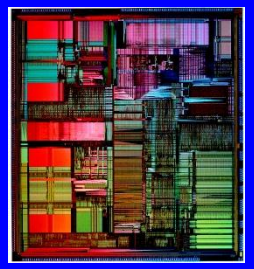
University of Thessaly
Department of Electrical and Computer Engineering

Digital Design



Instructor: George Stamoulis

Lectures: Monday 16:00 – 19:00

Laboratories: TBA

Textbooks:

- Digital Design by M. Morris Mano
- Notes
- Instructions for laboratory exercises

https://eclass.uth.gr/courses/E-CE_U_196/

Purpose of the course

- This course aims to convey the principles of the operation of digital circuits and to present basic structures used in logical design
- Upon completion of the course, the goal is for the students to have acquired fluency in binary logic and the basic structures of logical design

Course outline

- The syllabus includes:
  - Introduction to Boolean Algebra
  - Basic combinatorial and sequential structures
  - Subsystems
- We will follow the textbook:
  - Boolean codes and algebra
  - Combinatorial logic
  - Sequential logic
  - Algorithmic state machines

Grading

8 sets of exercises	10%
Laboratory exercises	30%
Advances	15%
Final test	45%

A prerequisite for successful completion of the course is a grade of >2.9 in progress and >4.9 in laboratory exercises and the final exam

Homework sets

Eight sets of exercises will be given with 3 or 4 exercises each. These exercises will be based on the material covered and aim to understand basic concepts.

Binary numbers

A general representation of a number is:

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \cdots + a_2 \cdot r^2 + a_1 \cdot r + a_0 + a_{-1} \cdot r^{-1} + \cdots + a_{-m} \cdot r^{-m}$$

where a_i are the coefficients and r is the base

e.g. the number 26.75 is described as:

$$2x10^1 + 6x10^0 + 7x10^{-1} + 5x10^{-2}$$

but also as:

$$1x2^4 + 1x2^3 + 0x2^2 + 1x2^1 + 0x2^0 + 1x2^{-1} + 1x2^{-2}$$

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Operations with binary numbers

addition

```
  101101
+ 100111
--------
 1010100
```

subtraction

```
  101101
- 100111
--------
  000110
```

multiplication

```
   1011
x  101
------
   1011
  0000
 1011
------
 110111
```

Addition

$$\begin{array}{r} 101101 \\ +\ 100111 \\ \hline \end{array}$$

Addition

$$
\begin{array}{rl}
 101101 & 1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0 \\
+\ 100111 & 1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0
\end{array}
$$

Addition

$$
\begin{array}{r}
101101 \\
+\ 100111 \\
\hline
\end{array}
$$

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

Addition

$$
\begin{array}{r}
101101 \\
+\ 100111 \\
\hline
\end{array}
$$

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

2

1 carry

Addition

$$
\begin{array}{rl}
101101 & 1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0 \\
+\ 100111 & 1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0 \\
\hline
0 &
\end{array}
$$

1 carry

Addition

```
  101101
+ 100111
--------
       0
```

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

1 carry

Addition

$$
\begin{array}{r}
10111\,01 \\
+\ 10011\,1 \\
\hline
0
\end{array}
$$

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

1

Addition

$$
\begin{array}{r}
101101 \\
+\ 100111 \\
\hline
\end{array}
$$

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

0

2

1 carry

Addition

101101	$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$
+ 100111	$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$
00	

1 carry

Addition

$$
\begin{array}{r}
101101 \\
+\ 100111 \\
\hline
00
\end{array}
$$

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

1 carry

Addition

$$
\begin{array}{r}
101101 \\
+\ 100111 \\
\hline
00
\end{array}
$$

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

2

Addition

$$
\begin{array}{r}
101101 \\
+\ 100111 \\
\hline
00
\end{array}
$$

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

3

1 carry

Addition

101101	$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$
+ 100111	$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$
100	

1 carry

Addition

```
  101101
+ 100111
--------
    100
```

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

1 carry

Addition

$$
\begin{array}{r}
101101 \\
+\ 100111 \\
\hline
100
\end{array}
$$

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

1

Addition

$$
\begin{array}{r}
101101 \\
+\ 100111 \\
\hline
100
\end{array}
$$

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

2

1 carry

Addition

$$
\begin{array}{rl}
 & 101101 \\
+ & 100111 \\
\hline
 & 0100
\end{array}
$$

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

1 carry

Addition

```
  101101
+ 100111
--------
 1 0100
```

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

1 carry

Addition

$$
\begin{array}{rl}
 & 101101 \\
+ & 100111 \\
\hline
 & 0100
\end{array}
\qquad
\begin{array}{l}
1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0 \\
1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0 \\
0
\end{array}
$$

Addition

$$
\begin{array}{r}
101101 \\
+\ 100111 \\
\hline
0100
\end{array}
$$

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

1

Addition

$$
\begin{array}{rl}
 101101 & 1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0 \\
+\ 100111 & 1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0 \\
\hline
 10100 &
\end{array}
$$

Addition

$$
\begin{array}{r}
101101 \\
+\ 100111 \\
\hline
10100
\end{array}
$$

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$

$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$

Addition

101101	$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$
+ 100111	$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$
10100	2

1 carry

Addition

101101	$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$
+ 100111	$1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$
010100	

Addition

$$
\begin{array}{rl}
101101 & 1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0 \\
+\ 100111 & 1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0 \\
\hline
1010100 & 1x2^6+0x2^5+1x2^4+0x2^3+1x2^2+0x2^1+0x2^0
\end{array}
$$

Addition

$$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0 \qquad 45$$

$$+\quad 1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$$

$$1x2^6+0x2^5+1x2^4+0x2^3+1x2^2+0x2^1+0x2^0$$

Addition

$1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0$ 45

\+ $1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0$ 39

$1x2^6+0x2^5+1x2^4+0x2^3+1x2^2+0x2^1+0x2^0$

Addition

$$
\begin{array}{rll}
 & 1x2^5+0x2^4+1x2^3+1x2^2+0x2^1+1x2^0 & 45 \\
+ & 1x2^5+0x2^4+0x2^3+1x2^2+1x2^1+1x2^0 & 39 \\
\hline
 & 1x2^6+0x2^5+1x2^4+0x2^3+1x2^2+0x2^1+0x2^0 & 84
\end{array}
$$

Number Base Conversion

integer	residual
41	

Number Base Conversion

20x2+1

integer	residual
41	

Number Base Conversion

20x2+1

integer	residual
41	
20	1

Number Base Conversion

10x2+0

integer	residual
41	
20	1

Number Base Conversion

10x2+0

integer	residual
41	
20	1
10	0

Number Base Conversion

5x2+0

integer	residual
41	
20	1
10	0

Number Base Conversion

5x2+0

integer	residual
41	
20	1
10	0
5	0

Number Base Conversion

2x2+1

integer	residual
41	
20	1
10	0
5	0

Number Base Conversion

2x2+1

integer	residual
41	
20	1
10	0
5	0
2	1

Number Base Conversion

1x2+0

integer	residual
41	
20	1
10	0
5	0
2	1

Number Base Conversion

1x2+0

integer	residual
41	
20	1
10	0
5	0
2	1
1	0

Number Base Conversion

0x2+1

integer	residual
41	
20	1
10	0
5	0
2	1
1	0

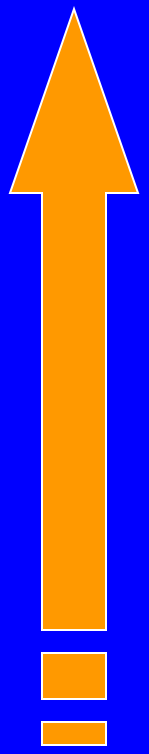
Number Base Conversion

0x2+1

integer	residual
41	
20	1
10	0
5	0
2	1
1	0
0	1

Number Base Conversion

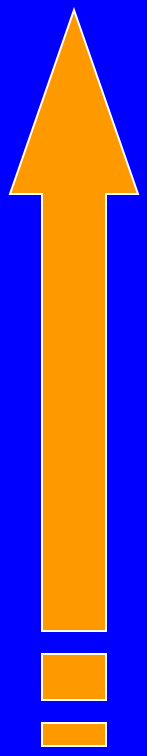
integer	residual
41	
20	1
10	0
5	0
2	1
1	0
0	1



=101001

Number Base Conversion

103	
51	1
25	1
12	1
6	0
3	0
1	1
0	1



Number Base Conversion

103	
51	1
25	1
12	1
6	0
3	0
1	1
0	1

$1X2^6+1X2^5+0X2^4+0X2^3+1X2^2+1X2^1+1X2^0$

Number Base Conversion

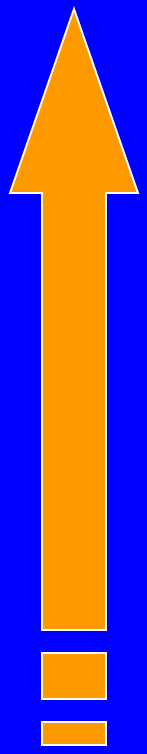
103	
51	1
25	1
12	1
6	0
3	0
1	1
0	1

$1X2^6+1X2^5+0X2^4+0X2^3+1X2^2+1X2^1+1X2^0$

64+ 32+ 0+ 0+ 4+ 2+ 1

Number Base Conversion (Base 3)

103	
34	1
11	1
3	2
1	0
0	1



Number Base Conversion (Base 3)

103		
34	1	
11	1	
3	2	$1X3^4+0X3^3+2X3^2+1X3^1+1X3^0$
1	0	
0	1	

Number Base Conversion (Base 3)

103

34 1

11 1

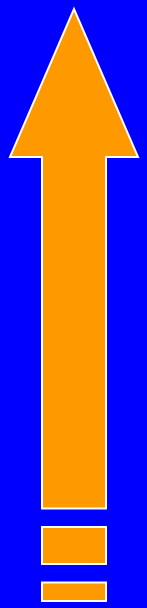
3 2 $1X3^4+0X3^3+2X3^2+1X3^1+1X3^0$

1 0 81+ 0+ 18+ 3+ 1

0 1

Number Base Conversion (Base 4)

103	
25	3
6	1
1	2
0	1



Number Base Conversion (Base 4)

103	
25	3
6	1
1	2
0	1

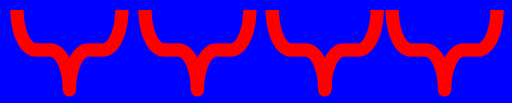
$1X4^3+2X4^2+1X4^1+3X4^0$

Number Base Conversion (Base 4)

103		
25	3	
6	1	
1	2	$1X4^3+2X4^2+1X4^1+3X4^0$
0	1	64+ 32+ 4+ 3

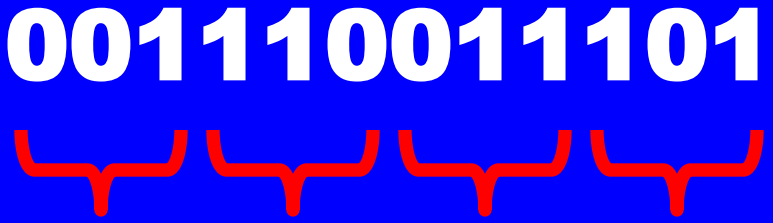
Conversion from binary to quaternary

01100111



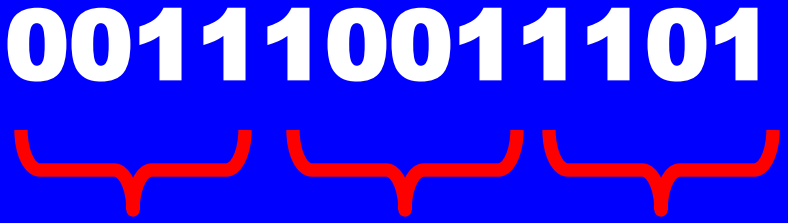
1 2 1 3

Conversion from binary to octal



1 6 3 5

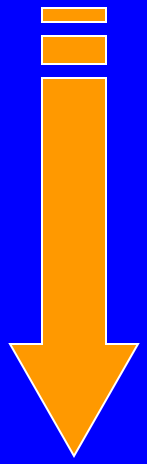
Conversion from binary to hexadecimal



3 9 D

Number Base Conversion

Fraction	Integer Part
0.6875	1
0.3750	0
0.7500	1
0.5000	1



=.1011

Complement

- In terms of r-1 (*diminished radix complement*) it is r^n-1-N
  - in base10 we have a complement to 9 e.g. the complement to 9 of 245 is the754
- In terms of r (*radix complement*) it is r^n-N
  - in base10 we have a complement to 10 e.g. the complement to 10 of 245 is 755

It is used in subtraction

- We add to the minuend the complement to the r of the subtrahend
- If $M \geq N$ the sum will have a carry that we ignore
- If $M < N$ then the result is the complement to r of M-N

Example

Let's try 76 - 23

- ✓ The complement to 10 of 23 is 77
- ✓ We calculate 76 + 77 = 153
- ✓ We ignore the carry 153
- ✓ The final result is 53

Example

Let's try 103 - 23

- ✓ The complement to 10 of 23 is 977
- ✓ We calculate 103 + 977 = 1080
- ✓ We ignore the carry 1080
- ✓ The final result is 80

Example

Let’s now try 23 - 76

- ✓ The complement to 10 of 76 είναι 24
- ✓ We calculate 23 + 24 = 47
- ✓ There is no carry so we calculate the complement to 10 of 47
- ✓ The final result is -53

Example

Let’s now try 23 - 103

- ✓ The complement to 10 of 103 is 897
- ✓ We calculate 23 + 897 = 920
- ✓ There is no carry so we calculate the complement to 10 of 920
- ✓ The final result is -80

Example with binaries

78=1001110

23=0010111

78	1001110
+23	+0010111
101	1100101

Example with binaries

78=1001110 23=0010111

Complement to 2 of 23=1101001

78	1001110
-23	+1101001
	10110111

Example with binaries

78=1001110 23=0010111

Complement to 2 of 23=1101001

78	1001110
-23	+1101001
	10110111
55	0110111

Example with binaries

78=1001110 23=0010111

Complement to 2 of 78=0110010

23	0010111
-78	+0110010
	1001001
-55	-0110111

Signed Binary Numbers

- signed magnitude representation
  - the first digit represents the sign and the rest the absolute value
- **2's complement representation**
  - friendlier representation to binary operations

Mixed-base systems

They are systems where each digit does not express forces of the same number (base).

Example: (years, months, days, hours, minutes, seconds)

(48,3,22,0,15,30)

Negative Base Systems

They are systems in which the base is a negative number e.g. -2 (negbinary).

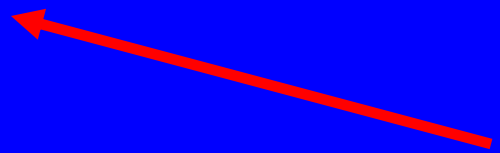
Conversions are made as in positive-based systems.

The numbers from 1-10 in negbinary

Decimal	Binary	Negbinary
0	0000	00000
1	0001	00001
2	0010	00110
3	0011	00111
4	0100	00100
5	0101	00101
6	0110	11010
7	0111	11011
8	1000	11000
9	1001	11001
10	1010	11110

Binary Codes

- Decimal Codes
  - BCD (8421)
  - excess-3
  - 84-2-1
  - 2421
  - 5043210



self-complementary

Decimal	BCD	Excess-3	84-2-1
0	0000	0011	0000
1	0001	0100	0111
2	0010	0101	0110
3	0011	0110	0101
4	0100	0111	0100
5	0101	1000	1011
6	0110	1001	1010
7	0111	1010	1001
8	1000	1011	1000
9	1001	1100	1111

Δεκαδικό	BCD	Biquinary	2421
00	0000	0100001	0000
01	0001	0100010	0001
02	0010	0100100	0010
03	0011	0101000	0011
04	0100	0110000	0100
05	0101	1000001	1011
06	0110	1000010	1100
07	0111	1000100	1101
08	1000	1001000	1110
09	1001	1010000	1111

Binary Codes

- Error Detection Codes
  - Parity bit
  - 5043210 code (biquinary)
- Gray code
- Alphanumeric codes
  - American Standard Code for Information Interchange (ASCII)
  - Extended Binary Coded Decimal Interchange Code (EBCDIC)

Decimal	Binary	even parity	odd parity
0	0000	0	1
1	0001	1	0
2	0010	1	0
3	0011	0	1
4	0100	1	0
5	0101	0	1
6	0110	0	1
7	0111	1	0
8	1000	1	0
9	1001	0	1
10	1010	0	1
11	1011	1	0
12	1100	0	1
13	1101	1	0
14	1110	1	0
15	1111	0	1

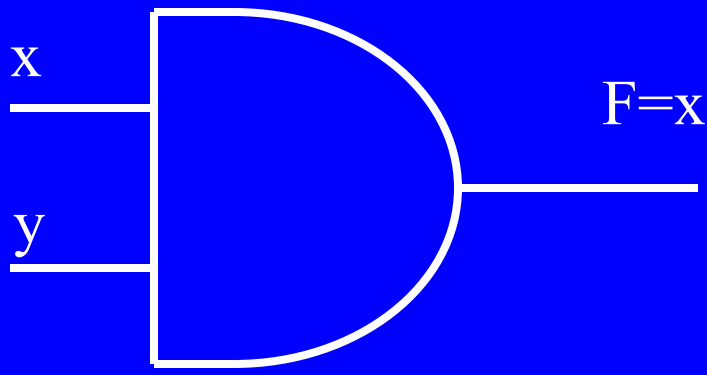
Decimal	Binary	Gray
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

Binary logic

- AND
- OR
- NOT

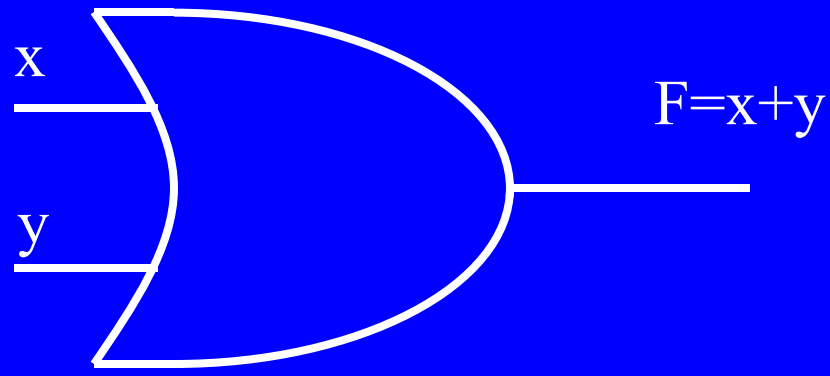
x	y	AND (x·y)	OR (x+y)	NOT (x')
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

AND gate



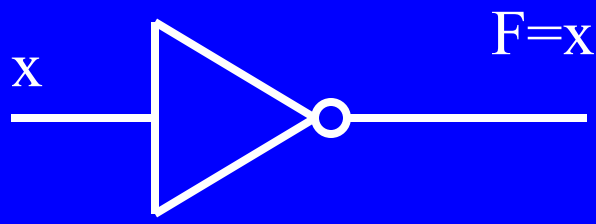
x	y	F
0	0	0
0	1	0
1	0	0
1	1	1

OR gate



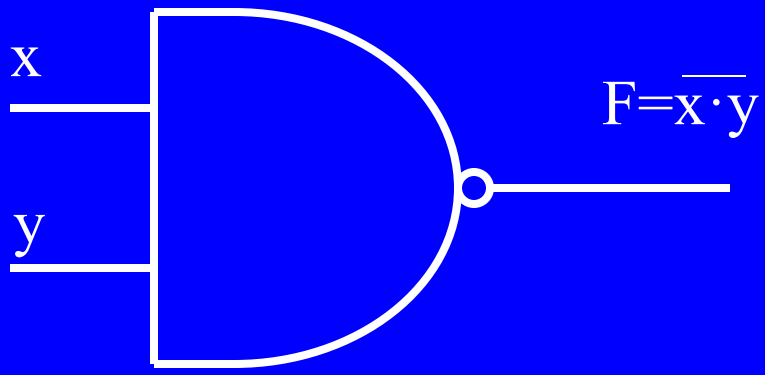
x	y	F
0	0	0
0	1	1
1	0	1
1	1	1

NOT gate



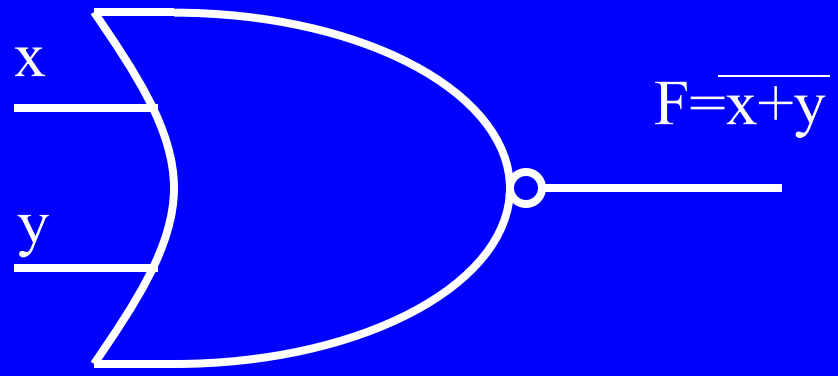
x	F
0	1
1	0

NAND gate



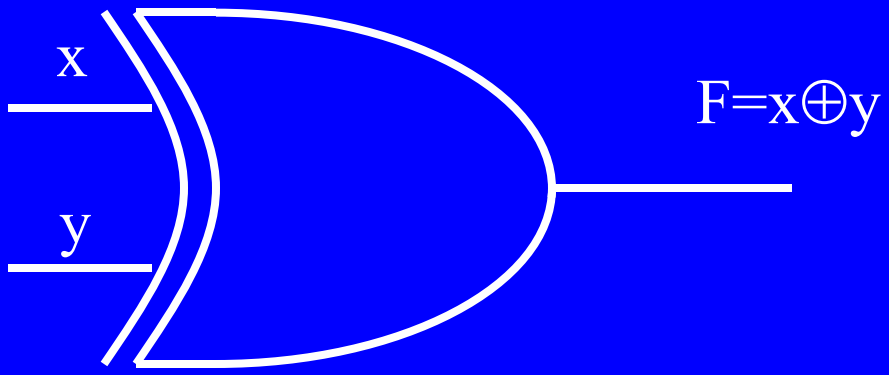
x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

NOR gate



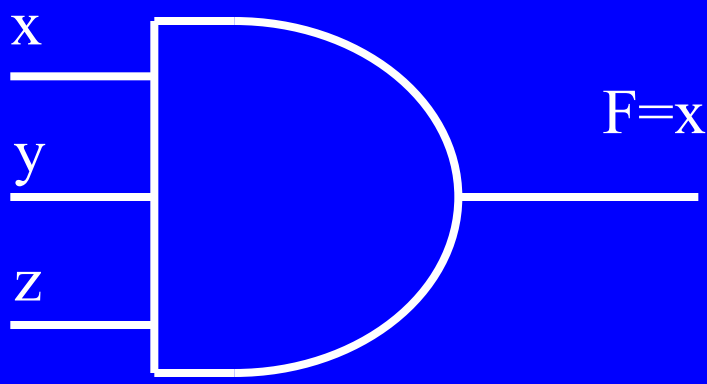
x	y	F
0	0	1
0	1	0
1	0	0
1	1	0

EXCLUSIVE-OR (XOR) gate

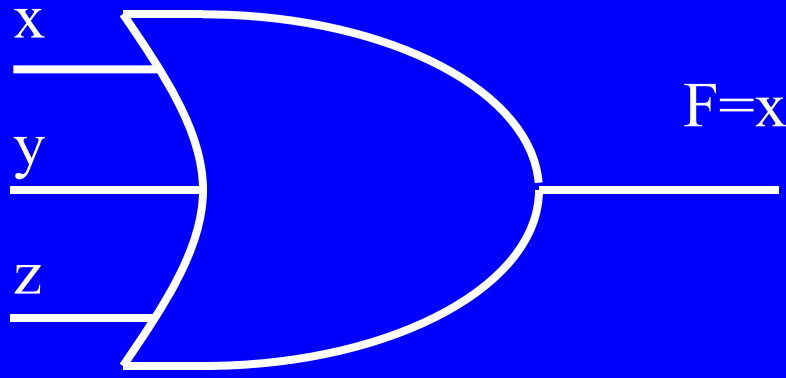


x	y	F
0	0	0
0	1	1
1	0	1
1	1	0

Other gates



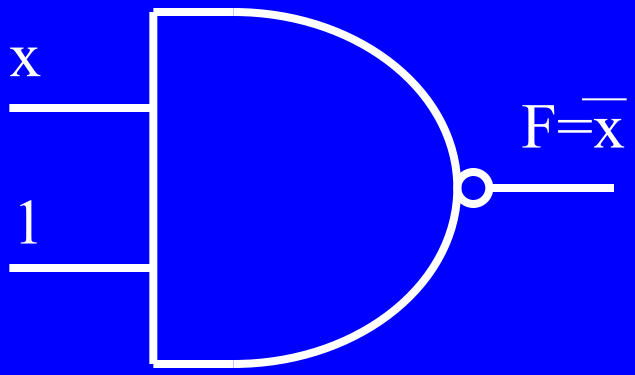
$F = x \cdot y \cdot z$

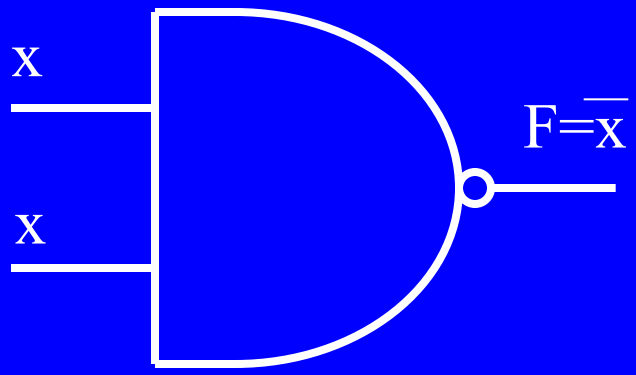


$F = x + y + z$

Universal gate

With the two-input NAND or NOR gate we can implement any other gate

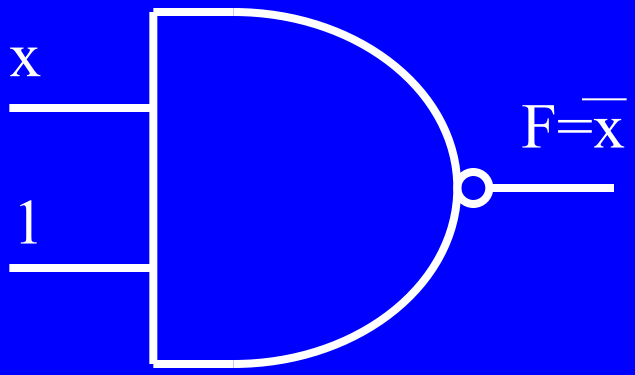


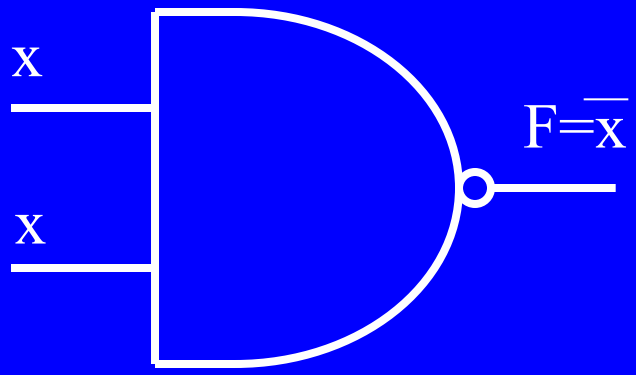


Universal gate

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

With the two-input NAND or NOR gate we can implement any other gate

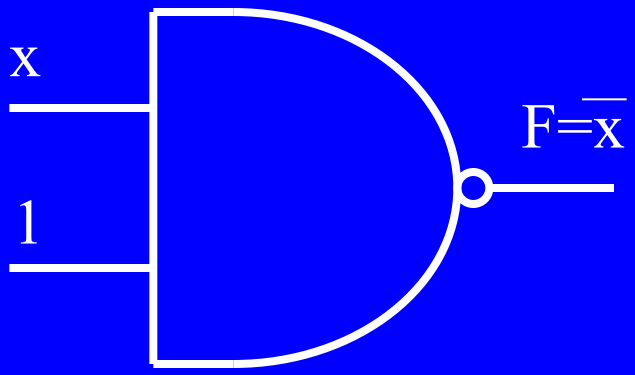


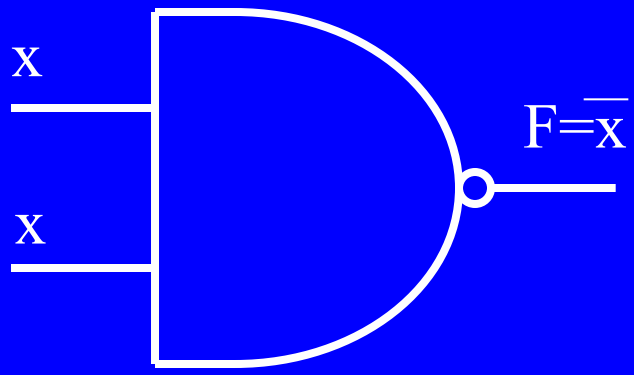


Universal gate

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

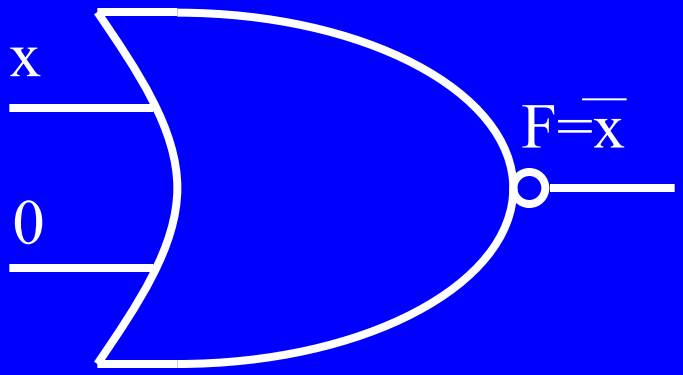
With the two-input NAND or NOR gate we can implement any other gate

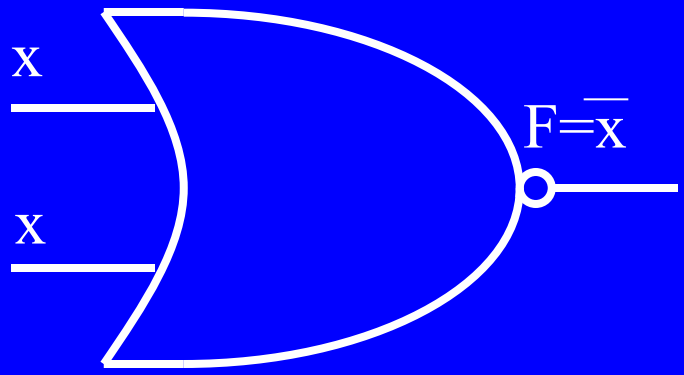




Universal gate

With the two-input NAND or NOR gate we can implement any other gate

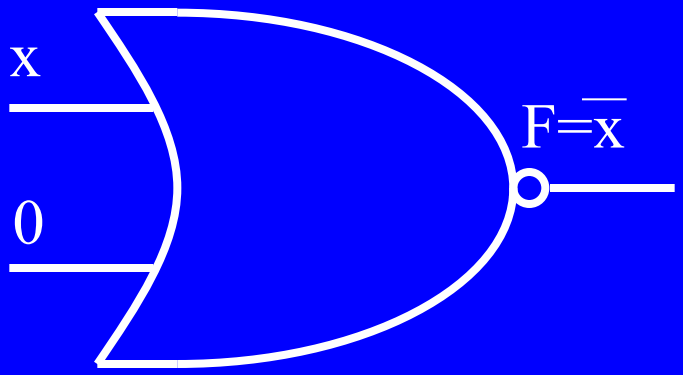


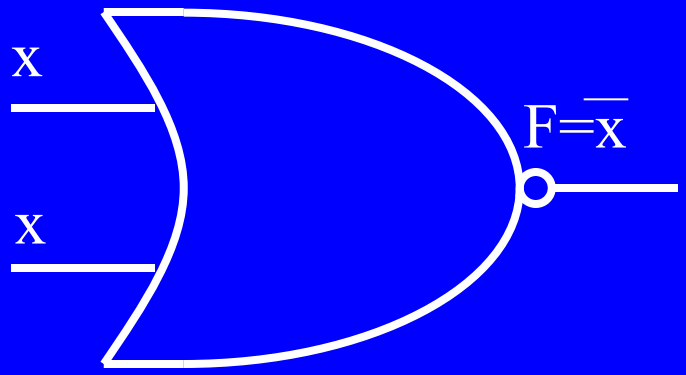


Universal gate

x	y	F
0	0	1
0	1	0
1	0	0
1	1	0

With the two-input NAND or NOR gate we can implement any other gate

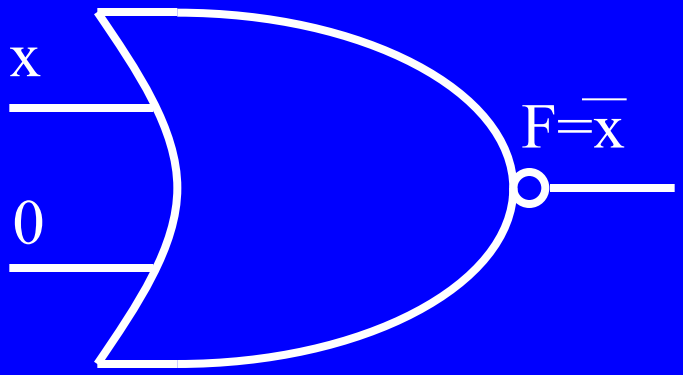


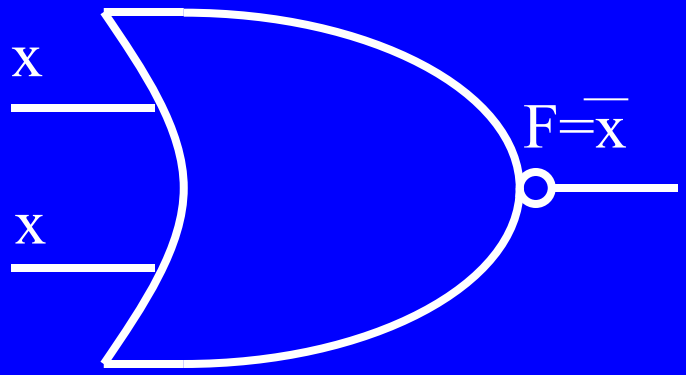


Universal gate

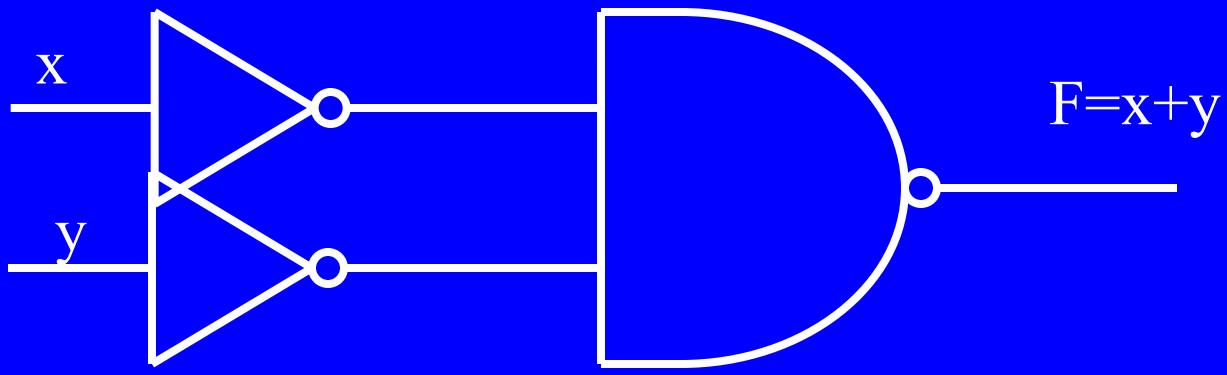
x	y	F
0	0	1
0	1	0
1	0	0
1	1	0

With the two-input NAND or NOR gate we can implement any other gate



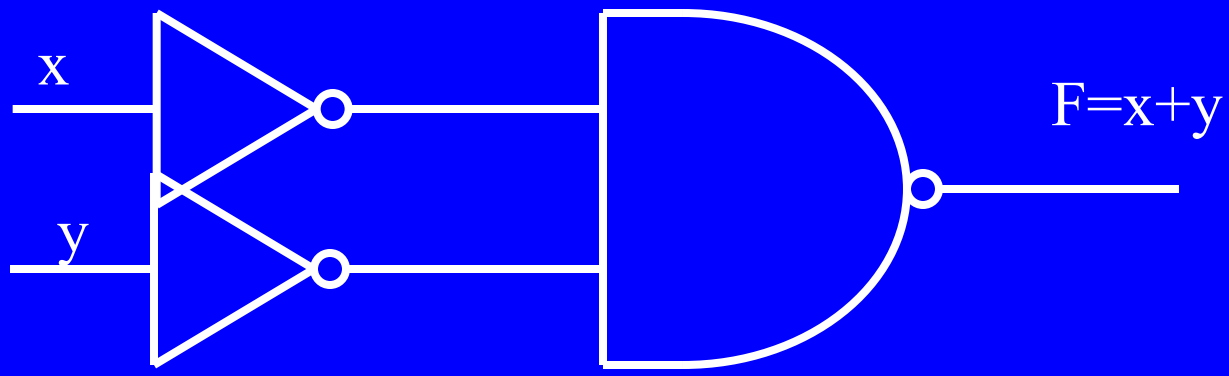


OR with NAND



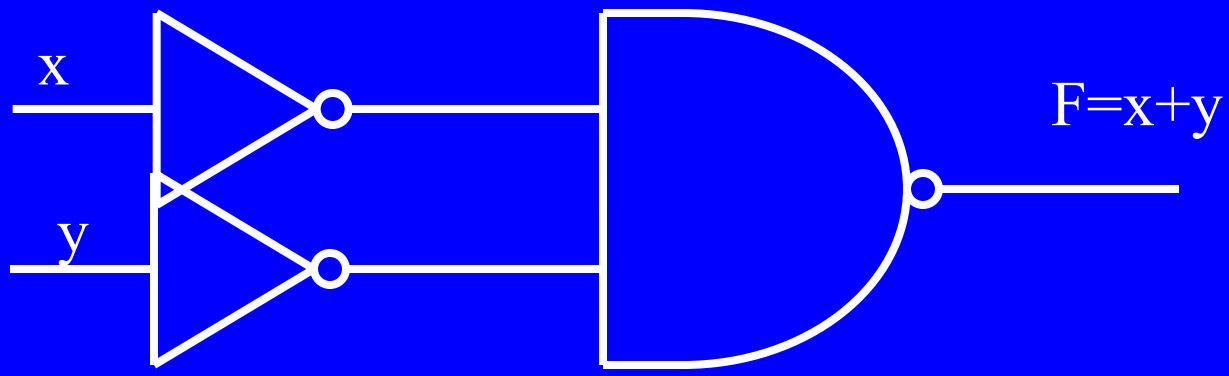
OR with NAND

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0



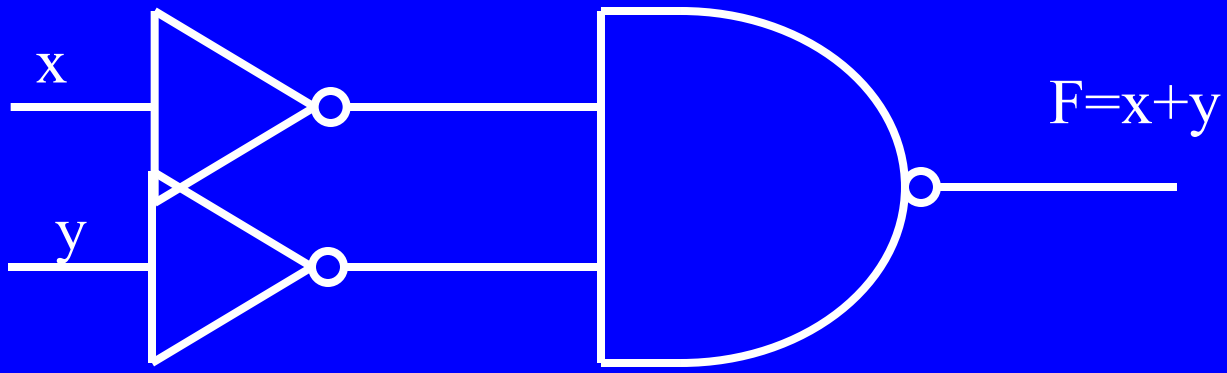
OR with NAND

x	y	F
1	1	1
1	0	1
0	1	1
0	0	0

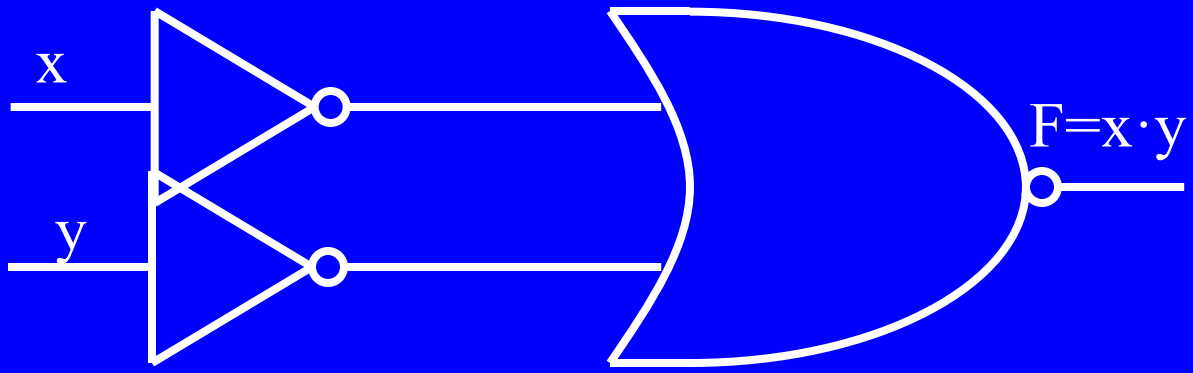


OR with NAND

x	y	F
0	0	0
0	1	1
1	0	1
1	1	1

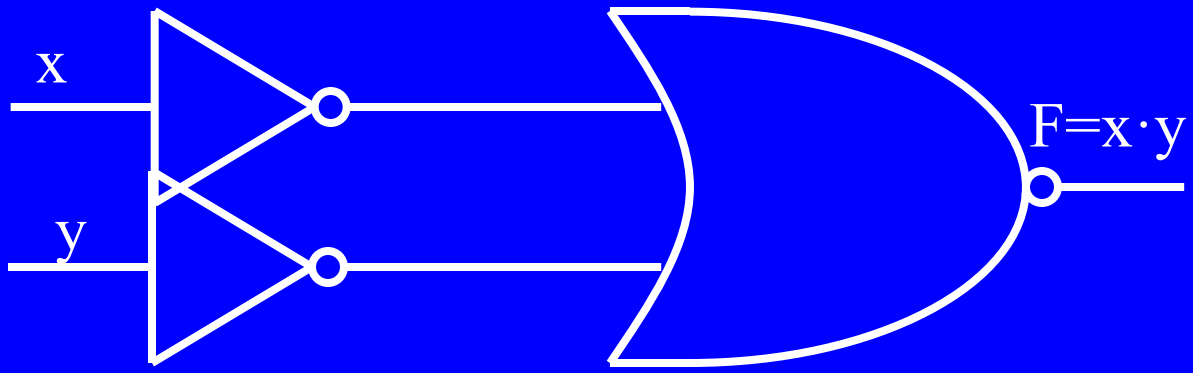


AND with NOR



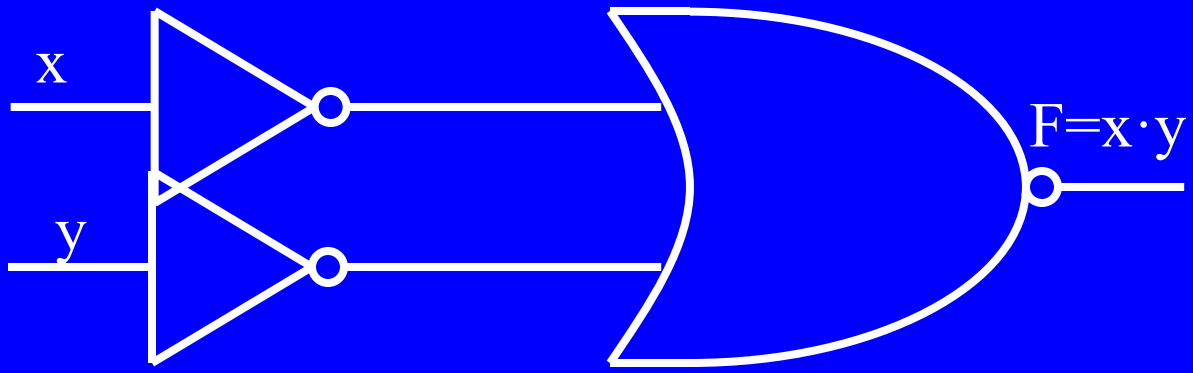
AND with NOR

x	y	F
0	0	1
0	1	0
1	0	0
1	1	0



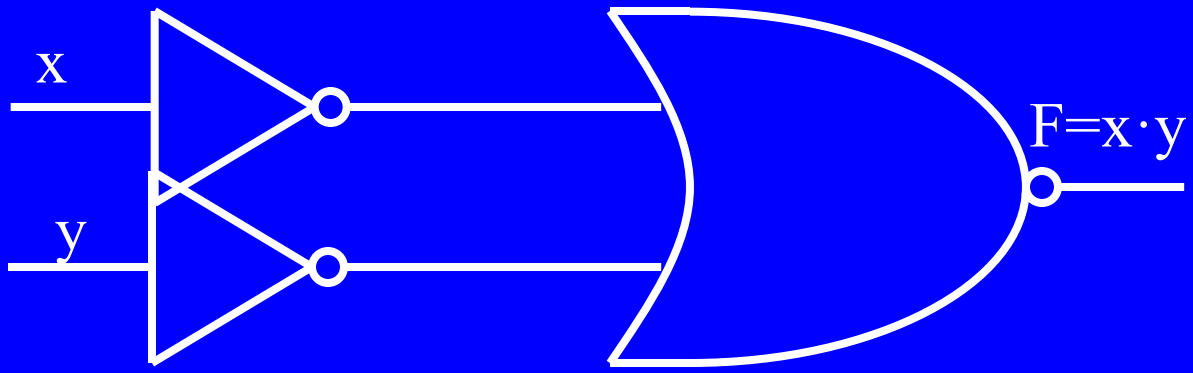
AND with NOR

x	y	F
1	1	1
1	0	0
0	1	0
0	0	0



AND with NOR

x	y	F
0	0	0
0	1	0
1	0	0
1	1	1



Coefficients Calculation Process

$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \cdots + a_2 \cdot r^2 + a_1 \cdot r + a_0$

$(a_n \cdot r^{n-1} + a_{n-1} \cdot r^{n-2} + \cdots + a_2 \cdot r^1 + a_1) \cdot r + a_0$

$a_n \cdot r^{n-1} + a_{n-1} \cdot r^{n-2} + \cdots + a_2 \cdot r^1 + a_1$ $\quad a_0$

$(a_n \cdot r^{n-2} + a_{n-1} \cdot r^{n-3} + \cdots + a_2) \cdot r + a_1$

$a_n \cdot r^{n-2} + a_{n-1} \cdot r^{n-3} + \cdots + a_2$ $\quad a_1$

$\vdots$

$a_n \cdot r + a_{n-1}$

a_n $\quad a_{n-1}$

1 $\quad a_n$