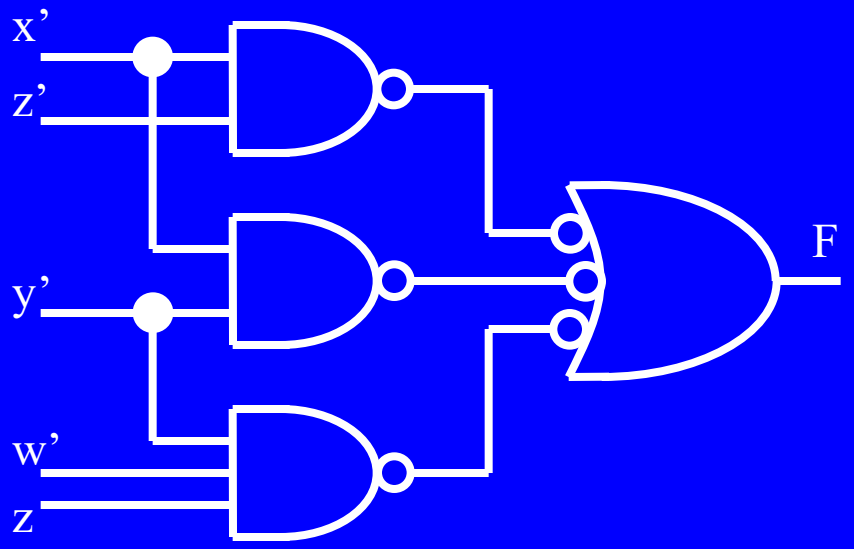


Implementation with NAND gates

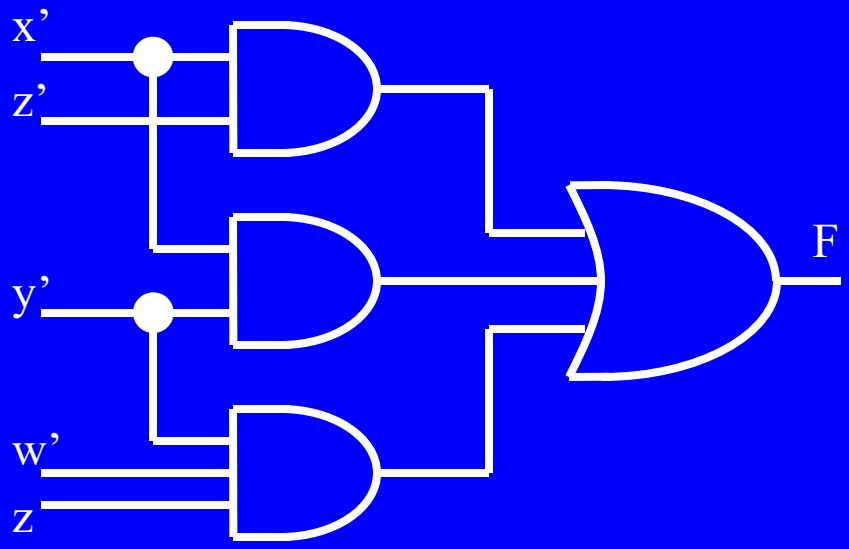


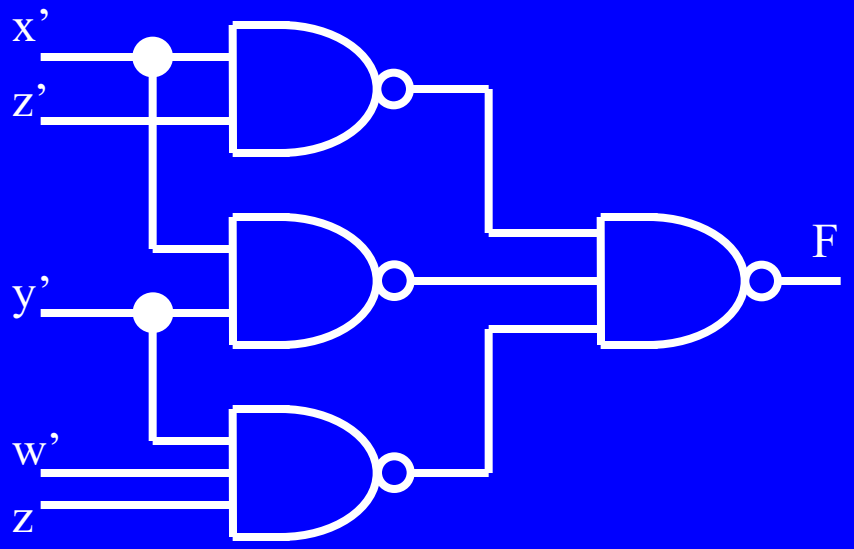
Implementation with NAND gates



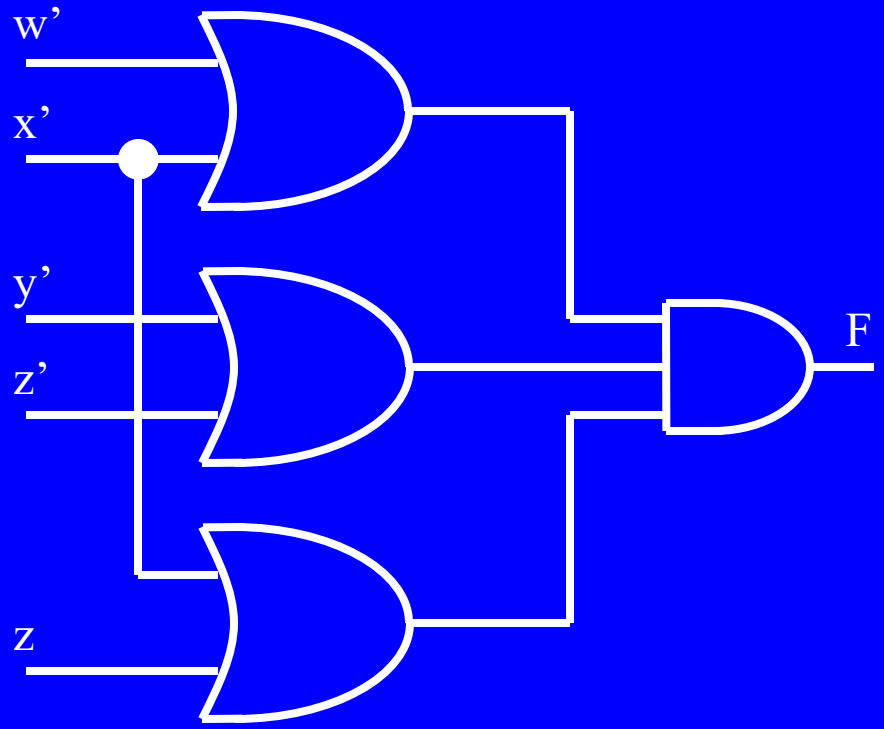


Implementation with NAND gates

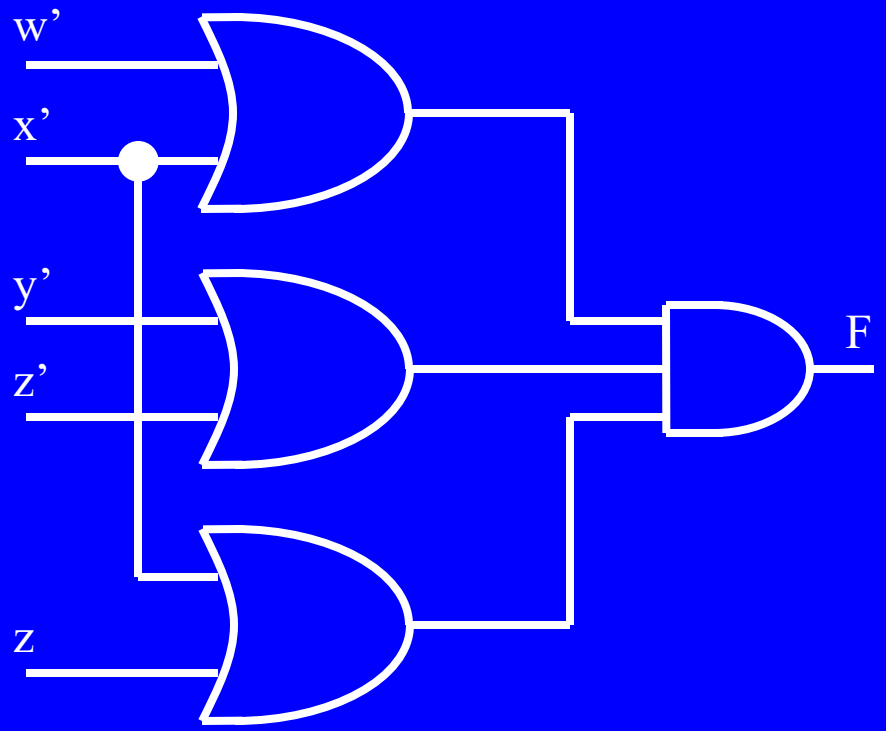


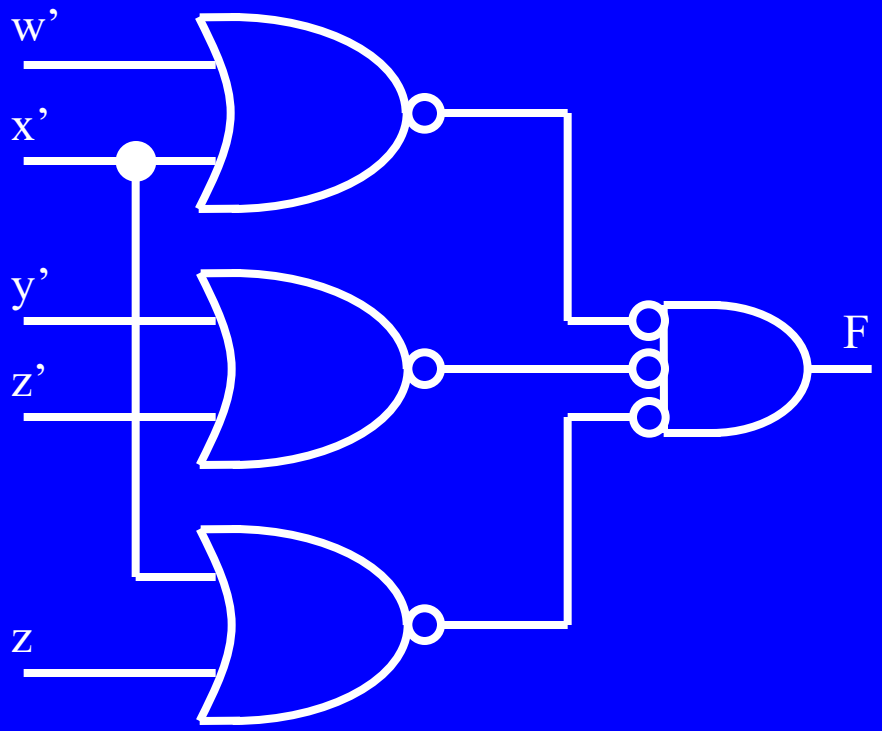


Implementation with NOR gates

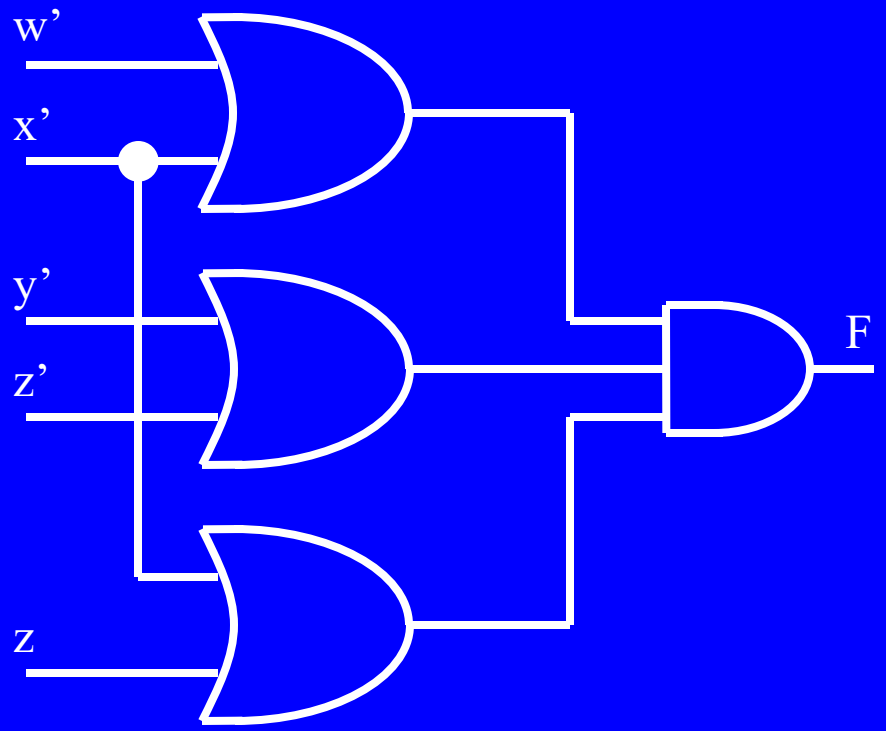


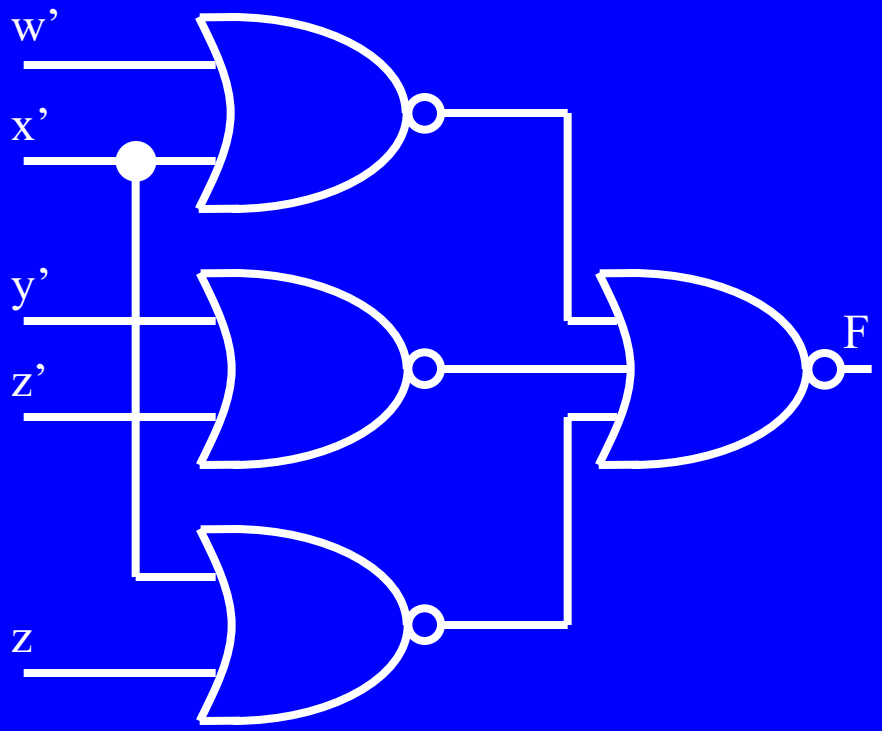
Implementation with NOR gates



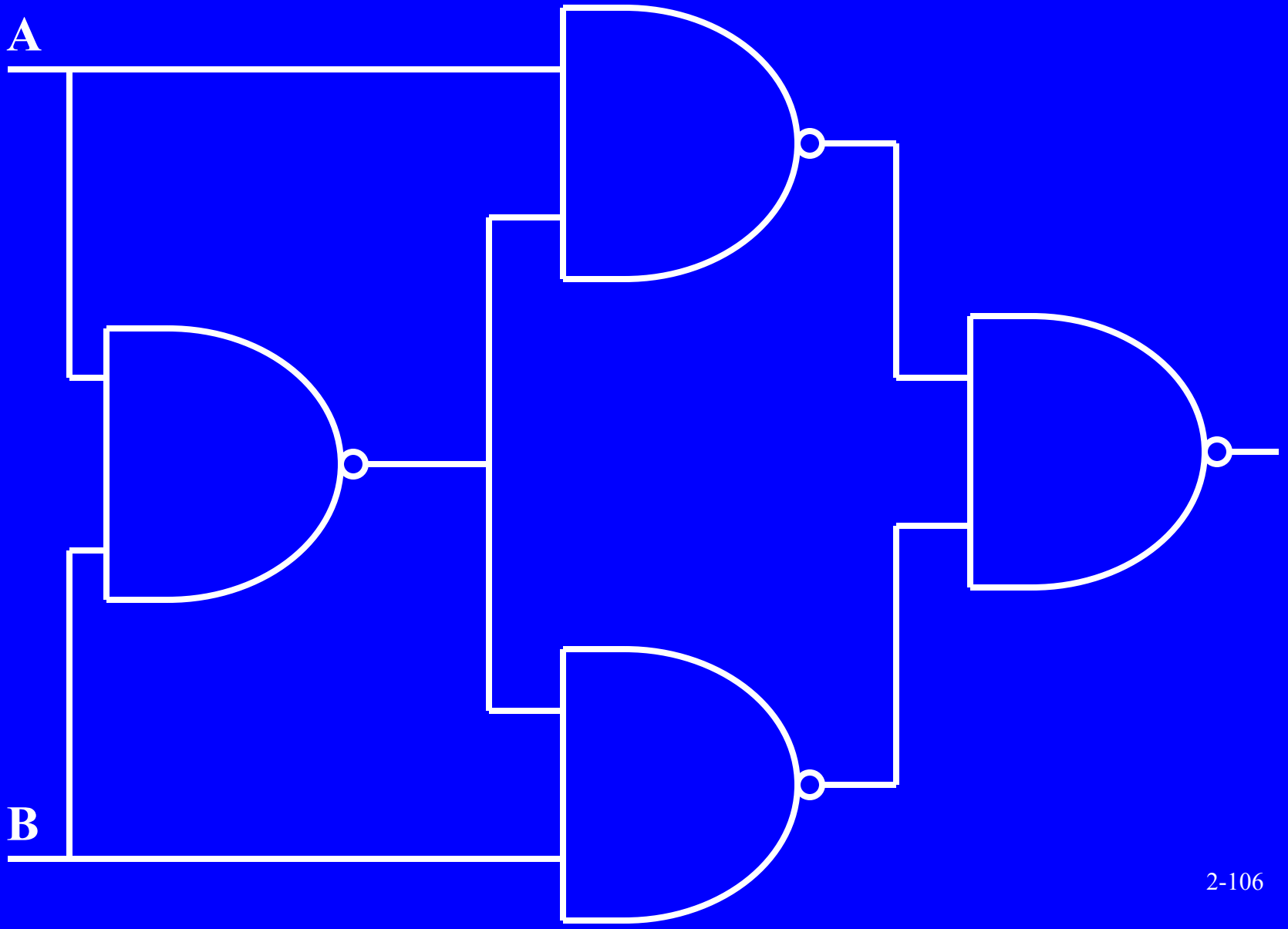


Implementation with NOR gates

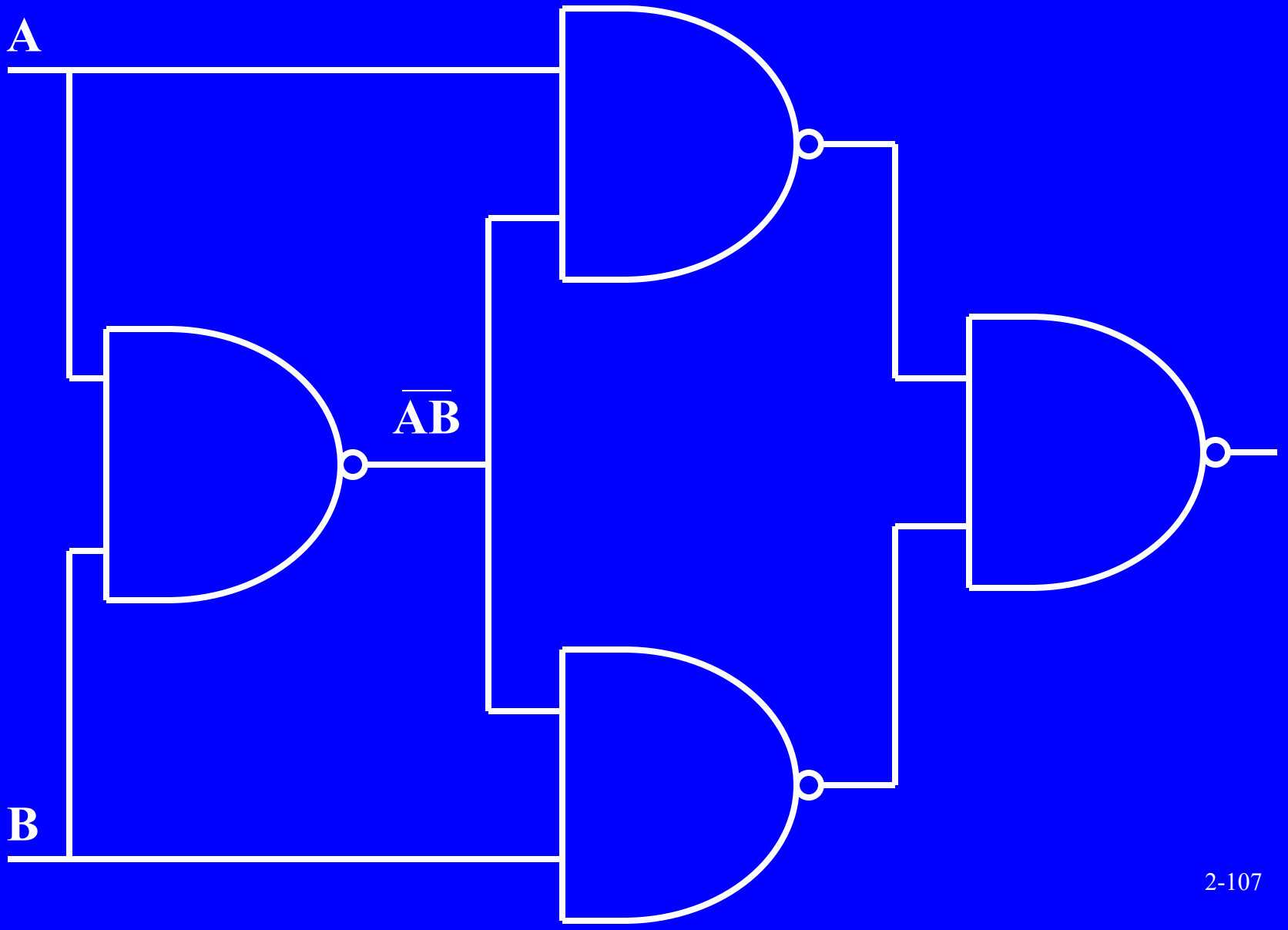




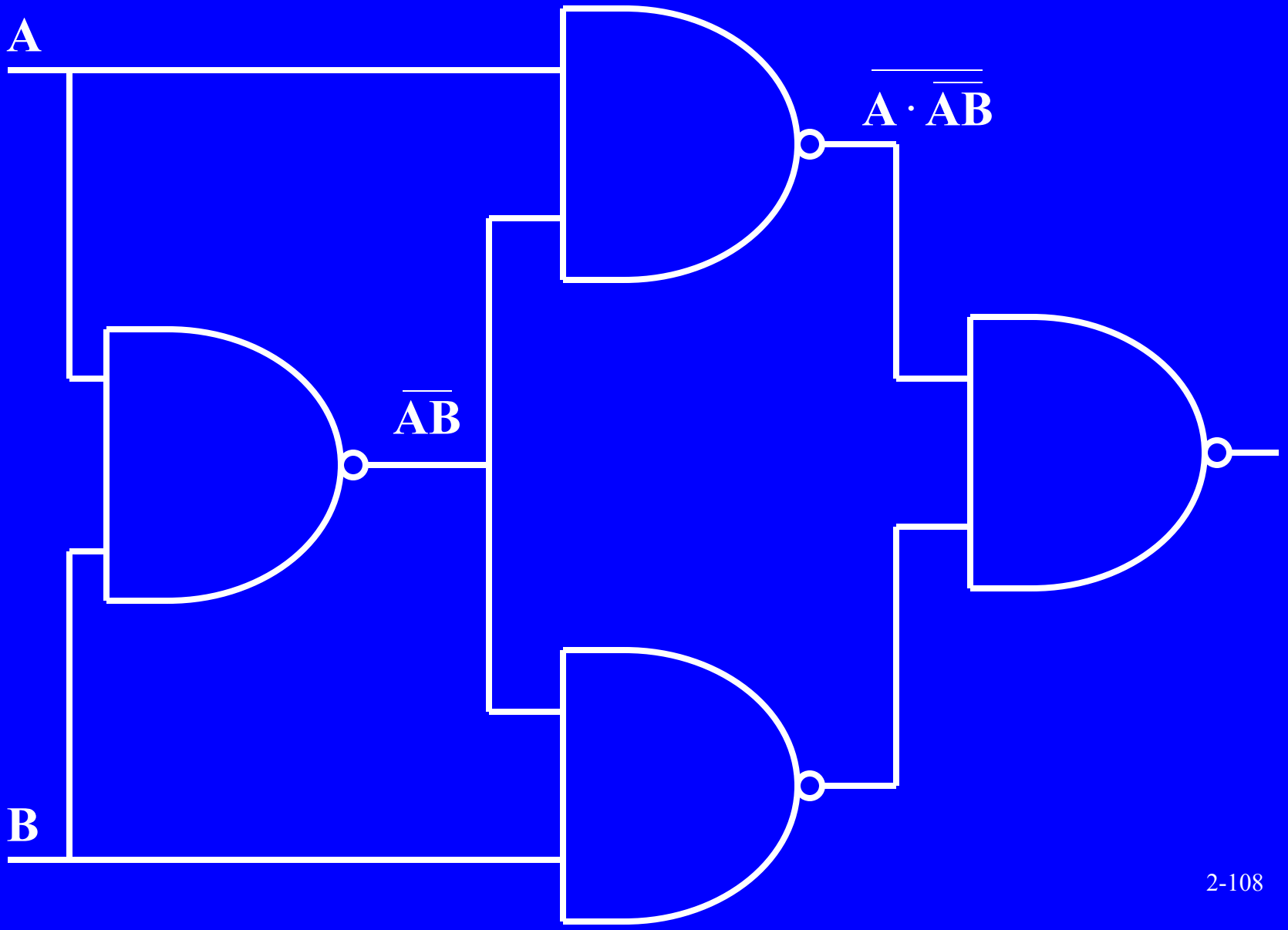
Finding the boolean function



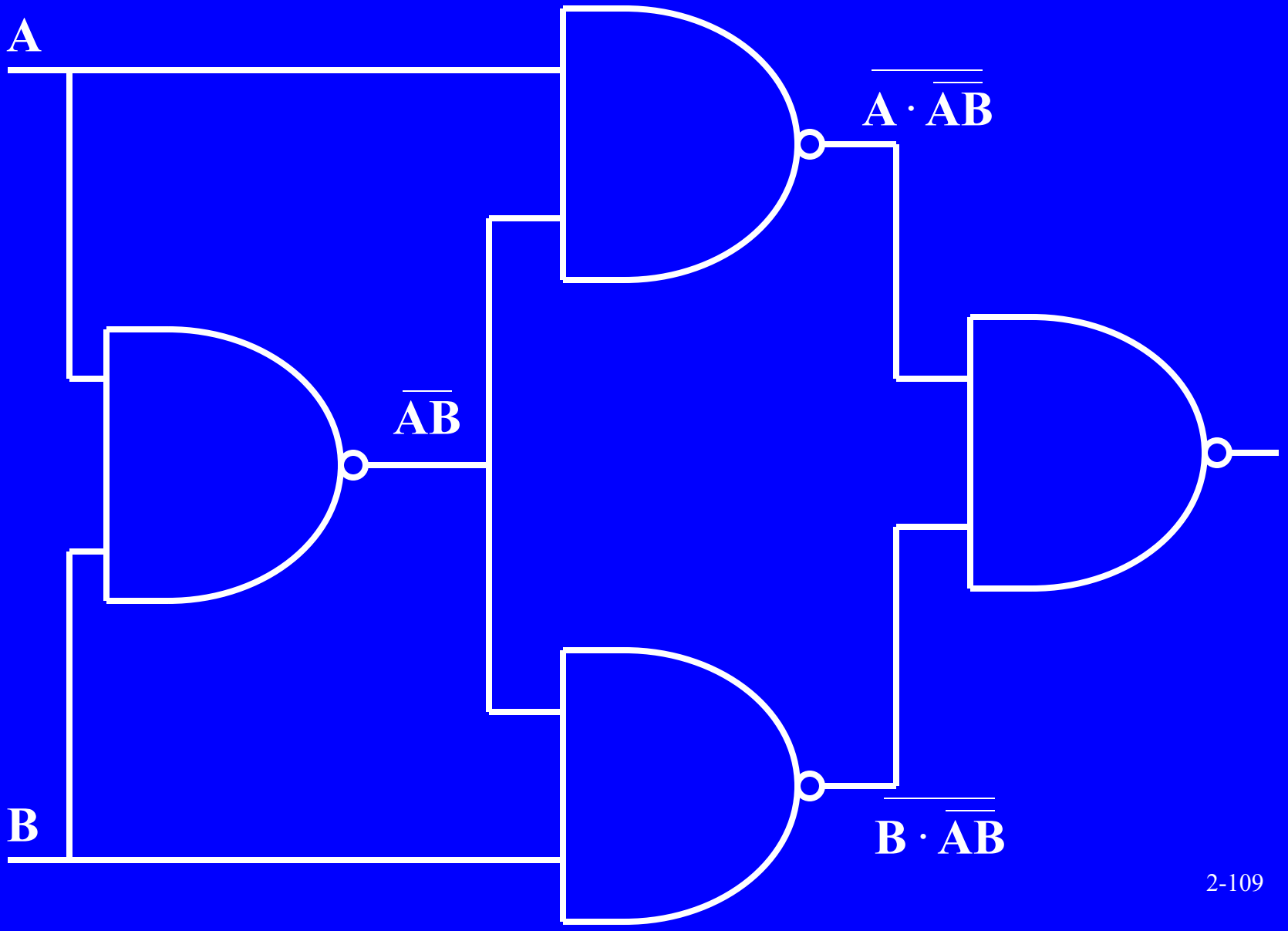
Finding the boolean function



Finding the boolean function



Finding the boolean function



Finding the boolean function

