

## Μερικοί ορισμοί

Μια συνάρτηση  $f$  λέμε ότι καλύπτει μια συνάρτηση  $g$ , αν η  $f$  παίρνει την τιμή 1 όταν το ίδιο συμβαίνει με την  $g$ .

Prime implicant  $p$  μιας συναρτήσεως  $f$  είναι ένας όρος γινομένου που καλύπτεται από την  $f$  και η απαλοιφή οποιουδήποτε παράγοντα από την  $p$  δημιουργεί μια συνάρτηση που δεν καλύπτεται από την  $f$ .

# Παράδειγμα

wx \ yz	00	01	11	10
00	0	1	1	1
01	0	1	1	0
11	0	0	1	0
10	0	0	1	0

$$F = w'z + yz + w'x'y$$

# Χάρτης Karnaugh τεσσάρων μεταβλητών

wx \ yz	00	01	11	10
00	0	1	1	1
01	0	1	1	0
11	0	0	1	0
10	0	0	1	0

A green rectangular box highlights the four cells in the third column (yz = 11), where the values are 1, 1, 1, and 1.

# Χάρτης Karnaugh τεσσάρων μεταβλητών

wx \ yz	00	01	11	10
00	0	1	1	1
01	0	1	1	0
11	0	0	1	0
10	0	0	1	0

A green rounded rectangle highlights the 1s in the yz=01 and yz=11 columns, representing the prime implicant  $yz$ .

# Χάρτης Karnaugh τεσσάρων μεταβλητών

wx \ yz	00	01	11	10
00	0	1	1	1
01	0	1	1	0
11	0	0	1	0
10	0	0	1	0

A green rounded rectangle highlights the four cells in the third column (yz = 11), representing a prime implicant.

Άρα το yz είναι prime implicant

# Χάρτης Karnaugh τεσσάρων μεταβλητών

wx \ yz	00	01	11	10
00	0	1	1	1
01	0	1	1	0
11	0	0	1	0
10	0	0	1	0

The image shows a 4x4 Karnaugh map for four variables (w, x, y, z). The vertical axis is labeled 'wx' and the horizontal axis is labeled 'yz'. The values for wx are 00, 01, 11, 10 and for yz are 00, 01, 11, 10. The map contains the following values: (00,00)=0, (00,01)=1, (00,11)=1, (00,10)=1, (01,00)=0, (01,01)=1, (01,11)=1, (01,10)=0, (11,00)=0, (11,01)=0, (11,11)=1, (11,10)=0, (10,00)=0, (10,01)=0, (10,11)=1, (10,10)=0. A red box highlights the two cells at (11,11) and (10,11), which both contain the value 1.

# Χάρτης Karnaugh τεσσάρων μεταβλητών

wx \ yz	00	01	11	10	
00	0	1	1	1	WZ
01	0	1	1	0	
11	0	0	1	0	
10	0	0	1	0	

# Χάρτης Karnaugh τεσσάρων μεταβλητών

wx \ yz	00	01	11	10	wy
00	0	1	1	1	
01	0	1	1	0	
11	0	0	1	0	
10	0	0	1	0	



# Χάρτης Karnaugh τεσσάρων μεταβλητών

wx \ yz	00	01	11	10
00	0	1	1	1
01	0	1	1	0
11	0	0	1	0
10	0	0	1	0

Άρα το  $wyz$  δεν είναι prime implicant

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

	w	x	y	z
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
8	1	0	0	0
5	0	1	0	1
9	1	0	0	1
10	1	0	1	0
7	0	1	1	1
13	1	1	0	1
15	1	1	1	1

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	<del>1</del> <sub>4</sub>

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

	w	x	y	z	
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0	1	0	
8	1	0	0	0	
5	0	1	0	1	
9	1	0	0	1	
10	1	0	1	0	
7	0	1	1	1	
13	1	1	0	1	
15	1	1	1	1	

0,1      w x y z  
0 0 0 -

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

	w	x	y	z	
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0	1	0	✓
8	1	0	0	0	
5	0	1	0	1	
9	1	0	0	1	
10	1	0	1	0	
7	0	1	1	1	
13	1	1	0	1	
15	1	1	1	1	

	w	x	y	z
0,1	0	0	0	-
0,2	0	0	-	0

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z		w	x	y	z	
0	0	0	0	✓	0,1	0	0	-	
1	0	0	1	✓	0,2	0	0	-	0
2	0	0	1	✓	0,8	-	0	0	0
8	1	0	0	✓					
5	0	1	0						
9	1	0	0						
10	1	0	1						
7	0	1	1						
13	1	1	0						
15	1	1	1						

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

Red circles highlight the 1s in the (00,00) and (10,00) cells.

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	0	
10	1	0	1	
7	0	1	1	
13	1	1	0	
15	1	1	1	

w	x	y	z
0,1	0	0	-
0,2	0	0	-0
0,8	-	0	00
1,5	0	-	01

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

4-14

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	1	✓
10	1	0	1	
7	0	1	1	
13	1	1	0	
15	1	1	1	

w	x	y	z
0,1	0	0	-
0,2	0	0	-0
0,8	-	0	00
1,5	0	-	01
1,9	-	0	01

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1 <sub>4-15</sub>

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	1	✓
10	1	0	1	✓
7	0	1	1	
13	1	1	0	
15	1	1	1	

w	x	y	z
0,1	0	0	-
0,2	0	0	-0
0,8	-	0	00
1,5	0	-	01
1,9	-	0	01
2,10	-	0	10

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

Red circles highlight the 1s in the (00,10) and (10,10) cells, with a '4-16' label below the bottom-right circle.



$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

	w	x	y	z	
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
5	0	1	0	1	✓
9	1	0	0	1	✓
10	1	0	1	0	✓
7	0	1	1	1	
13	1	1	0	1	
15	1	1	1	1	

	w	x	y	z
0,1	0	0	0	-
0,2	0	0	-	0
0,8	-	0	0	0
1,5	0	-	0	1
1,9	-	0	0	1
2,10	-	0	1	0
8,9	1	0	0	-

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	4-17

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	1	✓
10	1	0	1	✓
7	0	1	1	
13	1	1	0	
15	1	1	1	

w	x	y	z
0,1	0	0	-
0,2	0	0	-0
0,8	-	0	00
1,5	0	-	01
1,9	-	0	01
2,10	-	0	10
8,9	1	0	-
8,10	1	0	-0

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

Red markings in the Karnaugh map: a red 'D' shape around the '1' in cell (10,00) and a red 'C' shape around the '1' in cell (10,10). The '1' in cell (10,10) is labeled with '4-18'.

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	1	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	
15	1	1	1	

w	x	y	z
0,1	0	0	-
0,2	0	0	-0
0,8	-	0	00
1,5	0	-	01
1,9	-	0	01
2,10	-	0	10
8,9	1	0	-
8,10	1	0	-0
5,7	0	1	-1

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

4-19

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

<u>0</u> 0000 ✓	0,1	000-
1 0001 ✓	0,2	00-0
2 0010 ✓	<u>0,8</u>	-000
<u>8</u> 1000 ✓	1,5	0-01
5 0101 ✓	1,9	-001
9 1001 ✓	2,10	-010
<u>10</u> 1010 ✓	8,9	100-
7 0111 ✓	8,10	10-0
<u>13</u> 1101 ✓	5,7	01-1
15 1111	5,13	-101

		yz			
		00	01	11	10
wx	00	1	1	0	1
	01	0	1	1	0
	11	0	1	1	0
	10	1	1	0	<del>1</del> <sub>4-10</sub>

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	0	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	✓
15	1	1	1	

w	x	y	z
0,1	0	0	-
0,2	0	0	-0
0,8	-	0	00
1,5	0	-	01
1,9	-	0	01
2,10	-	0	10
8,9	1	0	-
8,10	1	0	-0
5,7	0	1	-1
5,13	-	1	01
9,13	1	-	01

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

4-11

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	1	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	✓
15	1	1	1	✓

w	x	y	z
0,1	0	0	-
0,2	0	0	-0
0,8	-	0	0
1,5	0	-	0
1,9	-	0	0
2,10	-	0	1
8,9	1	0	-
8,10	1	0	-0
5,7	0	1	-
5,13	-	1	0
9,13	1	-	0
7,15	-	1	1

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

4-12

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

<u>0</u> 0000 ✓	0,1	000-
1 0001 ✓	0,2	00-0
<u>2</u> 0010 ✓	<u>0,8</u>	-000
<u>8</u> 1000 ✓	1,5	0-01
5 0101 ✓	1,9	-001
9 1001 ✓	2,10	-010
<u>10</u> 1010 ✓	8,9	100-
<u>7</u> 0111 ✓	8,10	10-0
<u>13</u> 1101 ✓	5,7	01-1
<u>15</u> 1111 ✓	5,13	-101
	9,13	1-01
	<u>7,15</u>	-111
	13,15	11-1

		yz			
		00	01	11	10
wx	00	1	1	0	1
	01	0	1	1	0
	11	0	1	1	0
	10	1	1	0	1

4-13

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	0	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	✓
15	1	1	1	✓

	w	x	y	z	
0,1	0	0	0	-	✓
0,2	0	0	-	0	
0,8	-	0	0	0	
1,5	0	-	0	1	
1,9	-	0	0	1	
2,10	-	0	1	0	
8,9	1	0	0	-	✓
8,10	1	0	-	0	
5,7	0	1	-	1	
5,13	-	1	0	1	
9,13	1	-	0	1	
7,15	-	1	1	1	
13,15	1	1	-	1	

0,1,8,9 -00-

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1



$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	1	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	✓
15	1	1	1	✓

	w	x	y	z
0,1	0	0	0	- ✓
0,2	0	0	-	0 ✓
0,8	-	0	0	0
1,5	0	-	0	1
1,9	-	0	0	1
2,10	-	0	1	0
8,9	1	0	0	- ✓
8,10	1	0	-	0 ✓
5,7	0	1	-	1
5,13	-	1	0	1
9,13	1	-	0	1
7,15	-	1	1	1
13,15	1	1	-	1

	w	x	y	z
0,1,8,9	-	0	0	-
0,2,8,10	-	0	-	0

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	0	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	✓
15	1	1	1	✓

	w	x	y	z
0,1	0	0	0	- ✓
0,2	0	0	-	0 ✓
0,8	-	0	0	0 ✓
1,5	0	-	0	1
1,9	-	0	0	1 ✓
2,10	-	0	1	0
8,9	1	0	0	- ✓
8,10	1	0	-	0 ✓
5,7	0	1	-	1
5,13	-	1	0	1
9,13	1	-	0	1
7,15	-	1	1	1
13,15	1	1	-	1

	w	x	y	z
0,1,8,9	-	0	0	-
0,2,8,10	-	0	-	0

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

Red circles highlight the groups (00,01) and (10,11) in the first and last rows respectively.

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	1	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	✓
15	1	1	1	✓

	w	x	y	z
0,1	0	0	0	- ✓
0,2	0	0	-	0 ✓
0,8	-	0	0	0 ✓
1,5	0	-	0	1
1,9	-	0	0	1 ✓
2,10	-	0	1	0 ✓
8,9	1	0	0	- ✓
8,10	1	0	-	0 ✓
5,7	0	1	-	1
5,13	-	1	0	1
9,13	1	-	0	1
7,15	-	1	1	1
13,15	1	1	-	1

	w	x	y	z
0,1,8,9	-	0	0	-
0,2,8,10	-	0	-	0

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	0	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	✓
15	1	1	1	✓

	w	x	y	z	
0,1	0	0	0	-	✓
0,2	0	0	-	0	✓
0,8	-	0	0	0	✓
1,5	0	-	0	1	✓
1,9	-	0	0	1	✓
2,10	-	0	1	0	✓
8,9	1	0	0	-	✓
8,10	1	0	-	0	✓
5,7	0	1	-	1	
5,13	-	1	0	1	
9,13	1	-	0	1	✓
7,15	-	1	1	1	
13,15	1	1	-	1	

	w	x	y	z	
0,1,8,9	-	0	0	-	
0,2,8,10	-	0	-	0	
1,5,9,13	-	-	0	1	

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

4-18

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	1	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	✓
15	1	1	1	✓

	w	x	y	z	
0,1	0	0	0	-	✓
0,2	0	0	-	0	✓
0,8	-	0	0	0	✓
1,5	0	-	0	1	✓
1,9	-	0	0	1	✓
2,10	-	0	1	0	✓
8,9	1	0	0	-	✓
8,10	1	0	-	0	✓
5,7	0	1	-	1	
5,13	-	1	0	1	✓
9,13	1	-	0	1	✓
7,15	-	1	1	1	
13,15	1	1	-	1	

	w	x	y	z	
0,1,8,9	-	0	0	-	
0,2,8,10	-	0	-	0	
1,5,9,13	-	-	0	1	

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	1	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	✓
15	1	1	1	✓

	w	x	y	z	
0,1	0	0	0	-	✓
0,2	0	0	-	0	✓
0,8	-	0	0	0	✓
1,5	0	-	0	1	✓
1,9	-	0	0	1	✓
2,10	-	0	1	0	✓
8,9	1	0	0	-	✓
8,10	1	0	-	0	✓
5,7	0	1	-	1	✓
5,13	-	1	0	1	✓
9,13	1	-	0	1	✓
7,15	-	1	1	1	
13,15	1	1	-	1	✓

	w	x	y	z	
0,1,8,9	-	0	0	-	
0,2,8,10	-	0	-	0	
1,5,9,13	-	-	0	1	
5,7,13,15	1	-	1	-	

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

4-30

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	1	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	✓
15	1	1	1	✓

	w	x	y	z	
0,1	0	0	0	-	✓
0,2	0	0	-	0	✓
0,8	-	0	0	0	✓
1,5	0	-	0	1	✓
1,9	-	0	0	1	✓
2,10	-	0	1	0	✓
8,9	1	0	0	-	✓
8,10	1	0	-	0	✓
5,7	0	1	-	1	✓
5,13	-	1	0	1	✓
9,13	1	-	0	1	✓
7,15	-	1	1	1	✓
13,15	1	1	-	1	✓

	w	x	y	z	
0,1,8,9	-	0	0	-	
0,2,8,10	-	0	-	0	
1,5,9,13	-	-	0	1	
5,7,13,15	-	1	-	1	

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

4-31

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	1	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	✓
15	1	1	1	✓

	w	x	y	z	
0,1	0	0	0	-	✓
0,2	0	0	-	0	✓
0,8	-	0	0	0	✓
1,5	0	-	0	1	✓
1,9	-	0	0	1	✓
2,10	-	0	1	0	✓
8,9	1	0	0	-	✓
8,10	1	0	-	0	✓
5,7	0	1	-	1	✓
5,13	-	1	0	1	✓
9,13	1	-	0	1	✓
7,15	-	1	1	1	✓
13,15	1	1	-	1	✓

	w	x	y	z	
0,1,8,9	-	0	0	-	
0,2,8,10	-	0	-	0	
1,5,9,13	-	-	0	1	
5,7,13,15	-	1	-	1	

		yz			
		00	01	11	10
wx	00	1	1	0	1
	01	0	1	1	0
	11	0	1	1	0
	10	1	1	0	1

4-32



$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

	w	x	y	z	
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
5	0	1	0	1	✓
9	1	0	0	1	✓
10	1	0	1	0	✓
7	0	1	1	1	✓
13	1	1	0	1	✓
15	1	1	1	1	✓

	w	x	y	z	
0,1	0	0	0	-	✓
0,2	0	0	-	0	✓
0,8	-	0	0	0	✓
1,5	0	-	0	1	✓
1,9	-	0	0	1	✓
2,10	-	0	1	0	✓
8,9	1	0	0	-	✓
8,10	1	0	-	0	✓
5,7	0	1	-	1	✓
5,13	-	1	0	1	✓
9,13	1	-	0	1	✓
7,15	-	1	1	1	✓
13,15	1	1	-	1	✓

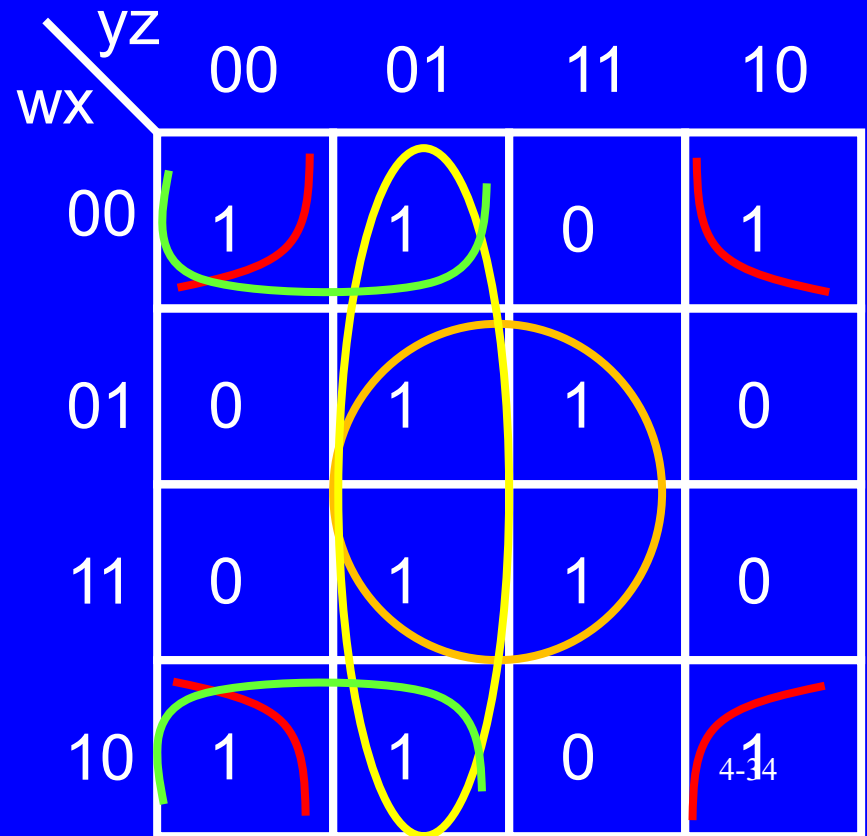
	w	x	y	z	
0,1,8,9	-	0	0	-	<b>A = x'y'</b>
0,2,8,10	-	0	-	0	<b>B = x'z'</b>
1,5,9,13	-	-	0	1	<b>C = y'z</b>
5,7,13,15	-	1	-	1	<b>D = xz</b>

$$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

w	x	y	z	
0	0	0	0	✓
1	0	0	1	✓
2	0	0	1	✓
8	1	0	0	✓
5	0	1	0	✓
9	1	0	1	✓
10	1	0	1	✓
7	0	1	1	✓
13	1	1	0	✓
15	1	1	1	✓

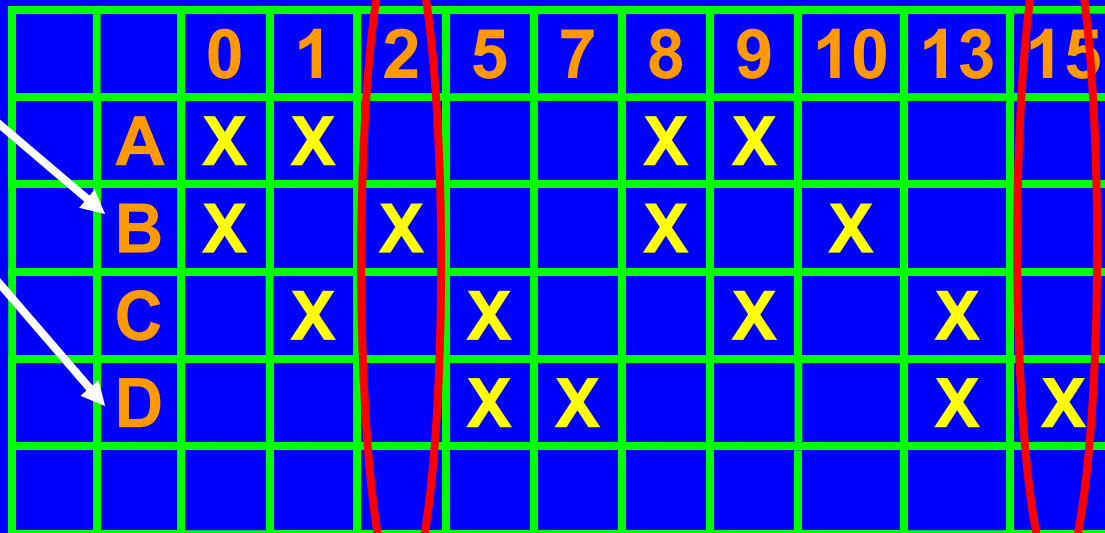
	w	x	y	z	
0,1	0	0	0	-	✓
0,2	0	0	-	0	✓
0,8	-	0	0	0	✓
1,5	0	-	0	1	✓
1,9	-	0	0	1	✓
2,10	-	0	1	0	✓
8,9	1	0	0	-	✓
8,10	1	0	-	0	✓
5,7	0	1	-	1	✓
5,13	-	1	0	1	✓
9,13	1	-	0	1	✓
7,15	-	1	1	1	✓
13,15	1	1	-	1	✓

	w	x	y	z	
0,1,8,9	-	0	0	-	<b>A = x'y'</b>
0,2,8,10	-	0	-	0	<b>B = x'z'</b>
1,5,9,13	-	-	0	1	<b>C = y'z</b>
5,7,13,15	-	1	-	1	<b>D = xz</b>



# Επιλογή prime implicants

Essential (ουσιώδεις)

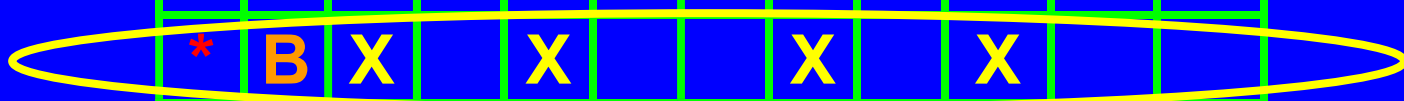


A Karnaugh map with 4 rows (A, B, C, D) and 12 columns (0, 1, 2, 5, 7, 8, 9, 10, 13, 15). The map contains 'X' marks in the following cells: (A,0), (A,1), (A,8), (A,9), (B,0), (B,2), (B,8), (B,10), (C,1), (C,5), (C,9), (C,13), (D,5), (D,7), (D,13), (D,15). Two red ovals highlight the prime implicants (A,0), (A,1), (B,0), (B,2) and (A,8), (A,9), (B,8), (B,10). Two white arrows point from the text 'Essential (ουσιώδεις)' to the cells (A,0) and (A,1).

	0	1	2	5	7	8	9	10	13	15
A	X	X				X	X			
B	X		X			X		X		
C		X		X			X		X	
D				X	X				X	X

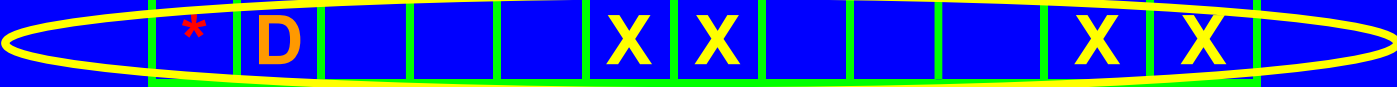
# Επιλογή prime implicants

		0	1	2	5	7	8	9	10	13	15
	A	X	X				X	X			
*	B	X		X			X		X		
	C		X		X			X		X	
*	D				X	X				X	X
		X		X			X		X		



# Επιλογή prime implicants

		0	1	2	5	7	8	9	10	13	15
	A	X	X				X	X			
*	B	X		X			X		X		
	C		X		X			X		X	
*	D				X	X				X	X
		X		X	X	X	X		X	X	X



# Επιλογή prime implicants

		0	1	2	5	7	8	9	10	13	15
✓	A	X	X				X	X			
*	B	X		X			X		X		
	C		X		X			X		X	
*	D				X	X				X	X
		X	X	X	X	X	X	X	X	X	X

# Επιλογή prime implicants

		0	1	2	5	7	8	9	10	13	15
√	A	X	X				X	X			
*	B	X		X			X		X		
	C		X		X			X		X	
*	D				X	X				X	X
		X	X	X	X	X	X	X	X	X	X

$$F = xz + x'z' + x'y'$$

# Επιλογή prime implicants

		0	1	2	5	7	8	9	10	13	15
	A	X	X				X	X			
*	B	X		X			X		X		
✓	C		X		X			X		X	
*	D				X	X				X	X
		X	X	X	X	X	X	X	X	X	X

$$F = xz + x'z' + y'z$$



# Επιλογή prime implicants

$$F = xz + x'z' + x'y'$$

$$F = xz + x'z' + y'z$$

# Επιλογή prime implicants

$$F = xz + x'z' + x'y'$$

$$F = xz + x'z' + y'z$$

$$F = \Sigma(0,1,2,5,7,8,9,10,13,15)$$

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1

2

12

17

18

20

24

13

19

21

25

15

23

27

29

31

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2		
<hr/>		
12		
17	✓	
18		
20		
<hr/>		
24		
13		
19		
21		
<hr/>		
25		
<hr/>		
15		
23		
27		
<hr/>		
29		
<hr/>		
31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<hr/> 12		
17	✓	
18	✓	
20		
<hr/> 24		
13		
19		
21		
<hr/> 25		
15		
23		
27		
<hr/> 29		
31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<u>12</u>	✓	<u>12,13 (1)</u>
17	✓	
18	✓	
20		
<u>24</u>		
13	✓	
19		
21		
<u>25</u>		
15		
23		
27		
<u>29</u>		
31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<hr/> 12	✓	<hr/> 12,13 (1)
17	✓	17,19 (2)
18	✓	
20		
<hr/> 24		
13	✓	
19	✓	
21		
<hr/> 25		
15		
23		
27		
<hr/> 29		
31		



$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<hr/> 12	✓	<hr/> 12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20		
<hr/> 24		
13	✓	
19	✓	
21	✓	
<hr/> 25		
15		
23		
27		
<hr/> 29		
31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<hr/>		<hr/>
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20		17,25 (8)
<hr/>		
24		
<hr/>		
13	✓	
19	✓	
21	✓	
25	✓	
<hr/>		
15		
23		
27		
<hr/>		
29		
<hr/>		
31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<hr/> 12	✓	<hr/> 12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20		17,25 (8)
<hr/> 24		<hr/> 18,19 (1)
13	✓	
19	✓	
21	✓	
<hr/> 25	✓	
15		
23		
27		
<hr/> 29		
31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<hr/> 12	✓	<hr/> 12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
<hr/> 24		18,19 (1)
13	✓	20,21 (1)
19	✓	
21	✓	
<hr/> 25	✓	
15		
23		
27		
<hr/> 29		
31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<hr/> 12	✓	<hr/> 12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
<hr/> 24	✓	<hr/> 18,19 (1)
13	✓	20,21 (1)
19	✓	24,25 (1)
21	✓	
<hr/> 25	✓	
<hr/> 15		
23		
27		
<hr/> 29		
<hr/> 31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<u>12</u>	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
<u>24</u>	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	<u>24,25 (1)</u>
21	✓	13,15 (2)
<u>25</u>	✓	
<u>15</u>	✓	
23		
27		
<u>29</u>		
31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<u>12</u>	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
<u>24</u>	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	<u>24,25 (1)</u>
21	✓	13,15 (2)
<u>25</u>	✓	13,29 (16)
<u>15</u>	✓	
23		
27		
<u>29</u>	✓	
31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<u>12</u>	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
<u>24</u>	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	<u>24,25 (1)</u>
21	✓	13,15 (2)
<u>25</u>	✓	13,29 (16)
15	✓	19,23 (4)
23	✓	
27		
<u>29</u>	✓	
31		



$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<u>12</u>	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
<u>24</u>	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	<u>24,25 (1)</u>
21	✓	13,15 (2)
<u>25</u>	✓	13,29 (16)
15	✓	19,23 (4)
23	✓	19,27 (8)
27	✓	
<u>29</u>	✓	
31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<u>12</u>	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
<u>24</u>	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	<u>24,25 (1)</u>
21	✓	13,15 (2)
<u>25</u>	✓	13,29 (16)
15	✓	19,23 (4)
23	✓	19,27 (8)
27	✓	21,23 (2)
<u>29</u>	✓	
31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<u>12</u>	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
<u>24</u>	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	<u>24,25 (1)</u>
21	✓	13,15 (2)
<u>25</u>	✓	13,29 (16)
15	✓	19,23 (4)
23	✓	19,27 (8)
27	✓	21,23 (2)
<u>29</u>	✓	21,29 (8)
31		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)
2	✓	2,18 (16)
<u>12</u>	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
<u>24</u>	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	<u>24,25 (1)</u>
21	✓	13,15 (2)
<u>25</u>	✓	13,29 (16)
15	✓	19,23 (4)
23	✓	19,27 (8)
27	✓	21,23 (2)
<u>29</u>	✓	21,29 (8)
31		25,27 (2)

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4)
2	✓	2,18 (16)	
<u>12</u>	✓	12,13 (1)	
17	✓	17,19 (2)	
18	✓	17,21 (4)	
20	✓	17,25 (8)	
<u>24</u>	✓	18,19 (1)	
13	✓	20,21 (1)	
19	✓	<u>24,25 (1)</u>	
21	✓	13,15 (2)	
<u>25</u>	✓	13,29 (16)	
15	✓	19,23 (4)	
23	✓	19,27 (8)	
27	✓	21,23 (2)	
<u>29</u>	✓	21,29 (8)	
31		25,27 (2)	

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	<u>25,29 (4)</u>
2	✓	<u>2,18 (16)</u>	15,31 (16)
12	✓	12,13 (1)	
17	✓	17,19 (2)	
18	✓	17,21 (4)	
20	✓	17,25 (8)	
<u>24</u>	✓	18,19 (1)	
13	✓	20,21 (1)	
19	✓	<u>24,25 (1)</u>	
21	✓	13,15 (2)	
<u>25</u>	✓	13,29 (16)	
15	✓	19,23 (4)	
23	✓	19,27 (8)	
27	✓	21,23 (2)	
<u>29</u>	✓	21,29 (8)	
31	✓	25,27 (2)	

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	<u>25,29 (4)</u>
2	✓	<u>2,18 (16)</u>	15,31 (16)
12	✓	12,13 (1)	23,31 (8)
17	✓	17,19 (2)	
18	✓	17,21 (4)	
20	✓	17,25 (8)	
<u>24</u>	✓	18,19 (1)	
13	✓	20,21 (1)	
19	✓	<u>24,25 (1)</u>	
21	✓	13,15 (2)	
<u>25</u>	✓	13,29 (16)	
15	✓	19,23 (4)	
23	✓	19,27 (8)	
27	✓	21,23 (2)	
<u>29</u>	✓	21,29 (8)	
31	✓	25,27 (2)	

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	<u>25,29 (4)</u>
2	✓	<u>2,18 (16)</u>	15,31 (16)
12	✓	12,13 (1)	23,31 (8)
17	✓	17,19 (2)	27,31 (4)
18	✓	17,21 (4)	
20	✓	17,25 (8)	
<u>24</u>	✓	18,19 (1)	
13	✓	20,21 (1)	
19	✓	<u>24,25 (1)</u>	
21	✓	13,15 (2)	
<u>25</u>	✓	13,29 (16)	
15	✓	19,23 (4)	
23	✓	19,27 (8)	
27	✓	21,23 (2)	
<u>29</u>	✓	21,29 (8)	
31	✓	25,27 (2)	



$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

<u>1</u>	✓	1,17 (16)	<u>25,29 (4)</u>
2	✓	<u>2,18 (16)</u>	15,31 (16)
12	✓	12,13 (1)	23,31 (8)
17	✓	17,19 (2)	27,31 (4)
18	✓	17,21 (4)	29,31 (2)
20	✓	17,25 (8)	
<u>24</u>	✓	18,19 (1)	
13	✓	20,21 (1)	
19	✓	<u>24,25 (1)</u>	
21	✓	13,15 (2)	
<u>25</u>	✓	13,29 (16)	
15	✓	19,23 (4)	
23	✓	19,27 (8)	
27	✓	21,23 (2)	
<u>29</u>	✓	21,29 (8)	
31	✓	25,27 (2)	

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	<u>25,29 (4)</u>
2	✓	<u>2,18 (16)</u>	15,31 (16)
12	✓	12,13 (1)	23,31 (8)
17	✓	17,19 (2) ✓	27,31 (4)
18	✓	17,21 (4)	29,31 (2)
20	✓	17,25 (8)	
<u>24</u>	✓	18,19 (1)	17,19,21,23 (2,4)
13	✓	20,21 (1)	
19	✓	<u>24,25 (1)</u>	
21	✓	13,15 (2)	
<u>25</u>	✓	13,29 (16)	
15	✓	19,23 (4)	
23	✓	19,27 (8)	
27	✓	21,23 (2) ✓	
<u>29</u>	✓	21,29 (8)	
31	✓	25,27 (2)	

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	<u>25,29 (4)</u>
2	✓	<u>2,18 (16)</u>	15,31 (16)
12	✓	12,13 (1)	23,31 (8)
17	✓	17,19 (2) ✓	27,31 (4)
18	✓	17,21 (4)	29,31 (2)
20	✓	17,25 (8)	
<u>24</u>	✓	18,19 (1)	17,19,21,23 (2,4)
13	✓	20,21 (1)	17,19,25,27 (2,8)
19	✓	<u>24,25 (1)</u>	
21	✓	13,15 (2)	
<u>25</u>	✓	13,29 (16)	
15	✓	19,23 (4)	
23	✓	19,27 (8)	
27	✓	21,23 (2) ✓	
<u>29</u>	✓	21,29 (8)	
31	✓	25,27 (2) ✓	

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)		<u>25,29 (4)</u>
2	✓	<u>2,18 (16)</u>		15,31 (16)
12	✓	12,13 (1)		23,31 (8)
17	✓	17,19 (2)	✓	27,31 (4)
18	✓	17,21 (4)	✓	29,31 (2)
20	✓	17,25 (8)		
<u>24</u>	✓	18,19 (1)		17,19,21,23 (2,4)
13	✓	20,21 (1)		17,19,25,27 (2,8)
19	✓	<u>24,25 (1)</u>		
21	✓	13,15 (2)		
<u>25</u>	✓	13,29 (16)		
15	✓	19,23 (4)	✓	
23	✓	19,27 (8)		
27	✓	21,23 (2)	✓	
<u>29</u>	✓	21,29 (8)		
31	✓	25,27 (2)	✓	

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4) ✓
2	✓	2,18 (16)	15,31 (16)
12	✓	12,13 (1)	23,31 (8)
17	✓	17,19 (2) ✓	27,31 (4)
18	✓	17,21 (4) ✓	29,31 (2)
20	✓	17,25 (8)	
24	✓	18,19 (1)	17,19,21,23 (2,4)
13	✓	20,21 (1)	17,19,25,27 (2,8)
19	✓	24,25 (1)	17,21,25,29 (4,8)
21	✓	13,15 (2)	
25	✓	13,29 (16)	
15	✓	19,23 (4) ✓	
23	✓	19,27 (8)	
27	✓	21,23 (2) ✓	
29	✓	21,29 (8)	
31	✓	25,27 (2) ✓	

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4) ✓
2	✓	2,18 (16)	15,31 (16)
12	✓	12,13 (1)	23,31 (8)
17	✓	17,19 (2) ✓	27,31 (4)
18	✓	17,21 (4) ✓	29,31 (2)
20	✓	17,25 (8) ✓	
24	✓	18,19 (1)	17,19,21,23 (2,4)
13	✓	20,21 (1)	17,19,25,27 (2,8)
19	✓	24,25 (1)	17,21,25,29 (4,8)
21	✓	13,15 (2)	
25	✓	13,29 (16)	
15	✓	19,23 (4) ✓	
23	✓	19,27 (8) ✓	
27	✓	21,23 (2) ✓	
29	✓	21,29 (8)	
31	✓	25,27 (2) ✓	

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4) ✓
2	✓	2,18 (16)	15,31 (16)
12	✓	12,13 (1)	23,31 (8)
17	✓	17,19 (2) ✓	27,31 (4)
18	✓	17,21 (4) ✓	29,31 (2)
20	✓	17,25 (8) ✓	
24	✓	18,19 (1)	17,19,21,23 (2,4)
13	✓	20,21 (1)	17,19,25,27 (2,8)
19	✓	24,25 (1)	17,21,25,29 (4,8)
21	✓	13,15 (2)	
25	✓	13,29 (16)	
15	✓	19,23 (4) ✓	
23	✓	19,27 (8) ✓	
27	✓	21,23 (2) ✓	
29	✓	21,29 (8) ✓	
31	✓	25,27 (2) ✓	

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4) ✓
2	✓	2,18 (16)	15,31 (16)
12	✓	12,13 (1)	23,31 (8)
17	✓	17,19 (2) ✓	27,31 (4)
18	✓	17,21 (4) ✓	29,31 (2) ✓
20	✓	17,25 (8) ✓	
24	✓	18,19 (1)	17,19,21,23 (2,4)
13	✓	20,21 (1)	17,19,25,27 (2,8)
19	✓	24,25 (1)	17,21,25,29 (4,8)
21	✓	13,15 (2) ✓	13,15,29,31 (2,16)
25	✓	13,29 (16)	
15	✓	19,23 (4) ✓	
23	✓	19,27 (8) ✓	
27	✓	21,23 (2) ✓	
29	✓	21,29 (8) ✓	
31	✓	25,27 (2) ✓	



$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4)	✓
2	✓	2,18 (16)	15,31 (16)	✓
12	✓	12,13 (1)	23,31 (8)	
17	✓	17,19 (2) ✓	27,31 (4)	
18	✓	17,21 (4) ✓	29,31 (2) ✓	
20	✓	17,25 (8) ✓		
24	✓	18,19 (1)	17,19,21,23 (2,4)	
13	✓	20,21 (1)	17,19,25,27 (2,8)	
19	✓	24,25 (1)	17,21,25,29 (4,8)	
21	✓	13,15 (2) ✓	13,15,29,31 (2,16)	
25	✓	13,29 (16) ✓		
15	✓	19,23 (4) ✓		
23	✓	19,27 (8) ✓		
27	✓	21,23 (2) ✓		
29	✓	21,29 (8) ✓		
31	✓	25,27 (2) ✓		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4)	✓
2	✓	2,18 (16)	15,31 (16)	✓
12	✓	12,13 (1)	23,31 (8)	
17	✓	17,19 (2) ✓	27,31 (4) ✓	
18	✓	17,21 (4) ✓	29,31 (2) ✓	
20	✓	17,25 (8) ✓		
24	✓	18,19 (1)	17,19,21,23 (2,4)	
13	✓	20,21 (1)	17,19,25,27 (2,8)	
19	✓	24,25 (1)	17,21,25,29 (4,8)	
21	✓	13,15 (2) ✓	13,15,29,31 (2,16)	
25	✓	13,29 (16) ✓	19,23,27,31 (4,8)	
15	✓	19,23 (4) ✓		
23	✓	19,27 (8) ✓		
27	✓	21,23 (2) ✓		
29	✓	21,29 (8) ✓		
31	✓	25,27 (2) ✓		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4)	✓
2	✓	2,18 (16)	15,31 (16)	✓
12	✓	12,13 (1)	23,31 (8)	✓
17	✓	17,19 (2) ✓	27,31 (4) ✓	
18	✓	17,21 (4) ✓	29,31 (2) ✓	
20	✓	17,25 (8) ✓		
24	✓	18,19 (1)	17,19,21,23 (2,4)	
13	✓	20,21 (1)	17,19,25,27 (2,8)	
19	✓	24,25 (1)	17,21,25,29 (4,8)	
21	✓	13,15 (2) ✓	13,15,29,31 (2,16)	
25	✓	13,29 (16) ✓	19,23,27,31 (4,8)	
15	✓	19,23 (4) ✓		
23	✓	19,27 (8) ✓		
27	✓	21,23 (2) ✓		
29	✓	21,29 (8) ✓		
31	✓	25,27 (2) ✓		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4)	✓
2	✓	2,18 (16)	15,31 (16)	✓
<u>12</u>	✓	12,13 (1)	23,31 (8)	✓
17	✓	17,19 (2) ✓	27,31 (4) ✓	
18	✓	17,21 (4) ✓	29,31 (2) ✓	
20	✓	17,25 (8) ✓		
<u>24</u>	✓	18,19 (1)	17,19,21,23 (2,4)	
13	✓	20,21 (1)	17,19,25,27 (2,8)	
19	✓	<u>24,25 (1)</u>	<u>17,21,25,29 (4,8)</u>	
21	✓	13,15 (2) ✓	13,15,29,31 (2,16)	
<u>25</u>	✓	13,29 (16) ✓	19,23,27,31 (4,8)	
15	✓	19,23 (4) ✓	21,23,29,31 (2,8)	
23	✓	19,27 (8) ✓		
27	✓	21,23 (2) ✓		
<u>29</u>	✓	21,29 (8) ✓		
31	✓	25,27 (2) ✓		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4)	✓
2	✓	2,18 (16)	15,31 (16)	✓
12	✓	12,13 (1)	23,31 (8)	✓
17	✓	17,19 (2) ✓	27,31 (4) ✓	
18	✓	17,21 (4) ✓	29,31 (2) ✓	
20	✓	17,25 (8) ✓		
24	✓	18,19 (1)	17,19,21,23 (2,4)	
13	✓	20,21 (1)	17,19,25,27 (2,8)	
19	✓	24,25 (1)	17,21,25,29 (4,8)	
21	✓	13,15 (2) ✓	13,15,29,31 (2,16)	
25	✓	13,29 (16) ✓	19,23,27,31 (4,8)	
15	✓	19,23 (4) ✓	21,23,29,31 (2,8)	
23	✓	19,27 (8) ✓	25,27,29,31 (2,4)	
27	✓	21,23 (2) ✓		
29	✓	21,29 (8) ✓		
31	✓	25,27 (2) ✓		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4)	✓
2	✓	2,18 (16)	15,31 (16)	✓
12	✓	12,13 (1)	23,31 (8)	✓
17	✓	17,19 (2) ✓	27,31 (4) ✓	
18	✓	17,21 (4) ✓	29,31 (2) ✓	
20	✓	17,25 (8) ✓		
24	✓	18,19 (1)	17,19,21,23 (2,4)	✓
13	✓	20,21 (1)	17,19,25,27 (2,8)	
19	✓	24,25 (1)	17,21,25,29 (4,8)	
21	✓	13,15 (2) ✓	13,15,29,31 (2,16)	
25	✓	13,29 (16) ✓	19,23,27,31 (4,8)	
15	✓	19,23 (4) ✓	21,23,29,31 (2,8)	
23	✓	19,27 (8) ✓	25,27,29,31 (2,4)	✓
27	✓	21,23 (2) ✓		
29	✓	21,29 (8) ✓	17,19,21,23,25,27,29,31 (2,4,8)	
31	✓	25,27 (2) ✓		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4)	✓
2	✓	2,18 (16)	15,31 (16)	✓
12	✓	12,13 (1)	23,31 (8)	✓
17	✓	17,19 (2) ✓	27,31 (4) ✓	
18	✓	17,21 (4) ✓	29,31 (2) ✓	
20	✓	17,25 (8) ✓		
24	✓	18,19 (1)	17,19,21,23 (2,4)	✓
13	✓	20,21 (1)	17,19,25,27 (2,8)	✓
19	✓	24,25 (1)	17,21,25,29 (4,8)	
21	✓	13,15 (2) ✓	13,15,29,31 (2,16)	
25	✓	13,29 (16) ✓	19,23,27,31 (4,8)	
15	✓	19,23 (4) ✓	21,23,29,31 (2,8)	✓
23	✓	19,27 (8) ✓	25,27,29,31 (2,4)	✓
27	✓	21,23 (2) ✓		
29	✓	21,29 (8) ✓	17,19,21,23,25,27,29,31 (2,4,8)	
31	✓	25,27 (2) ✓		

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	25,29 (4)	✓
2	✓	2,18 (16)	15,31 (16)	✓
<u>12</u>	✓	12,13 (1)	23,31 (8)	✓
17	✓	17,19 (2) ✓	27,31 (4) ✓	
18	✓	17,21 (4) ✓	29,31 (2) ✓	
20	✓	17,25 (8) ✓		
<u>24</u>	✓	18,19 (1)	17,19,21,23 (2,4)	✓
13	✓	20,21 (1)	17,19,25,27 (2,8)	✓
19	✓	<u>24,25 (1)</u>	<u>17,21,25,29 (4,8)</u>	✓
21	✓	13,15 (2) ✓	13,15,29,31 (2,16)	
<u>25</u>	✓	13,29 (16) ✓	19,23,27,31 (4,8)	✓
15	✓	19,23 (4) ✓	21,23,29,31 (2,8)	✓
23	✓	19,27 (8) ✓	25,27,29,31 (2,4)	✓
27	✓	21,23 (2) ✓		
<u>29</u>	✓	21,29 (8) ✓	17,19,21,23,25,27,29,31 (2,4,8)	
31	✓	25,27 (2) ✓		



$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	H	25,29 (4)	✓	
2	✓	2,18 (16)	G	15,31 (16)	✓	
12	✓	12,13 (1)	F	23,31 (8)	✓	UWXYZ
17	✓	17,19 (2)	✓	27,31 (4)	✓	10001 (17)
18	✓	17,21 (4)	✓	29,31 (2)	✓	10011 (19)
20	✓	17,25 (8)	✓			10101 (21)
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	10111 (23)
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	11001 (25)
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	11011 (27)
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	11101 (29)
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	11111 (31)
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	H	25,29 (4)	✓	
2	✓	2,18 (16)	G	15,31 (16)	✓	
12	✓	12,13 (1)	F	23,31 (8)	✓	UWXYZ
17	✓	17,19 (2)	✓	27,31 (4)	✓	10001 (17)
18	✓	17,21 (4)	✓	29,31 (2)	✓	10011 (19)
20	✓	17,25 (8)	✓			10101 (21)
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	10111 (23)
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	11001 (25)
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	11011 (27)
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	11101 (29)
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	11111 (31)
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	H	25,29 (4)	✓	<b>A=uz</b> <u>UWXYZ</u> 10001 (17) 10011 (19) 10101 (21) 10111 (23) 11001 (25) 11011 (27) 11101 (29) 11111 (31)
2	✓	2,18 (16)	G	15,31 (16)	✓	
12	✓	12,13 (1)	F	23,31 (8)	✓	
17	✓	17,19 (2)	✓	27,31 (4)	✓	
18	✓	17,21 (4)	✓	29,31 (2)	✓	
20	✓	17,25 (8)	✓			
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz
2	✓	2,18 (16)	G	15,31 (16)	✓	B=wxz
12	✓	12,13 (1)	F	23,31 (8)	✓	
17	✓	17,19 (2)	✓	27,31 (4)	✓	
18	✓	17,21 (4)	✓	29,31 (2)	✓	
20	✓	17,25 (8)	✓			
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	uvwxyz
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	01101 (13)
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	01111 (15)
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	11101 (29)
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	11111 (31)
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz B=wxz C=uwx'y'
2	✓	2,18 (16)	G	15,31 (16)	✓	
12	✓	12,13 (1)	F	23,31 (8)	✓	
17	✓	17,19 (2)	✓	27,31 (4)	✓	
18	✓	17,21 (4)	✓	29,31 (2)	✓	
20	✓	17,25 (8)	✓			
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	UWXYZ
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	11000 (24)
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	11001 (25)
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz
2	✓	2,18 (16)	G	15,31 (16)	✓	B=wxz
12	✓	12,13 (1)	F	23,31 (8)	✓	C=uwx'y'
17	✓	17,19 (2)	✓	27,31 (4)	✓	D=uw'xy'
18	✓	17,21 (4)	✓	29,31 (2)	✓	
20	✓	17,25 (8)	✓			
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	UWXYZ
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	10100 (20)
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	10101 (21)
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz
2	✓	2,18 (16)	G	15,31 (16)	✓	B=wxz
12	✓	12,13 (1)	F	23,31 (8)	✓	C=uwx'y'
17	✓	17,19 (2)	✓	27,31 (4)	✓	D=uw'xy'
18	✓	17,21 (4)	✓	29,31 (2)	✓	E=uw'x'y
20	✓	17,25 (8)	✓			
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	UWXYZ
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	10010 (18)
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	10011 (19)
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz
2	✓	2,18 (16)	G	15,31 (16)	✓	B=wxz
12	✓	12,13 (1)	F	23,31 (8)	✓	C=uwx'y'
17	✓	17,19 (2)	✓	27,31 (4)	✓	D=uw'xy'
18	✓	17,21 (4)	✓	29,31 (2)	✓	E=uw'x'y
20	✓	17,25 (8)	✓			F=u'wxy'
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	UWXYZ
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	01100 (12)
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	01101 (13)
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			



$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz
2	✓	2,18 (16)	G	15,31 (16)	✓	B=wxz
12	✓	12,13 (1)	F	23,31 (8)	✓	C=uwx'y'
17	✓	17,19 (2)	✓	27,31 (4)	✓	D=uw'xy'
18	✓	17,21 (4)	✓	29,31 (2)	✓	E=uw'x'y
20	✓	17,25 (8)	✓			F=u'wxy'
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	G=w'x'yz'
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	UWXYZ
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	00010 (2)
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	10010 (18)
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz
2	✓	2,18 (16)	G	15,31 (16)	✓	B=wxz
12	✓	12,13 (1)	F	23,31 (8)	✓	C=uwx'y'
17	✓	17,19 (2)	✓	27,31 (4)	✓	D=uw'xy'
18	✓	17,21 (4)	✓	29,31 (2)	✓	E=uw'x'y
20	✓	17,25 (8)	✓			F=u'wxy'
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	G=w'x'yz'
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	H=w'x'y'z
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	UWXYZ
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	00001 (1)
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	10001 (17)
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

$$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\phi}(1,2,12,24)$$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz
2	✓	2,18 (16)	G	15,31 (16)	✓	B=wxz
12	✓	12,13 (1)	F	23,31 (8)	✓	C=uwx'y'
17	✓	17,19 (2)	✓	27,31 (4)	✓	D=uw'xy'
18	✓	17,21 (4)	✓	29,31 (2)	✓	E=uw'x'y
20	✓	17,25 (8)	✓			F=u'wxy'
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	G=w'x'yz'
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	H=w'x'y'z
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

# Επιλογή prime implicants

essential



A Karnaugh map with 14 columns (minterms 13, 15, 17, 18, 19, 20, 21, 23, 25, 27, 29, 31) and 8 rows (prime implicants A-H). The map is outlined in red. Three arrows labeled 'essential' point to the first three rows (A, B, D). The cells contain 'X' for minterms covered by the prime implicant.

		13	15	17	18	19	20	21	23	25	27	29	31
*	A			X		X		X	X	X	X	X	X
*	B	X	X									X	X
	C									X			
*	D						X	X					
✓	E				X	X							
	F	X											
	G				X								
	H			X									
		X	X	X	X	X	X	X	X	X	X	X	X

# Επιλογή prime implicants

essential

		13	15	17	18	19	20	21	23	25	27	29	31
*	A			X		X		X	X	X	X	X	X
*	B	X	X									X	X
	C									X			
*	D						X	X					
✓	E				X	X							
	F	X											
	G				X								
	H			X									
		X	X	X	X	X	X	X	X	X	X	X	X

$$F = uz + wxz + uw'xy' + uw'x'y$$

# Binary Decision Diagrams

Σαν binary decision diagram ορίζουμε κατευθυντικό ακυκλικό γράφο με μια αρχή (rooted, directed acyclic graph) ο οποίος έχει έναν ή δύο τερματικούς κόμβους που αντιστοιχούν στο λογικό 0 και στο λογικό 1 και ένα σύνολο από μη τερματικούς κόμβους από τους οποίους ξεκινούν δύο ακμές.

# Binary Decision Diagrams

Στις δύο αυτές ακμές αντιστοιχούν δυο συναρτήσεις  $Fx$  και  $Fx'$  από το θεώρημα του Shannon

$$F(x_1, x_2, \dots, x_i, \dots, x_n) = x_i F(x_1, x_2, \dots, 1, \dots, x_n) + x_i' F(x_1, x_2, \dots, 0, \dots, x_n)$$

Σε κάθε κόμβο αντιστοιχεί μια μεταβλητή

# Binary Decision Diagrams

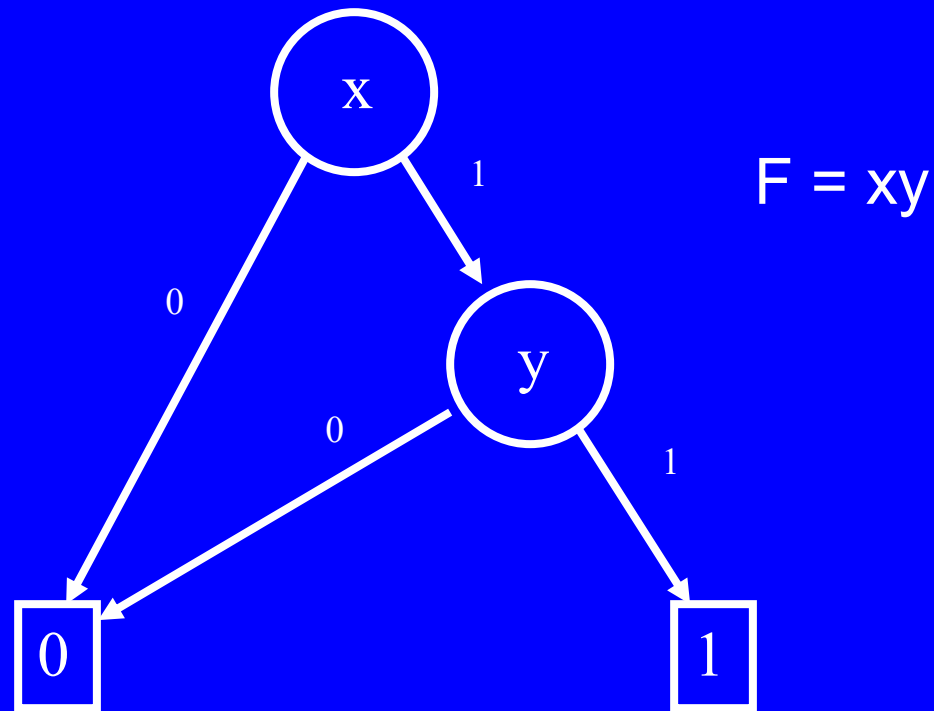
Ένα BDD λέγεται διατεταγμένο αν σε όλες τις διαδρομές μέσα στο γράφο οι μεταβλητές εμφανίζονται πάντα με την ίδια σειρά ( $x_1 < x_2 < \dots < x_i < \dots < x_n$ )

Ένα BDD λέγεται ελαχιστοποιημένο αν:

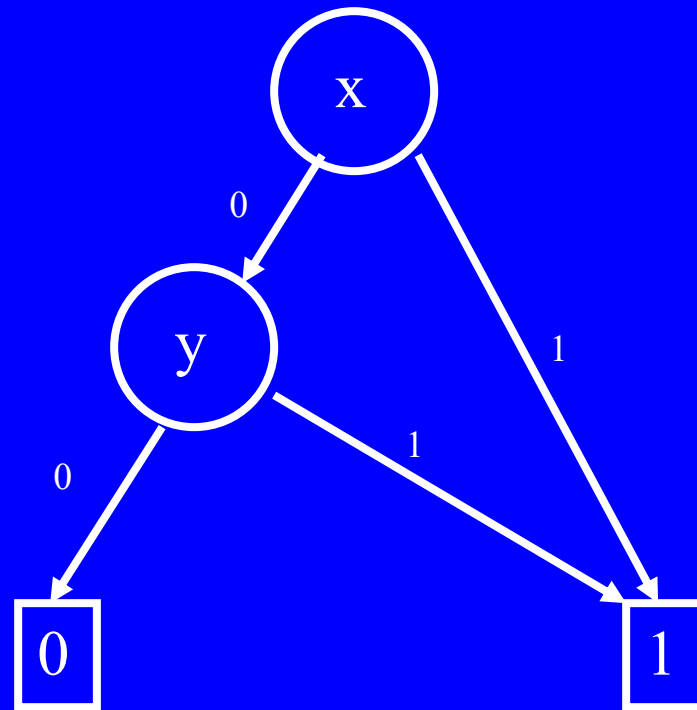
- δεν υπάρχουν δύο κόμβοι που να αντιστοιχούν στην ίδια μεταβλητή και να έχουν τους ίδιους υπογράφους
- κανένας κόμβος δεν έχει δύο ίδιους υπογράφους



# Binary Decision Diagrams



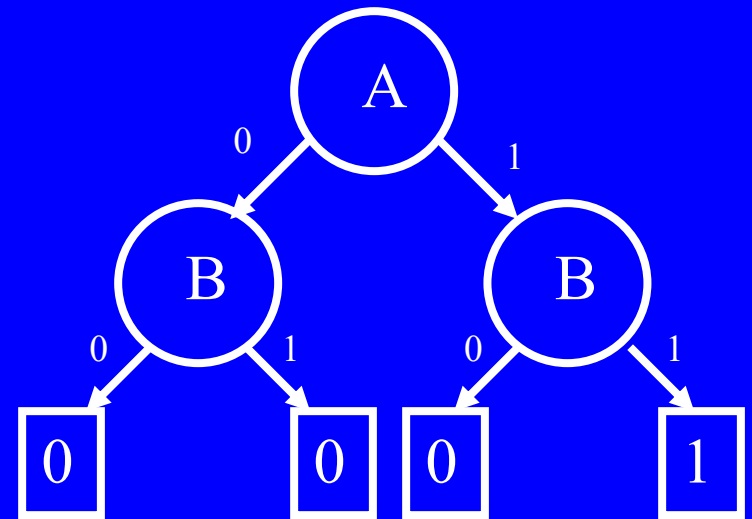
# Binary Decision Diagrams



$$F = x + y$$

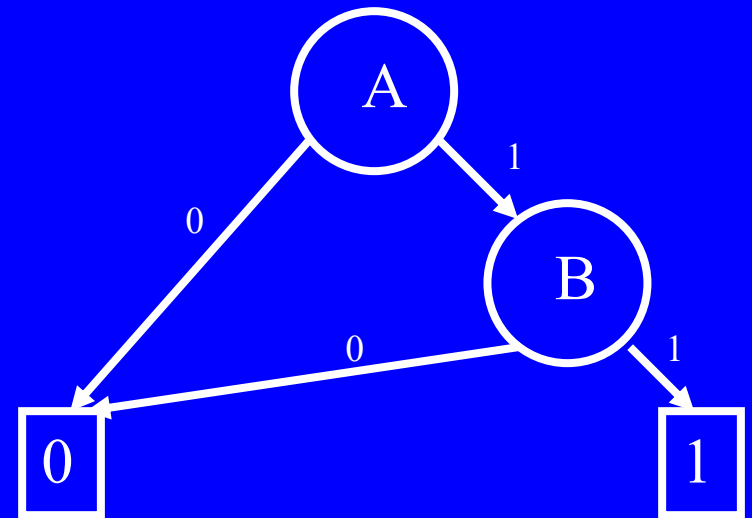
# Binary Decision Diagrams

<b>A</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>1</b>
<b>B</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>
<b>F</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>1</b>



# Binary Decision Diagrams

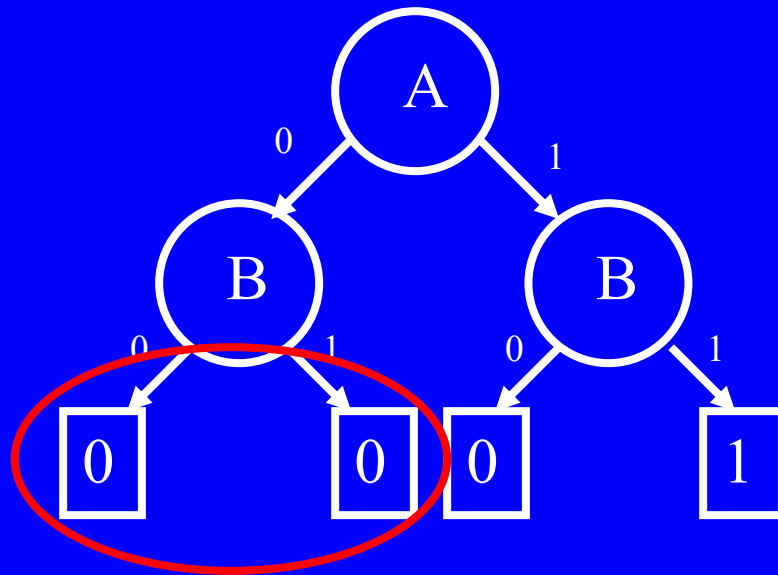
<b>A</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>1</b>
<b>B</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>
<b>F</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>1</b>



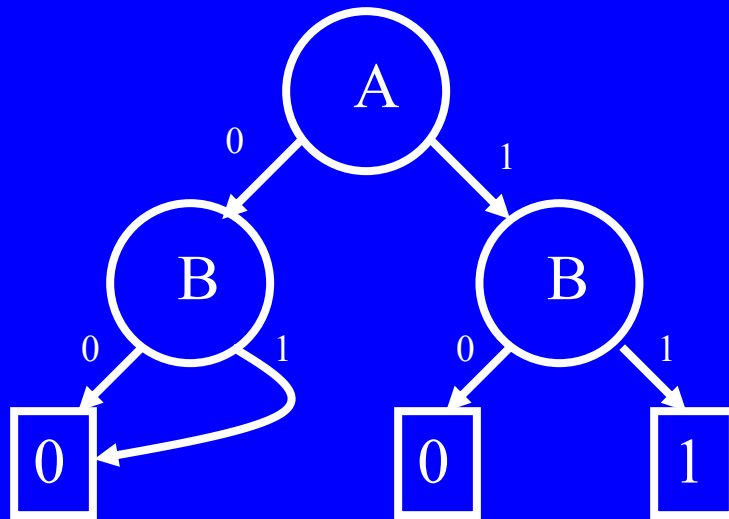
# Διαδικασία απλοποίησης

- ❖ Ενώνουμε τους ίδιους κόμβους
- ❖ Αν και οι δύο ακμές που φεύγουν από ένα κόμβο  $X$  δείχνουν τον ίδιο κόμβο  $Y$ , τότε αφαιρούμε τον κόμβο  $X$  και ενώνουμε κατευθείαν τον κόμβο ανώτερης βαθμίδας με τον  $Y$ .
- ❖ Επαναλαμβάνουμε μέχρι να μην μπορούμε να ενώσουμε ή να αφαιρέσουμε άλλους κόμβους.

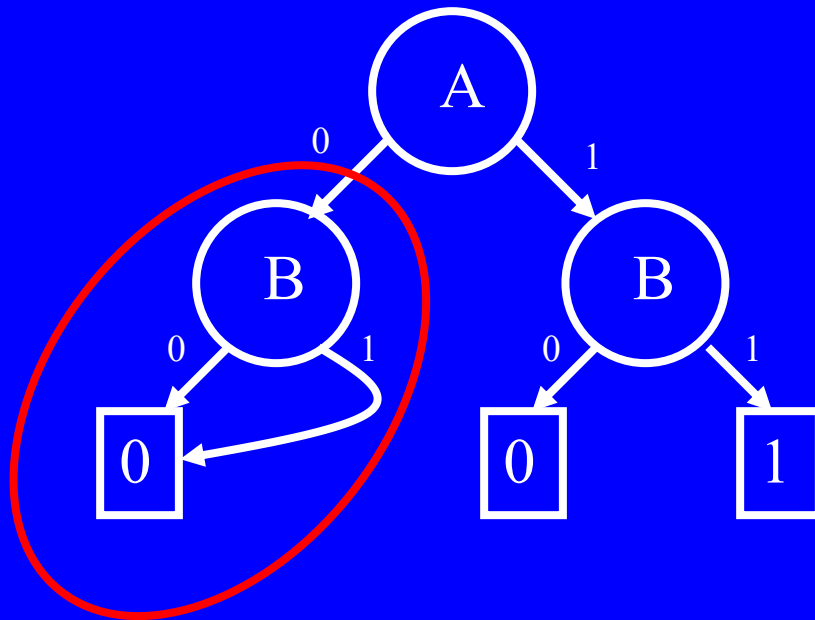
# Διαδικασία απλοποίησης



# Διαδικασία απλοποίησης

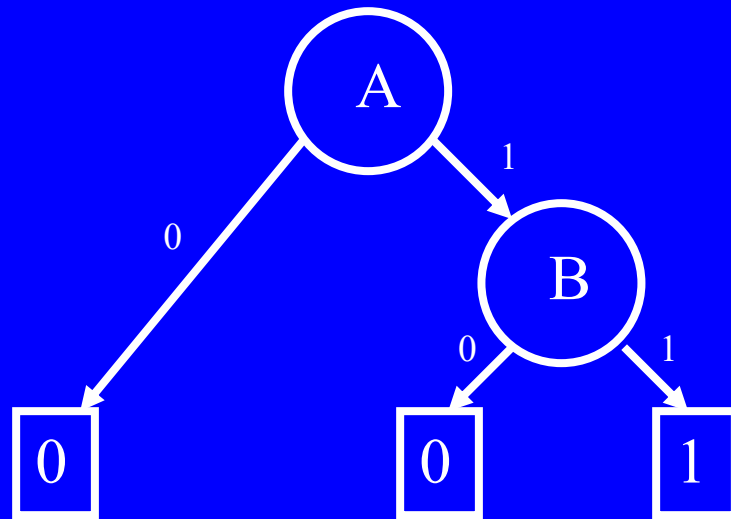


# Διαδικασία απλοποίησης

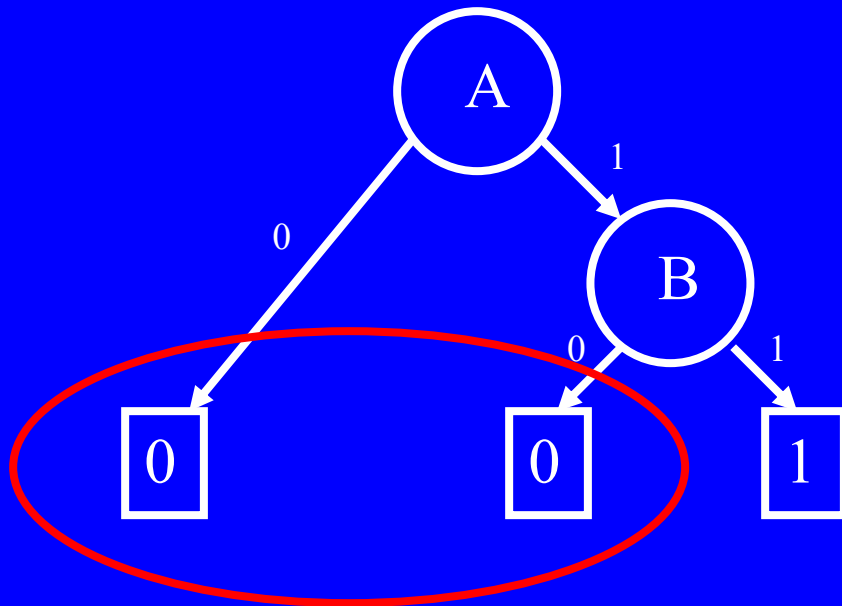




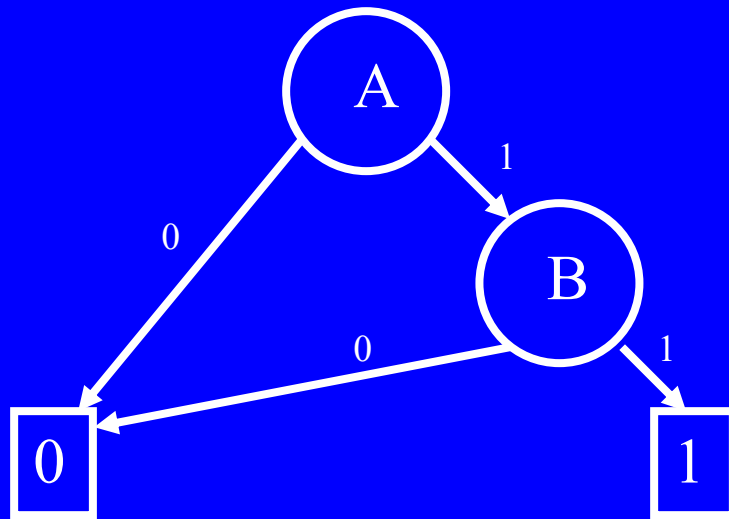
# Διαδικασία απλοποίησης



# Διαδικασία απλοποίησης



# Διαδικασία απλοποίησης

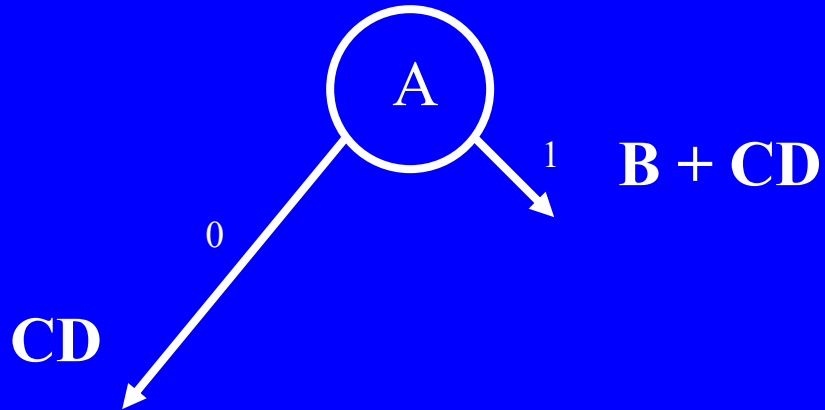


# Παράδειγμα



$$F = AB + CD$$
$$A < B < C < D$$

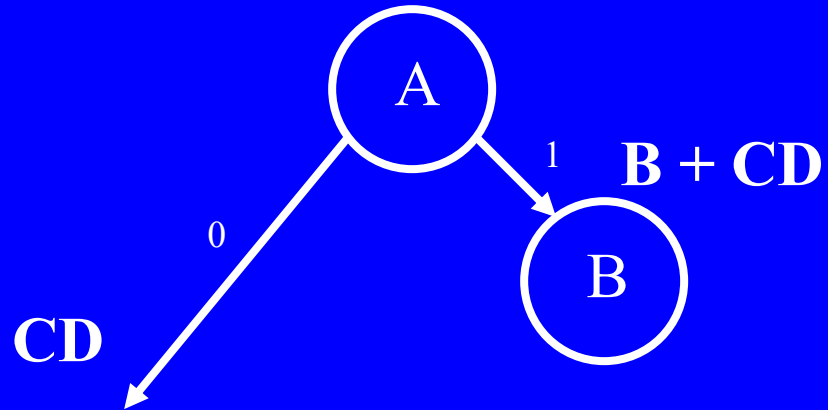
# Παράδειγμα



$$F = AB + CD$$

$$A < B < C < D$$

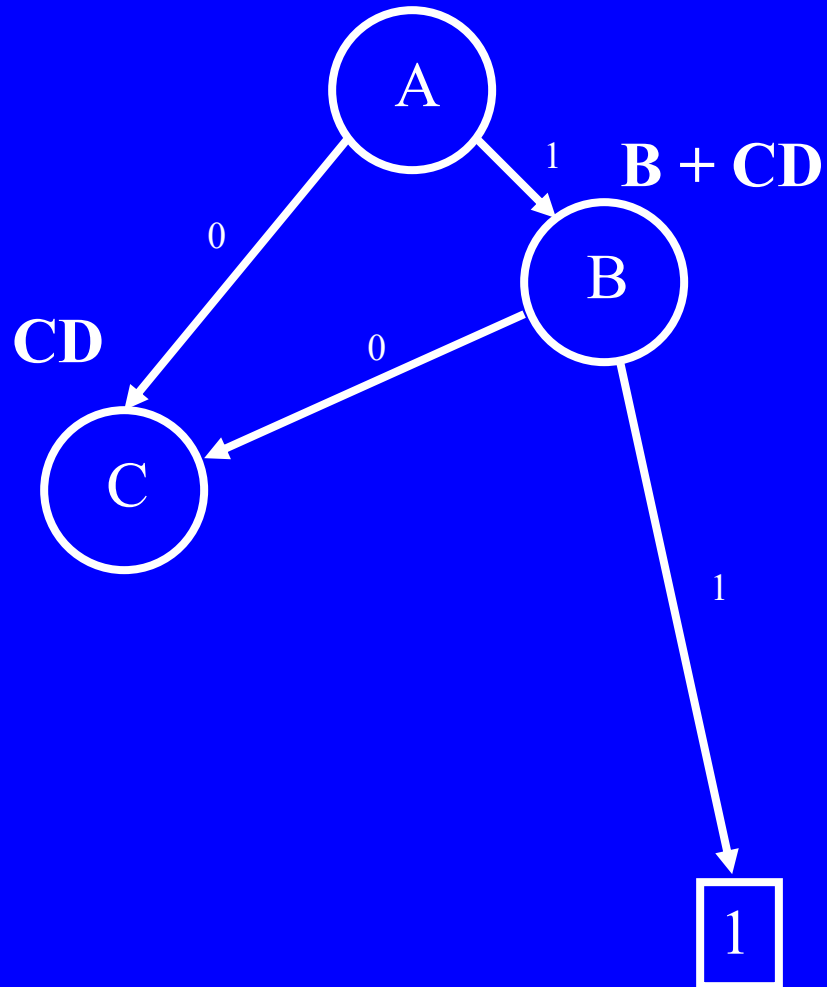
# Παράδειγμα



$$F = AB + CD$$

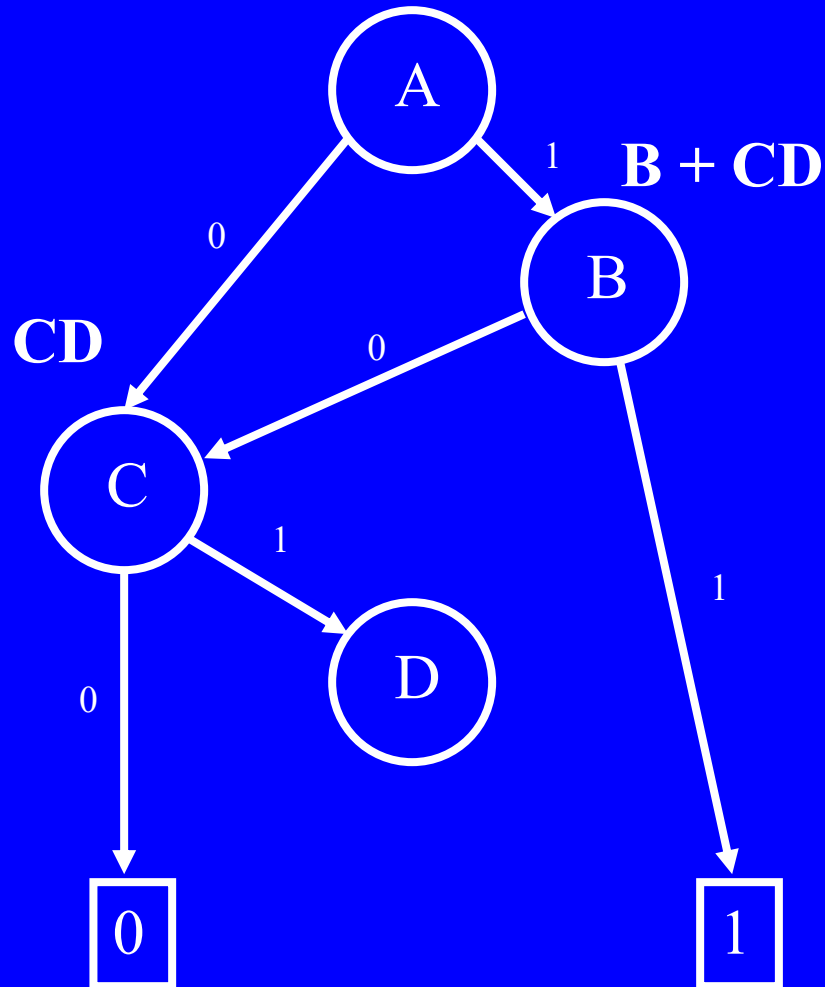
$$A < B < C < D$$

# Παράδειγμα



$$F = AB + CD$$
$$A < B < C < D$$

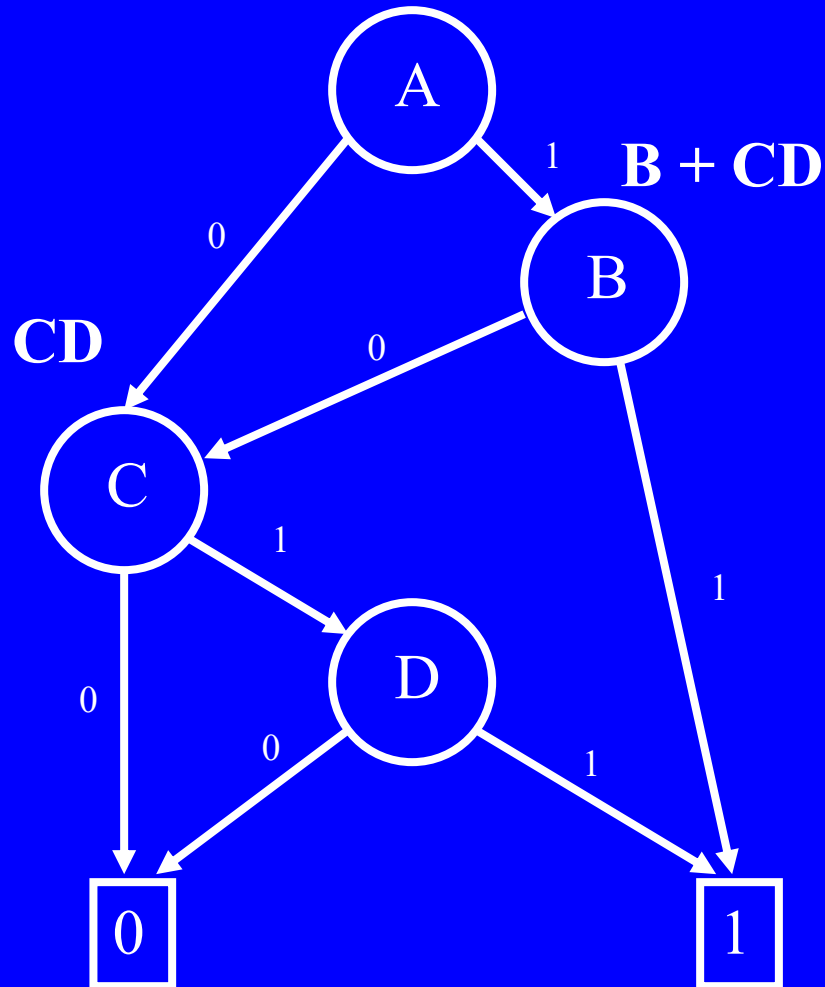
# Παράδειγμα



$$F = AB + CD$$
$$A < B < C < D$$



# Παράδειγμα



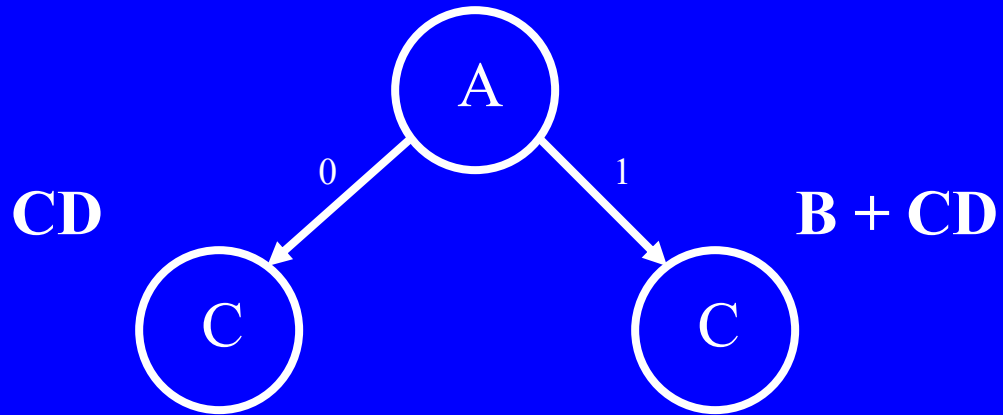
$$F = AB + CD$$
$$A < B < C < D$$

# Άλλο ένα παράδειγμα



$$F = AB + CD$$
$$A < C < B < D$$

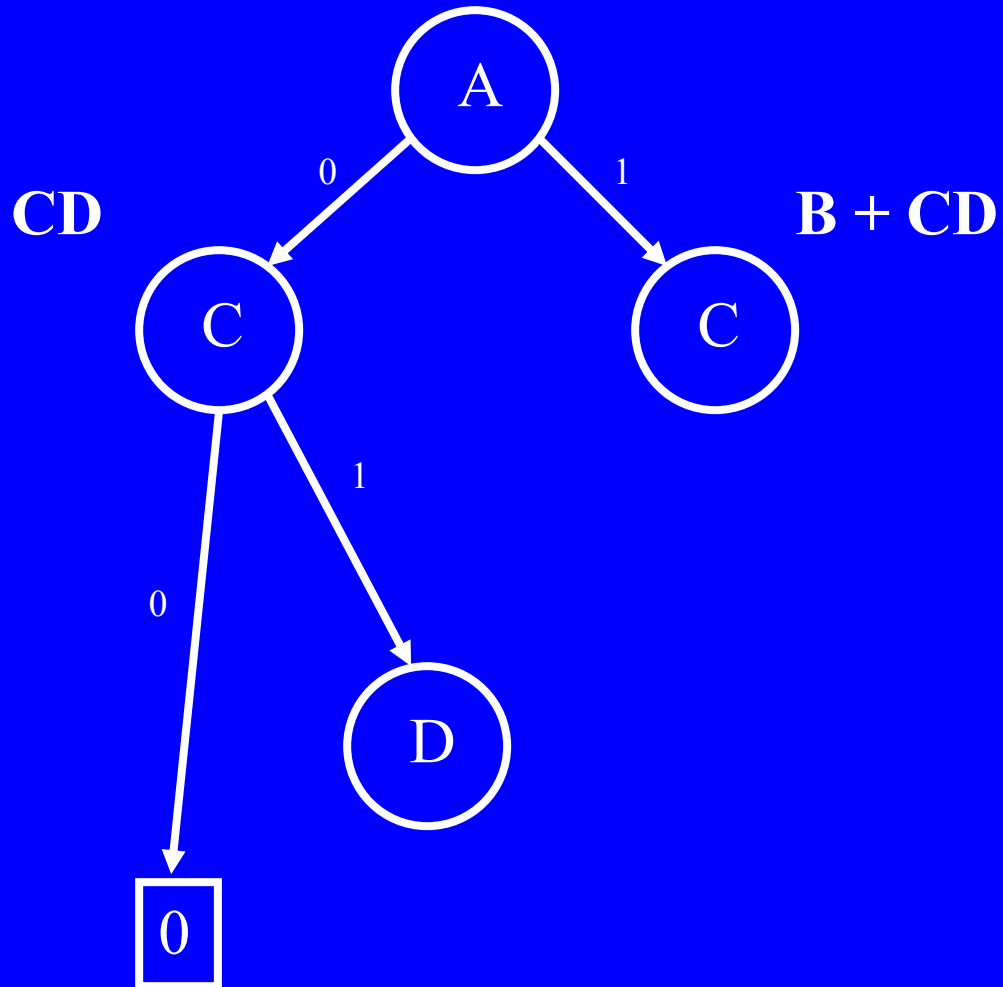
# Άλλο ένα παράδειγμα



$$F = AB + CD$$

$$A < C < B < D$$

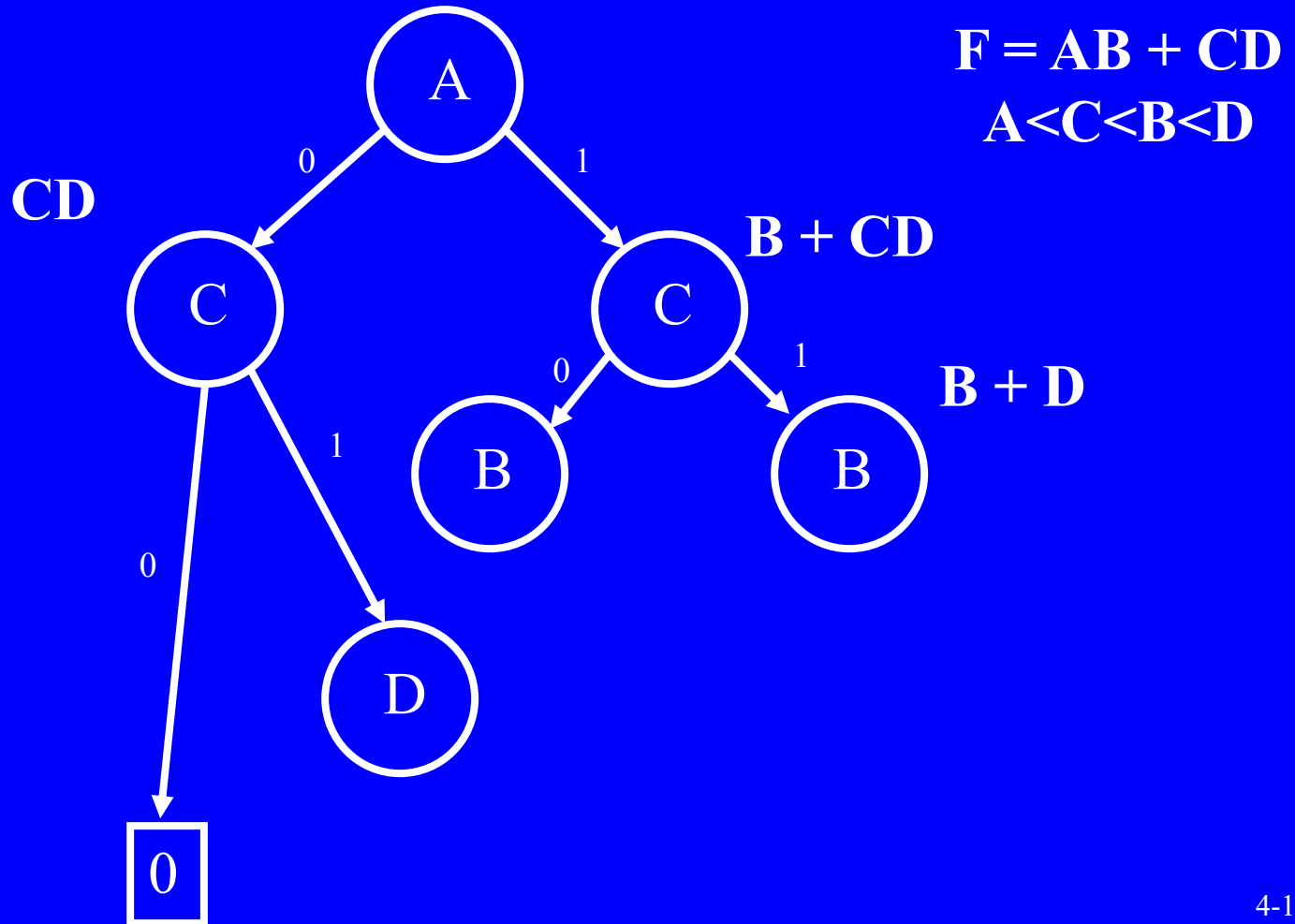
# Άλλο ένα παράδειγμα



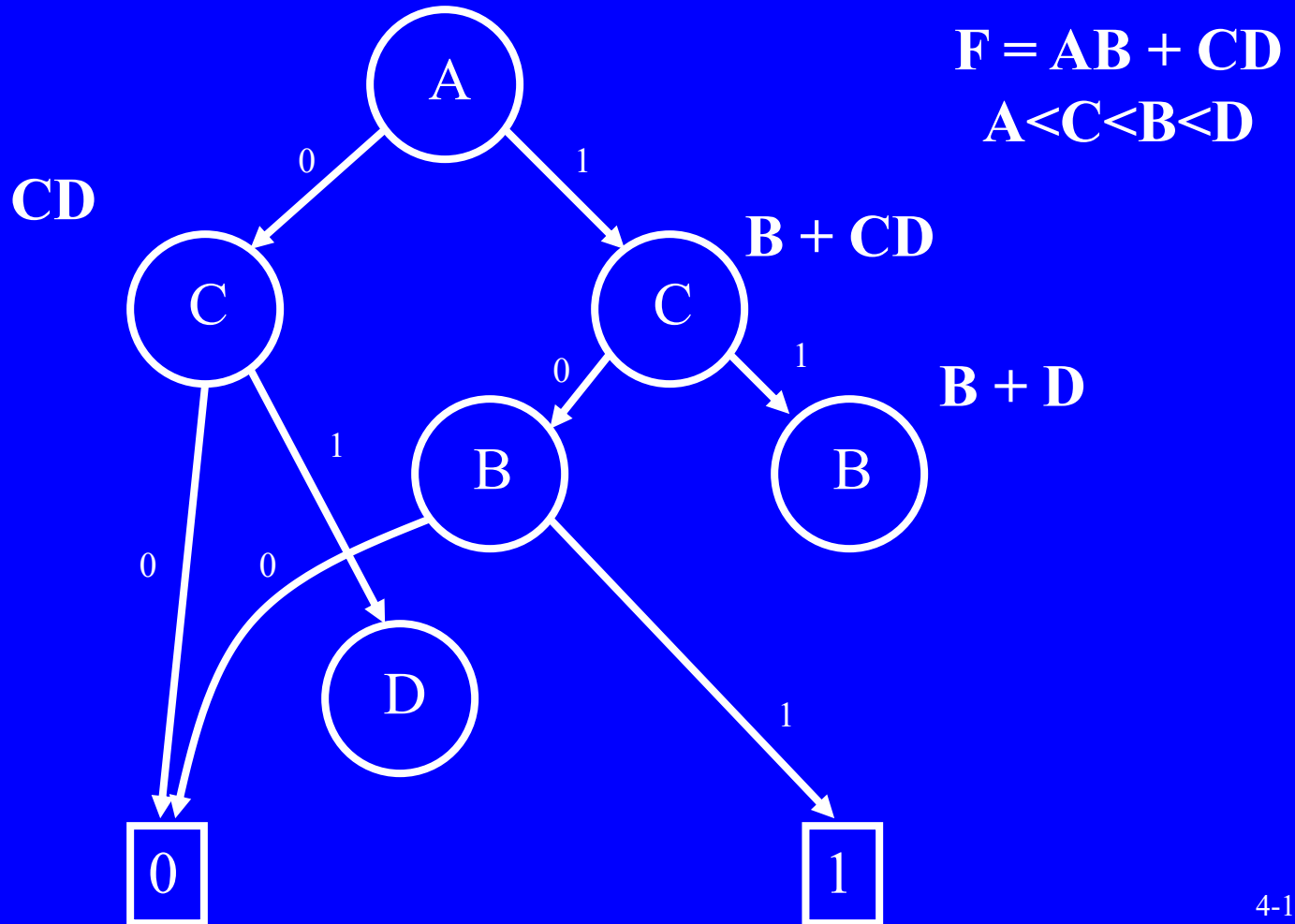
$$F = AB + CD$$

$$A < C < B < D$$

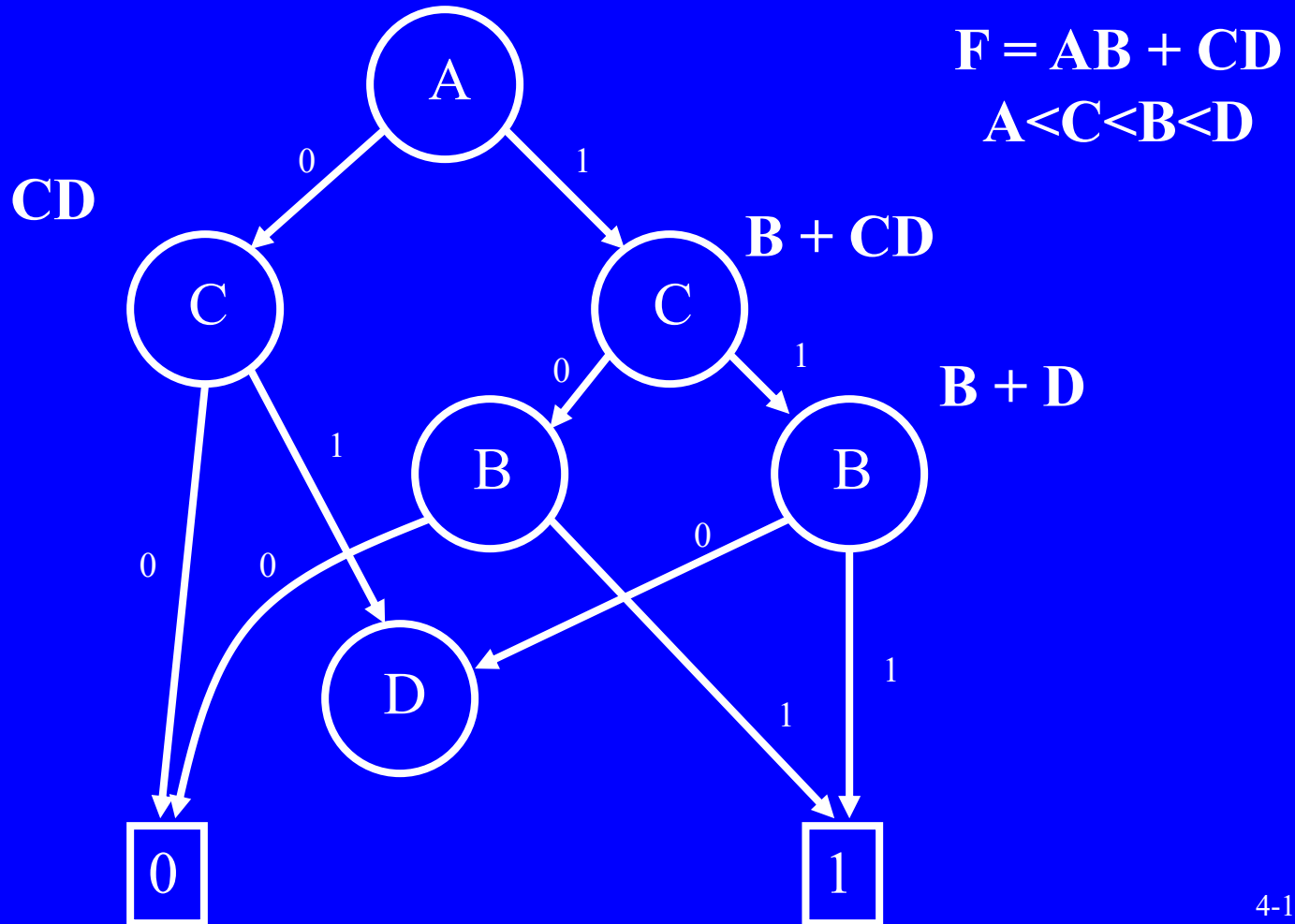
# Άλλο ένα παράδειγμα



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