Computational Intelligence & Machine Learning

Fuzzy Systems

Adopted from Debasis Samanta

Introduction

- Fuzzy logic is a mathematical language to express something.

  This means it has grammar, syntax, semantic like a language for communication.

- There are some other mathematical languages also known

  - **Relational algebra** (operations on sets)
  - **Boolean algebra** (operations on Boolean variables)
  - **Predicate logic** (operations on well formed formulae (wff), also called predicate propositions)

- **Fuzzy logic deals with Fuzzy set.**

Introduction

- First time introduced by Lotfi Abdelli Zadeh (1965), University of California, Berkley, USA (1965).

- He is fondly nick-named as **LAZ**

Introduction

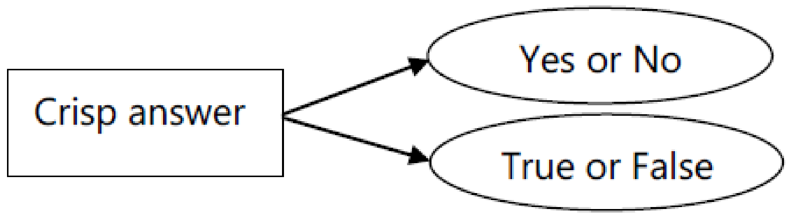
1. Dictionary meaning of **fuzzy** is not clear, noisy etc.

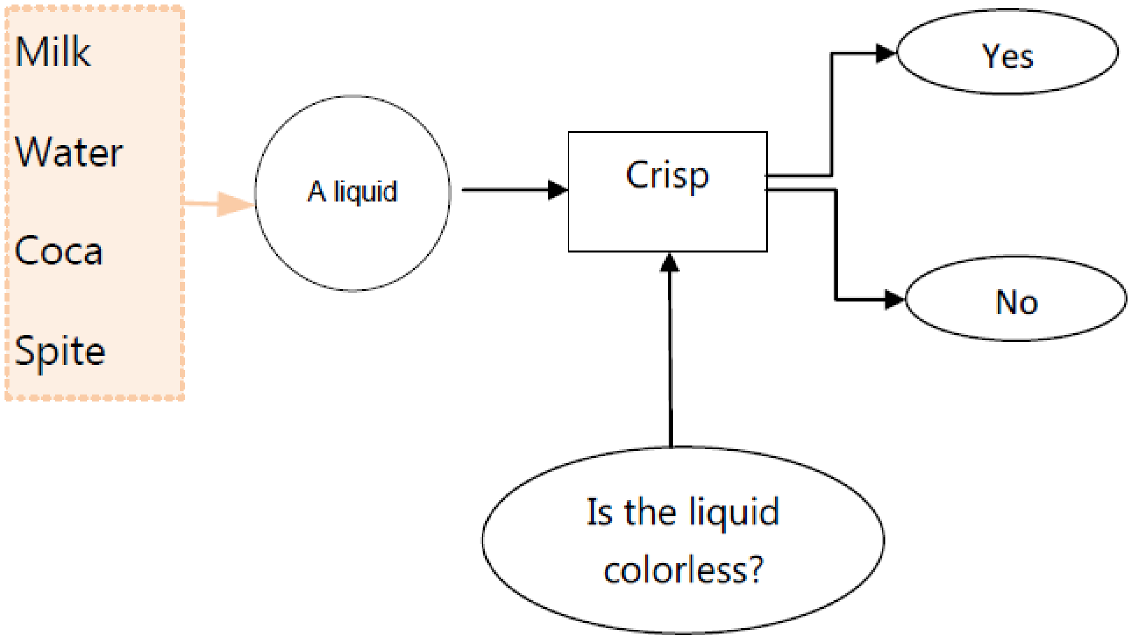
   Example: Is the picture on this slide is fuzzy?

2. Antonym of fuzzy is **crisp**

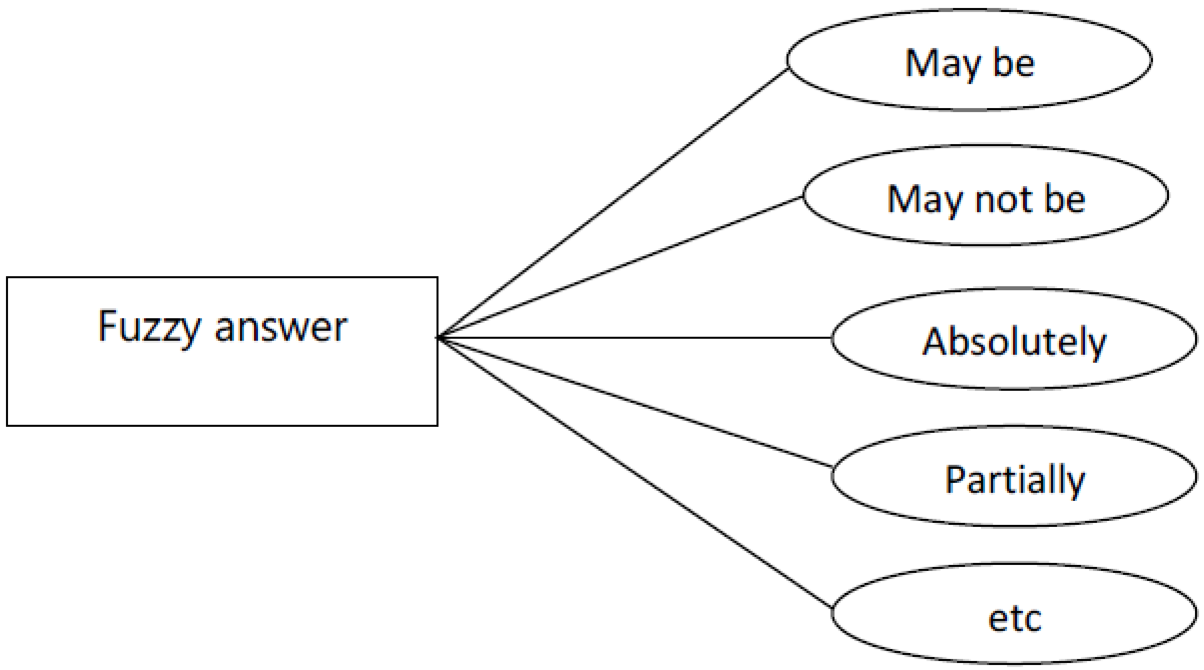
   Example: Are the chips crisp?

Introduction

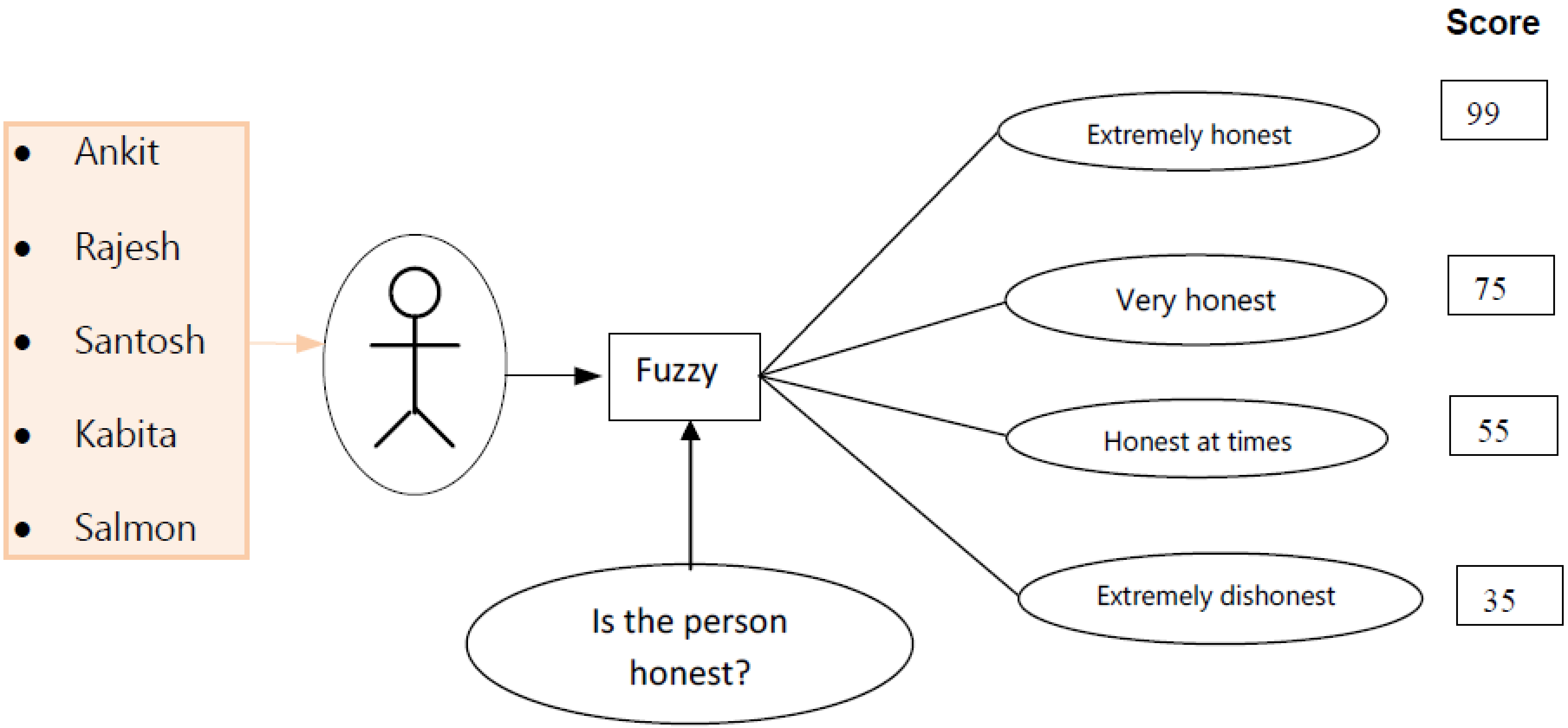




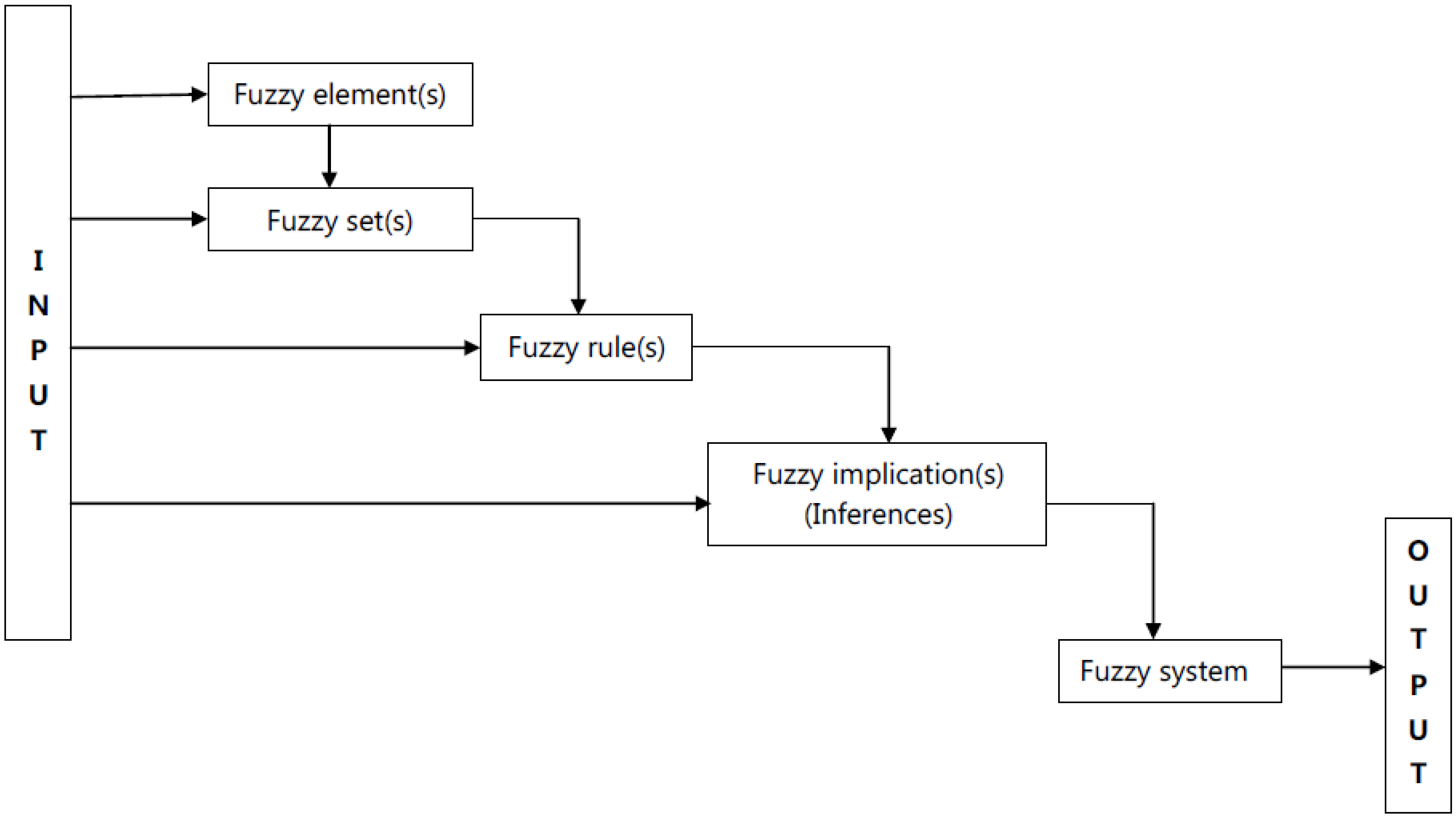
Introduction



Introduction

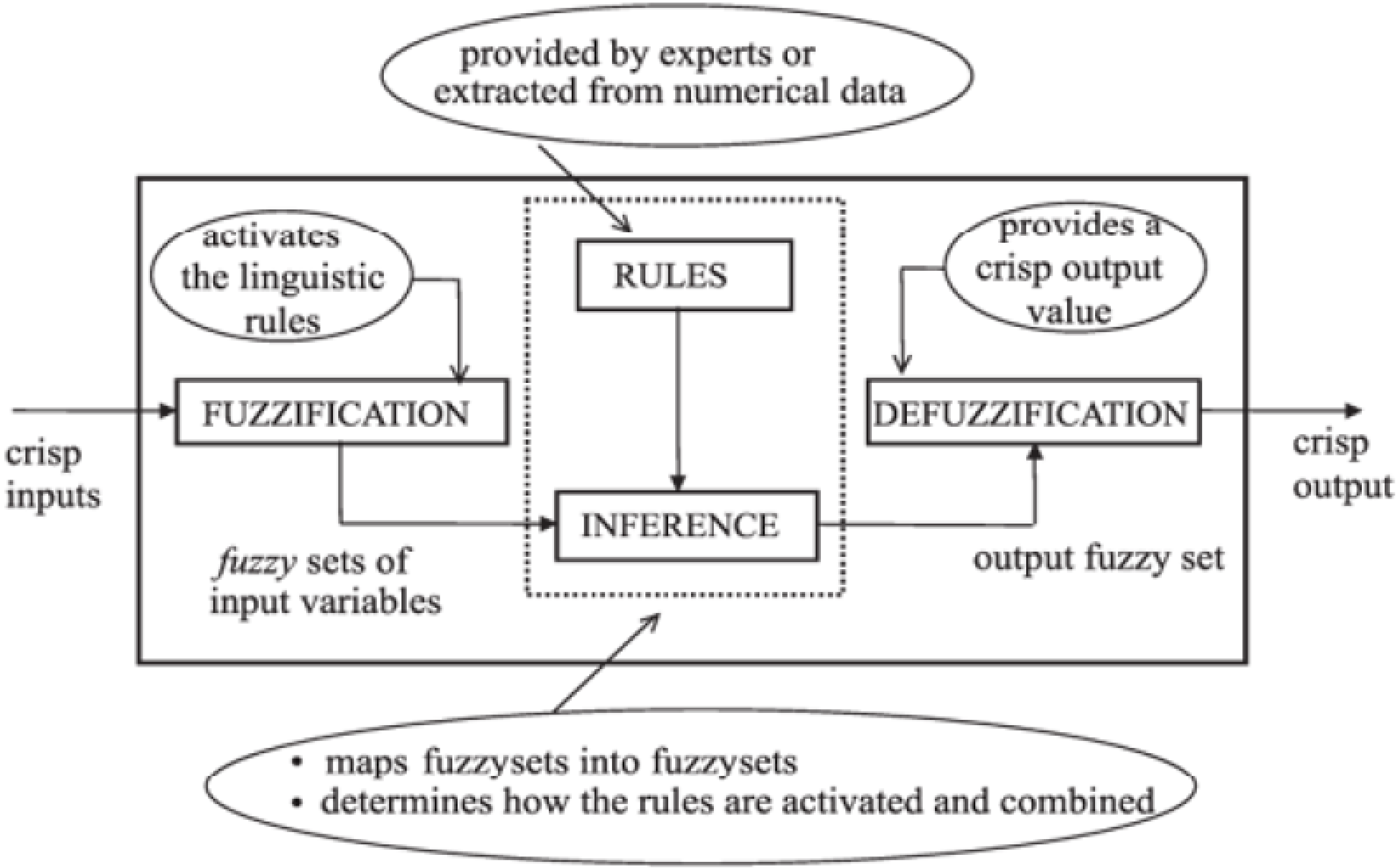


Phases



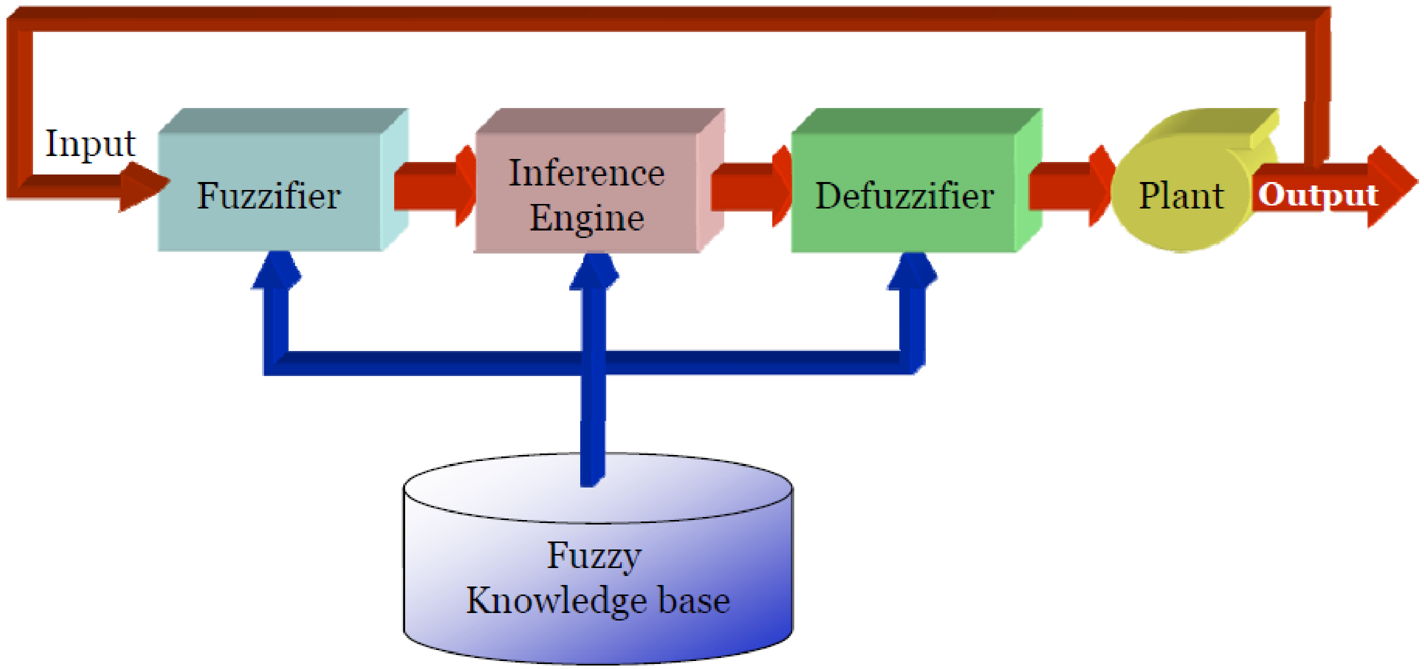
INPUT
Fuzzy element(s)
Fuzzy set(s)
Fuzzy rule(s)
Fuzzy implication(s)
(Inferences)
Fuzzy system
OUTPUT

System

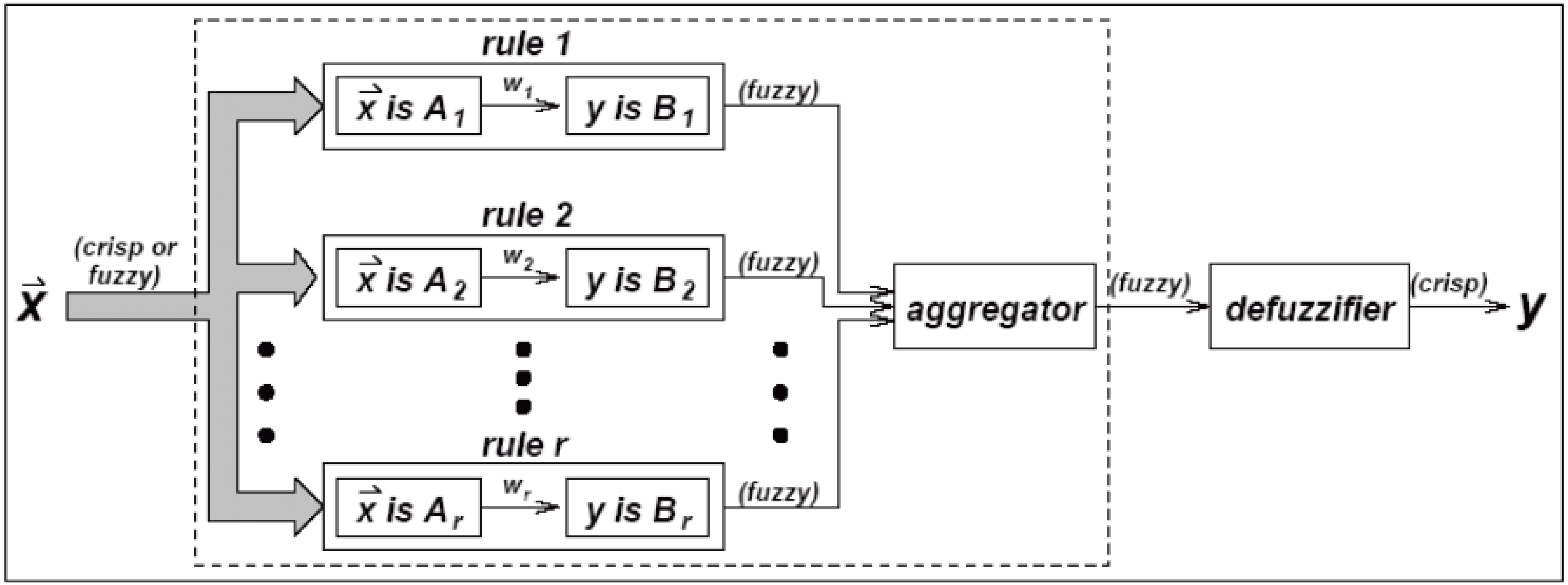


provided by experts or
extracted from numerical data
activates
the linguistic
rules
RULES
provides a
crisp output
value
FUZZIFICATION
DEFUZZIFICATION
crisp
inputs
crisp
output
INFERENCE
fuzzy sets of
input variables
output fuzzy set
• maps fuzzysets into fuzzysets
• determines how the rules are activated and combined

System


Input
Fuzzifier
Inference Engine
Defuzzifier
Plant
Output
Fuzzy Knowledge base

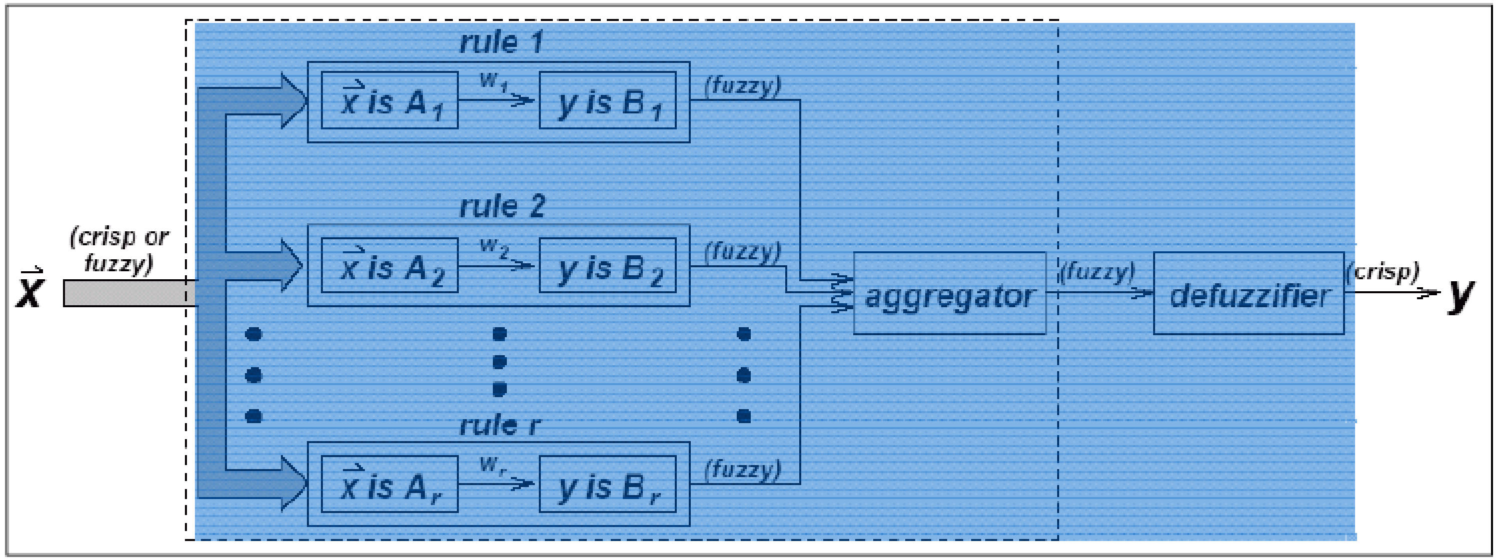
System



rule 1
x is A 1
w 1
y is B 1
(fuzzy)
rule 2
x is A 2
w 2
y is B 2
(fuzzy)
(crisp or
fuzzy)
x
rule r
x is A r
w r
y is B r
(fuzzy)
aggregator
(fuzzy)
defuzzifier
(crisp)
y

Mapping

In the case of crisp inputs & outputs, a fuzzy inference system implements a nonlinear mapping from its input space to output space.



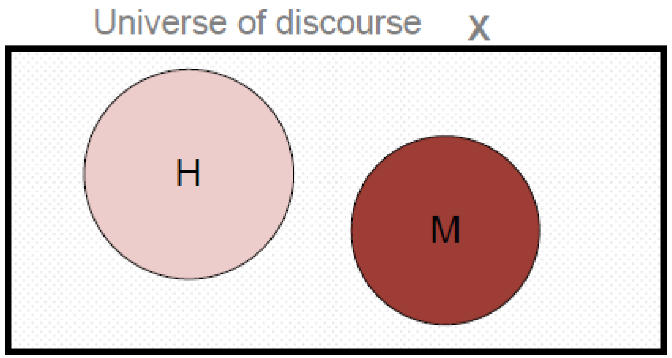
Fuzzy Sets

To understand the concept of **fuzzy set** it is better, if we first clear our idea of **crisp set**.

X = The entire population of India.

H = All Hindu population = $\{ h_1, h_2, h_3, \ldots, h_L \}$

M = All Muslim population = $\{ m_1, m_2, m_3, \ldots, m_N \}$



Here, All are the sets of finite numbers of individuals.

Such a set is called crisp set.

Fuzzy Sets

Let us discuss about fuzzy set.

X = All students in IT60108.

S = All **Good students**.

S = { (s, g) | s $\in$ X } and g(s) is a measurement of goodness of the student *s*.

Example:

S = { (Rajat, 0.8), (Kabita, 0.7), (Salman, 0.1), (Ankit, 0.9) } etc.

Fuzzy Sets

Crisp Set	Fuzzy Set
1. $S = \{ s \mid s \in X \}$	1. $F = (s, \mu) \mid s \in X$ and $\mu(s)$ is the degree of s.
2. It is a collection of elements.	2. It is collection of ordered pairs.
3. Inclusion of an element $s \in X$ into S is crisp, that is, has strict boundary **yes** or **no**.	3. Inclusion of an element $s \in X$ into F is fuzzy, that is, if present, then with a degree of **membership**.

Fuzzy Sets

Note: A crisp set is a fuzzy set, but, a fuzzy set is not necessarily a crisp set.

Example:
H = { (h_1, 1), (h_2, 1), ... , (h_L, 1) }

Person = { (p_1, 1), (p_2, 0), ... , (p_N, 1) }

In case of a crisp set, the elements are with extreme values of degree of membership namely either 1 or 0.

How to decide the degree of memberships of elements in a fuzzy set?

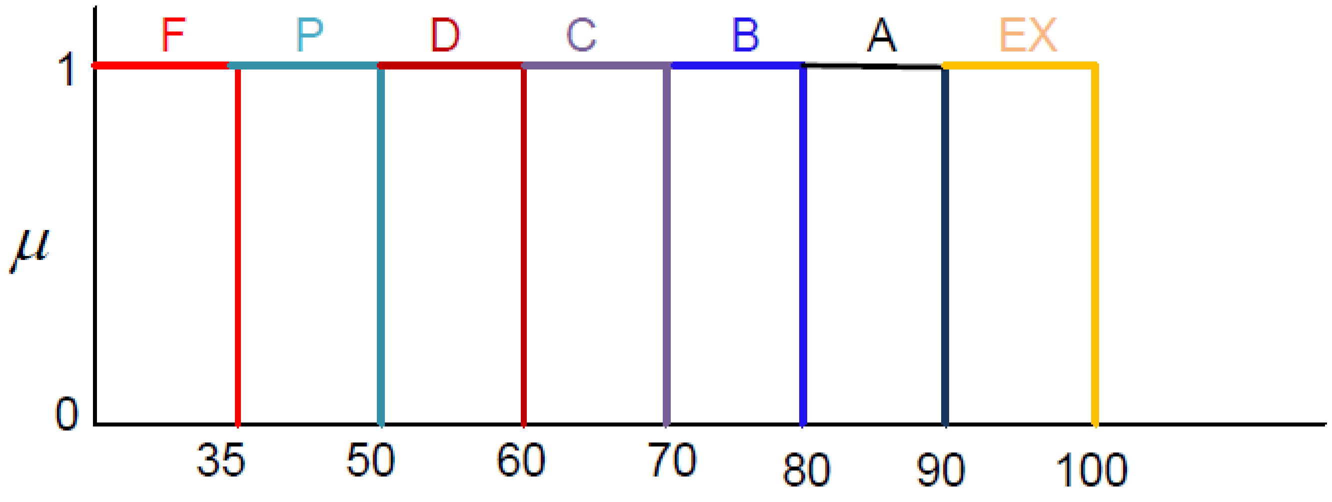
City	Bangalore	Bombay	Hyderabad	Kharagpur	Madras	Delhi
DoM	0.95	0.90	0.80	0.01	0.65	0.75

How the cities of comfort can be judged?

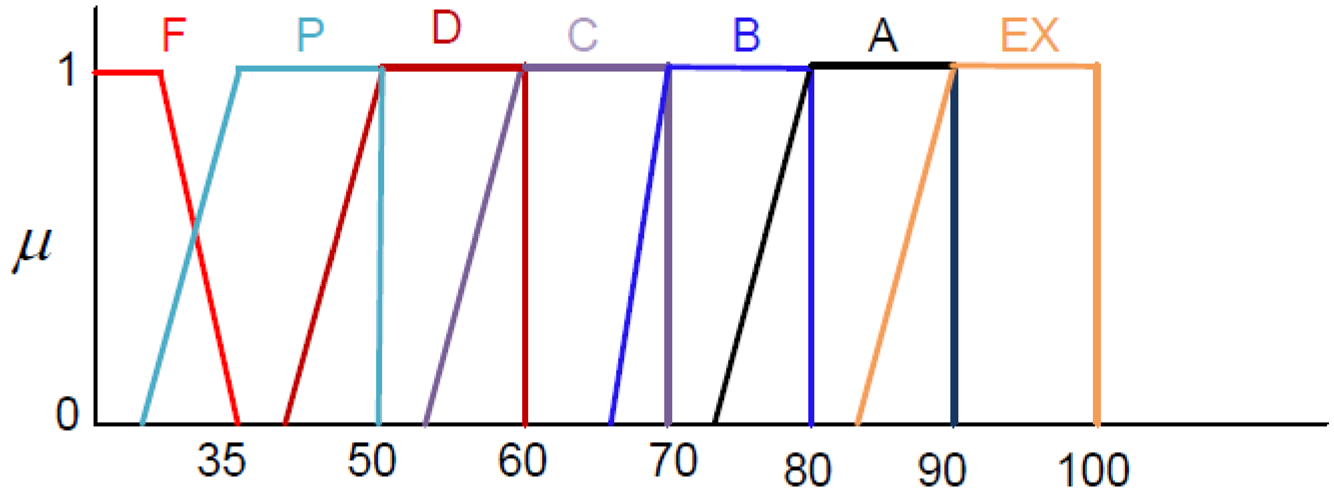
Fuzzy Sets

1. $EX = Marks \geq 90$
2. $A = 80 \leq Marks < 90$
3. $B = 70 \leq Marks < 80$
4. $C = 60 \leq Marks < 70$
5. $D = 50 \leq Marks < 60$
6. $P = 35 \leq Marks < 50$
7. $F = Marks < 35$

Fuzzy Sets



Fuzzy Sets



Examples

- High Temperature
- Low Pressure
- Color of Apple
- Sweetness of Orange
- Weight of Mango

Note: Degree of membership values lie in the range [0...1].

Fuzzy Sets

Definition 1: Membership function (and Fuzzy set)

If X is a universe of discourse and x $\in$ X, then a fuzzy set A in X is defined as a set of ordered pairs, that is

$A = \{(x, \mu_A(x)) | x \in X\}$ where μ_A(x) is called the membership function for the fuzzy set A.

Note:
μ_A(x) map each element of X onto a membership grade (or membership value) between 0 and 1 (both inclusive).

Question:
How (and who) decides μ_A(x) for a Fuzzy set A in X?

Fuzzy Sets

Example:

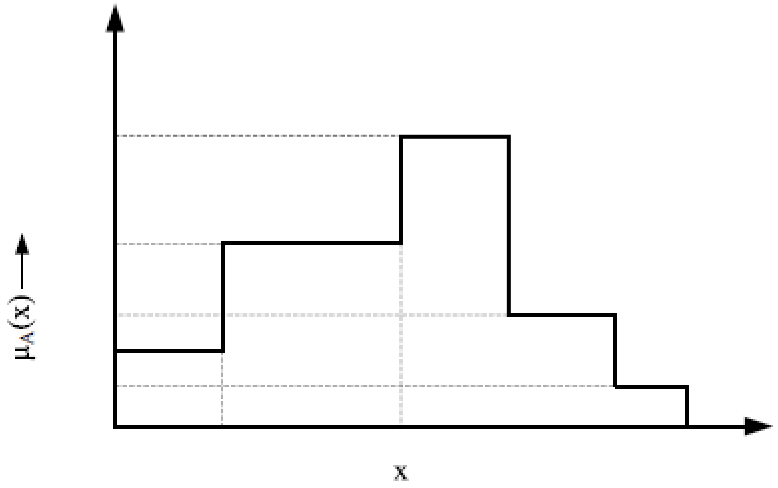
X = All cities in India

A = City of comfort

A={(New Delhi, 0.7), (Bangalore, 0.9), (Chennai, 0.8), (Hyderabad, 0.6), (Kolkata, 0.3), (Kharagpur, 0)}

Fuzzy Sets

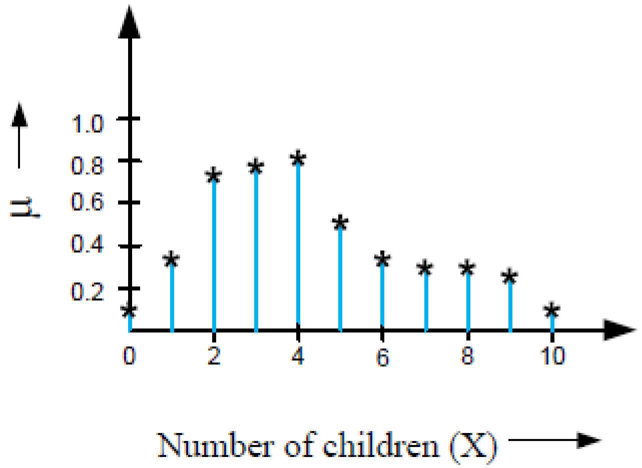
The membership values may be of discrete values.



A fuzzy set with discrete values of μ

Fuzzy Sets

Either elements or their membership values (or both) also may be of discrete values.



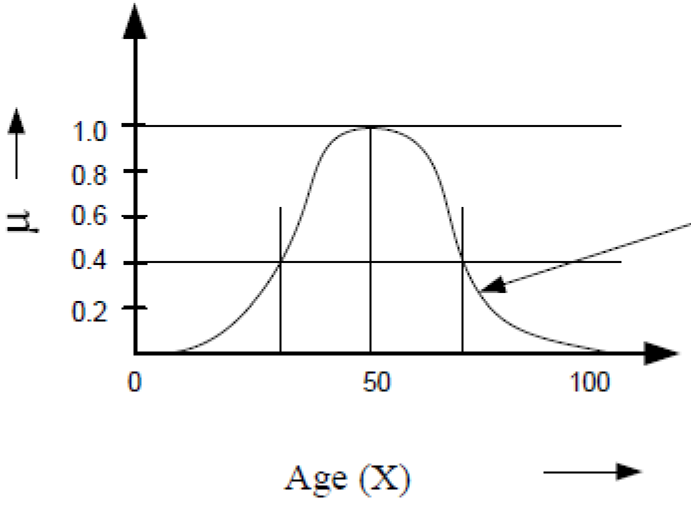
A = {(0,0.1),(1,0.30),(2,0.78)……(10,0.1)}

Note : X = discrete value

How you measure happiness ??

A = "Happy family"

Fuzzy Sets



$$\mu_B(x) = \frac{1}{1+\left(\frac{x-50}{10}\right)^4}$$

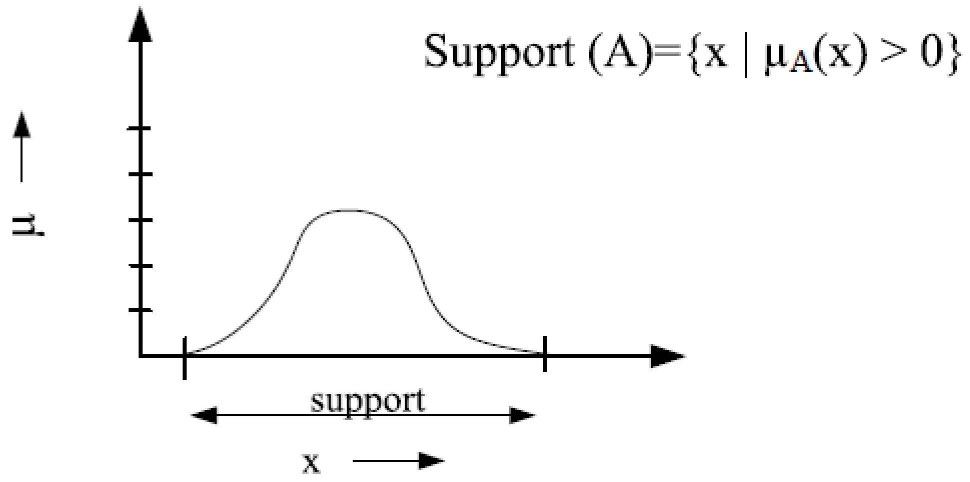
$$B=\{(x,\mu_B(x))\}$$

B = "Middle aged"

Note : x = real value
= R^+

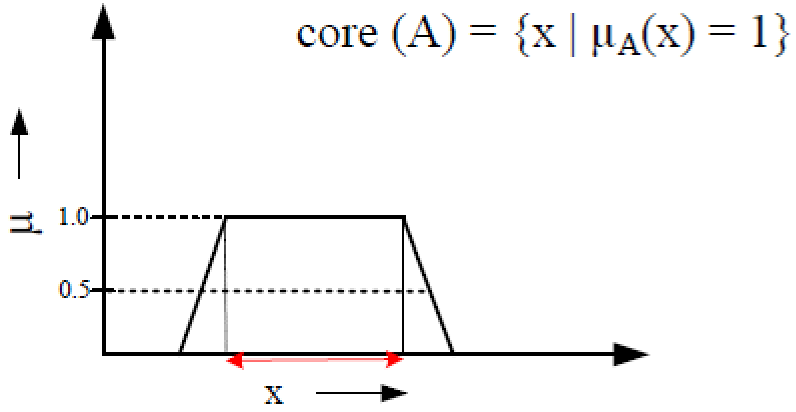
Fuzzy Sets

Support: The support of a fuzzy set A is the set of all points $x \in X$ such that $\mu_A(x) > 0$



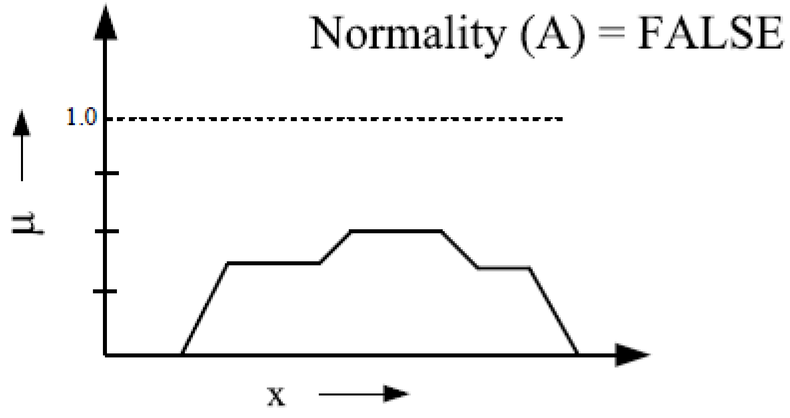
Fuzzy Sets

Core: The core of a fuzzy set A is the set of all points x in X such that $\mu_A(x) = 1$



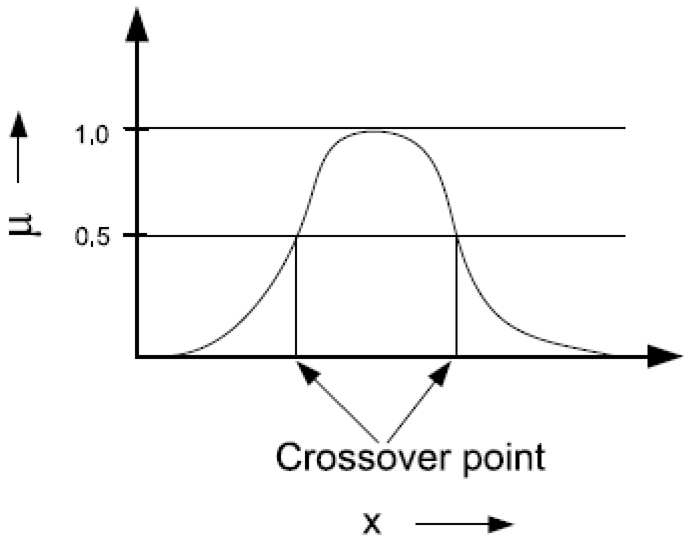
Fuzzy Sets

Normality : A fuzzy set A is a normal if its core is non-empty. In other words, we can always find a point $x \in X$ such that $\mu_A(x) = 1$.



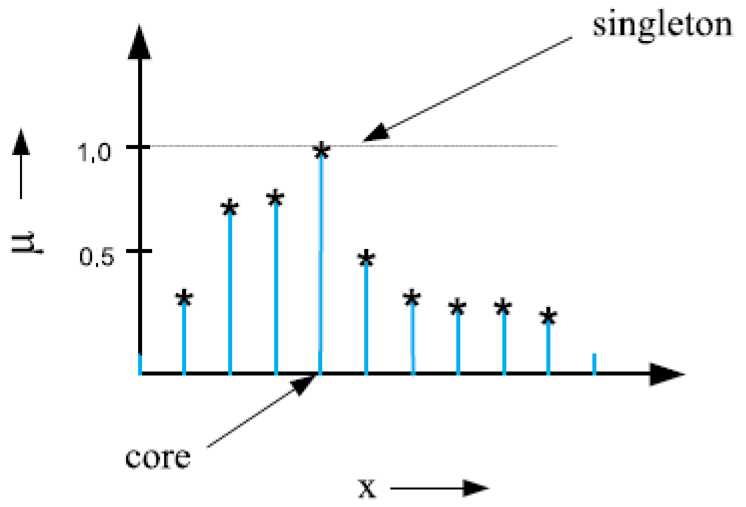
Fuzzy Sets

Crossover point : A crossover point of a fuzzy set A is a point $x \in X$ at which $\mu_A(x) = 0.5$. That is
Crossover $(A) = \{x | \mu_A(x) = 0.5\}$.



Fuzzy Sets

Fuzzy Singleton : A fuzzy set whose support is a single point in X with $\mu_A(x) = 1$ is called a fuzzy singleton. That is $|A| = |\{ x \mid \mu_A(x) = 1\}| = 1$. Following fuzzy set is not a fuzzy singleton.



Fuzzy Sets

α-cut and strong α-cut :

The α-cut of a fuzzy set A is a crisp set defined by

$$A_\alpha = \{x \mid \mu_A(x) \geq \alpha \}$$

Strong α-cut is defined similarly :

$$A_\alpha' = \{x \mid \mu_A(x) > \alpha \}$$

Note : Support(A) = A_0' and Core(A) = A_1.

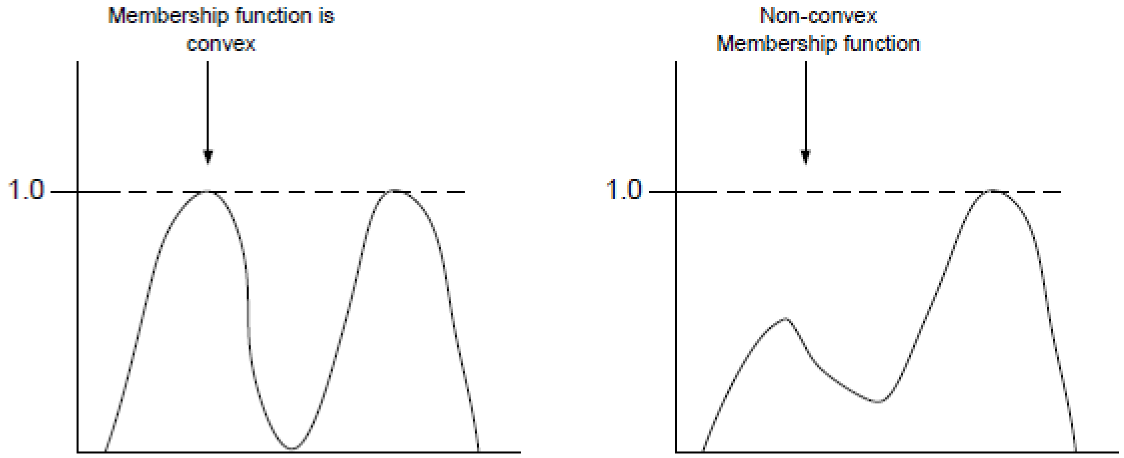
Fuzzy Sets

Convexity : A fuzzy set A is convex if and only if for any x_1 and $x_2 \in X$ and any $\lambda \in [0, 1]$

$$\mu_A\,(\lambda x_1 + (1 - \lambda) x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$

Note :

- A is convex if all its α- level sets are convex.
- Convexity $(A_\alpha) \Longrightarrow A_\alpha$ is composed of a single line segment only.



Fuzzy Sets

Bandwidth :

For a normal and convex fuzzy set, the bandwidth (or width) is defined as the distance the two unique crossover points:

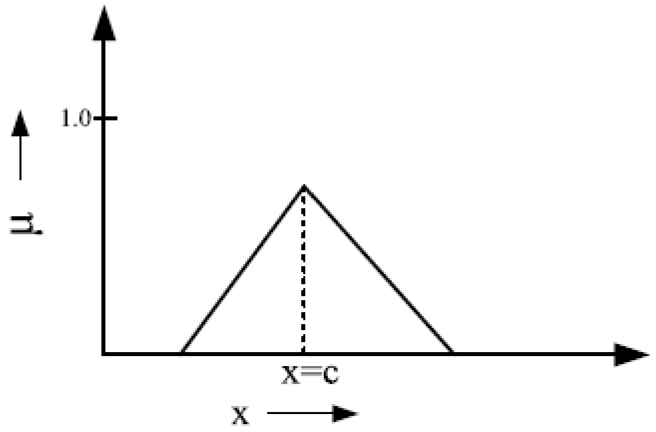
Bandwidth(A) = | x_1 - x_2 |

where $\mu_A(x_1) = \mu_A(x_2) = 0.5$

Fuzzy Sets

Symmetry :

A fuzzy set A is symmetric if its membership function around a certain point $x = c$, namely $\mu_A(x + c) = \mu_A(x - c)$ for all $x \in X$.



Fuzzy Sets

A fuzzy set A is

Open left

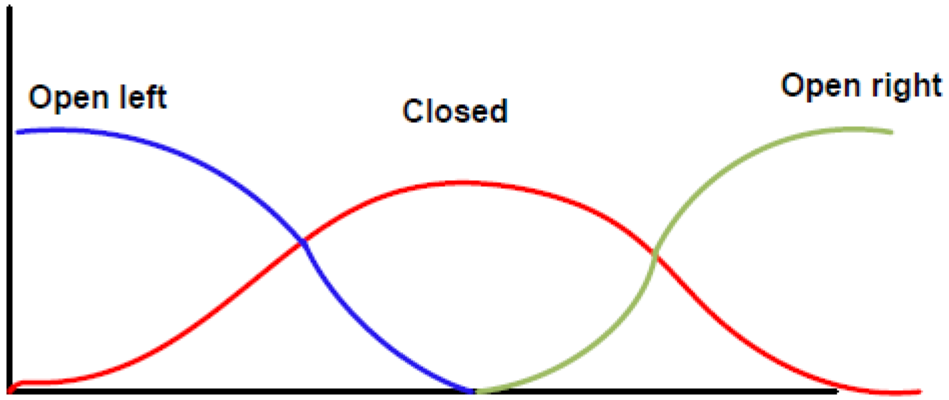
If $\lim_{x \to -\infty} \mu_A(x) = 1$ and $\lim_{x \to +\infty} \mu_A(x) = 0$

Open right:

If $\lim_{x \to -\infty} \mu_A(x) = 0$ and $\lim_{x \to +\infty} \mu_A(x) = 1$

Closed

If : $\lim_{x \to -\infty} \mu_A(x) = \lim_{x \to +\infty} \mu_A(x) = 0$



Fuzzy vs Probability

Fuzzy : When we say about certainty of a thing

Example: A patient come to the doctor and he has to diagnose so that medicine can be prescribed.
Doctor prescribed a medicine with certainty 60% that the patient is suffering from flue. So, the disease will be cured with certainty of 60% and uncertainty 40%. Here, in stead of flue, other diseases with some other certainties may be.

Probability: When we say about the chance of an event to occur

Example: India will win the T20 tournament with a chance 60% means that out of 100 matches, India own 60 matches.

Prediction vs Forecasting

The Fuzzy vs. Probability is analogical to Prediction vs. Forecasting

Prediction : When you start guessing about things.

Forecasting : When you take the information from the past job and apply it to new job.

The main difference:

Prediction is based on the best guess from experiences.
Forecasting is based on data you have actually recorded and packed from previous job.

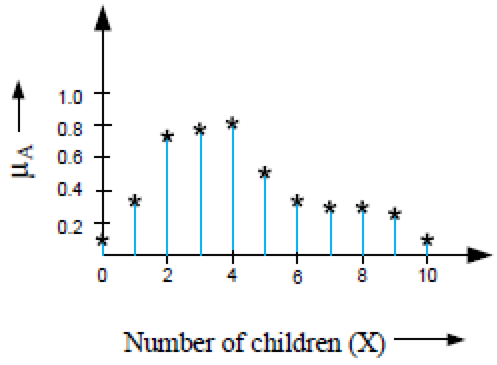
Membership Functions

A fuzzy set is completely characterized by its membership function (sometimes abbreviated as *MF* and denoted as μ). So, it would be important to learn how a membership function can be expressed (mathematically or otherwise).

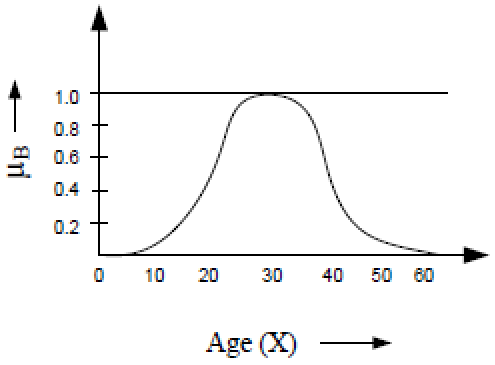
Note: A membership function can be on

(a) a discrete universe of discourse and
(b) a continuous universe of discourse.

Example:



A = Fuzzy set of "Happy family"

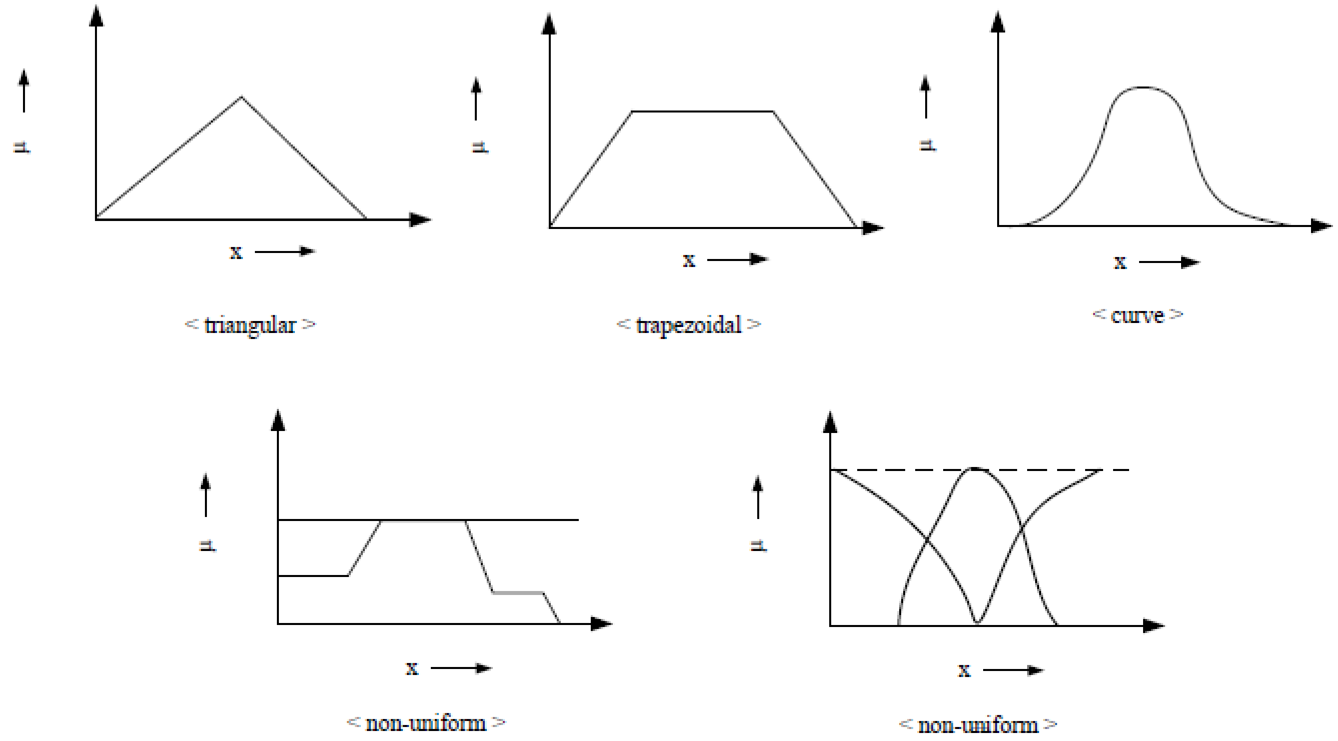


B = "Young age"

Membership Functions

So, membership function on a discrete universe of course is trivial. However, a membership function on a continuous universe of discourse needs a special attention.

Following figures shows a typical examples of membership functions.

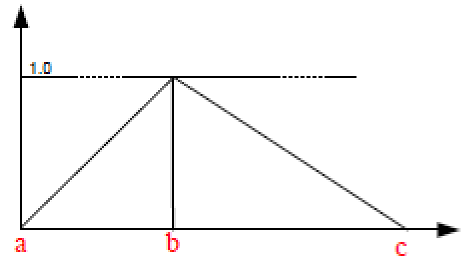


Membership Functions

In the following, we try to parameterize the different MFs on a continuous universe of discourse.

Triangular MFs : A triangular MF is specified by three parameters $\{a, b, c\}$ and can be formulated as follows.

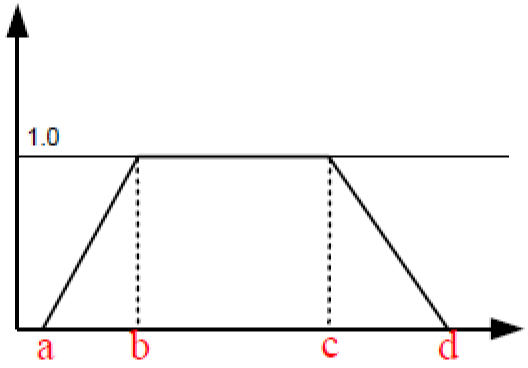
$$triangle(x; a, b, c) = \begin{cases} 0 & \text{if } x \leq a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \frac{c-x}{c-b} & \text{if } b \leq x \leq c \\ 0 & \text{if } c \leq x \end{cases}$$



Membership Functions

A trapezoidal MF is specified by four parameters $\{a, b, c, d\}$ and can be defined as follows:

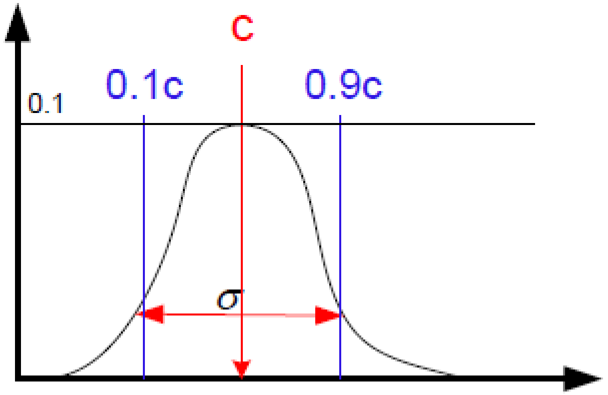
$$trapezoid(x; a, b, c, d) = \begin{cases} 0 & \text{if } x \leq a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } b \leq x \leq c \\ \frac{d-x}{d-c} & \text{if } c \leq x \leq d \\ 0 & \text{if } d \leq x \end{cases}$$



Membership Functions

A Gaussian MF is specified by two parameters $\{c, \sigma\}$ and can be defined as below:

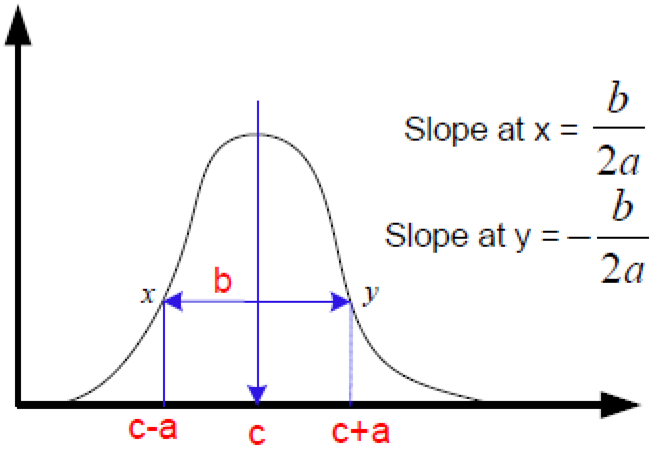
$$\text{gaussian}(x;c,\sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} .$$



Membership Functions

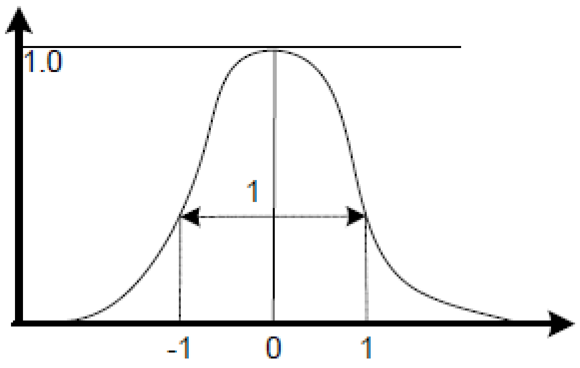
It is also called Cauchy MF. A generalized bell MF is specified by three parameters $\{a, b, c\}$ and is defined as:

$$\text{bell}(x; a, b, c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$

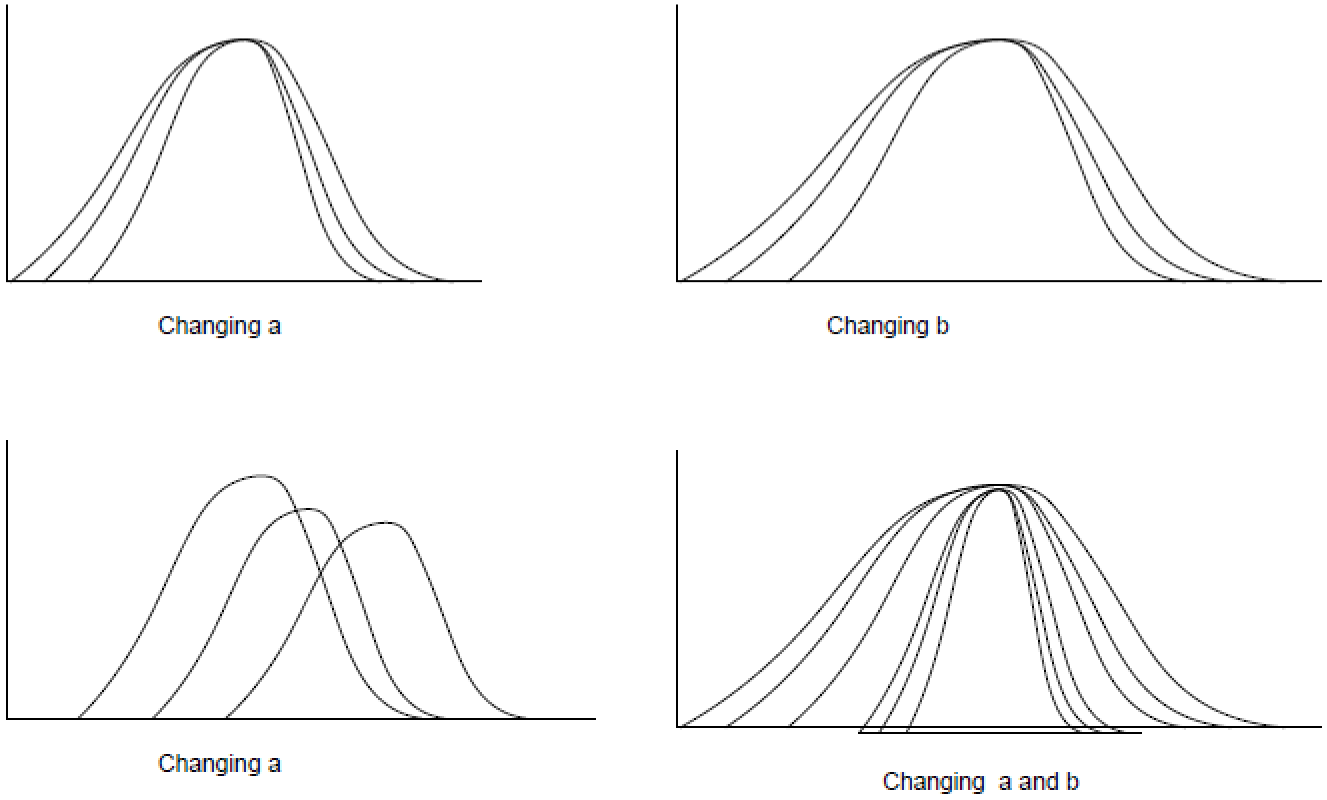


Membership Functions

Example: $\mu(x) = \frac{1}{1+x^2}$;
$a = b = 1$ and $c = 0$;



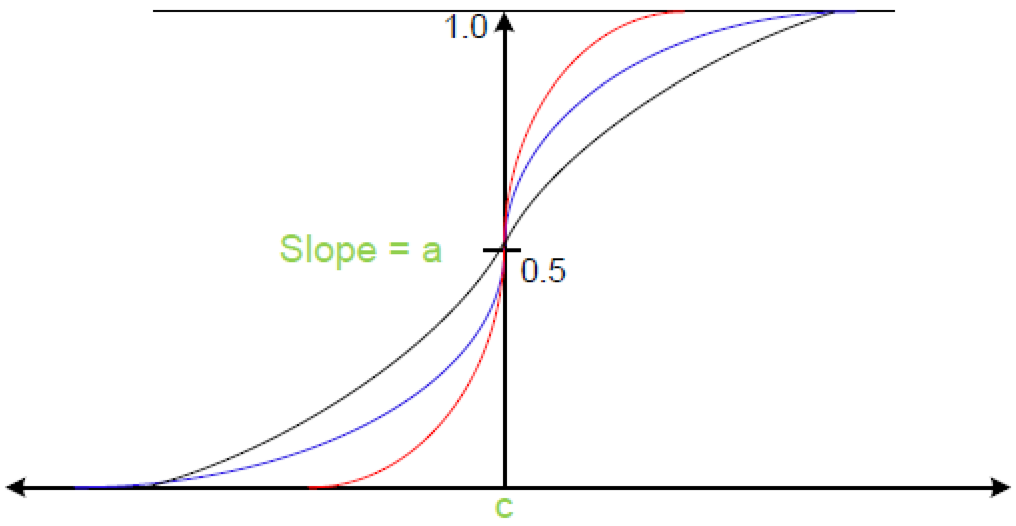
Membership Functions



Membership Functions

Parameters: $\{a, c\}$; where c = crossover point and a = slope at c;

$$\text{Sigmoid}(x;a,c)=\frac{1}{1+e^{-[\frac{a}{x-c}]}}$$



Membership Functions

Example : Consider the following grading system for a course.

Excellent = Marks $\leq$ 90

Very good = 75 $\leq$ Marks $\leq$ 90

Good = 60 $\leq$ Marks $\leq$ 75

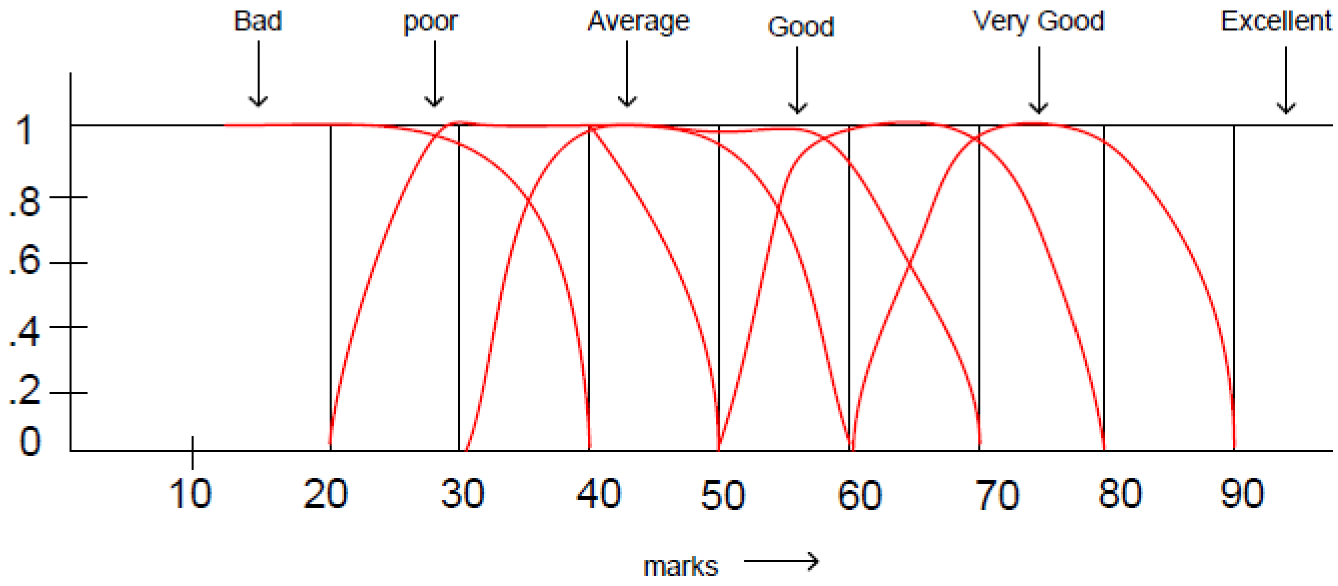
Average = 50 $\leq$ Marks $\leq$ 60

Poor = 35 $\leq$ Marks $\leq$ 50

Bad= Marks $\leq$ 35

Membership Functions

A fuzzy implementation will look like the following.

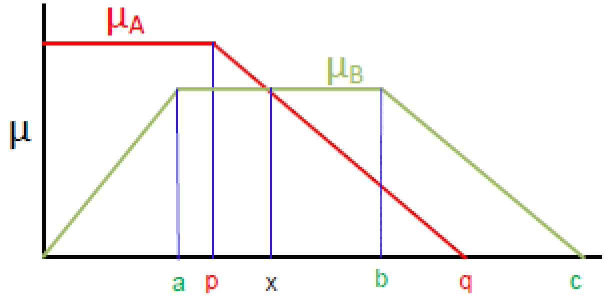


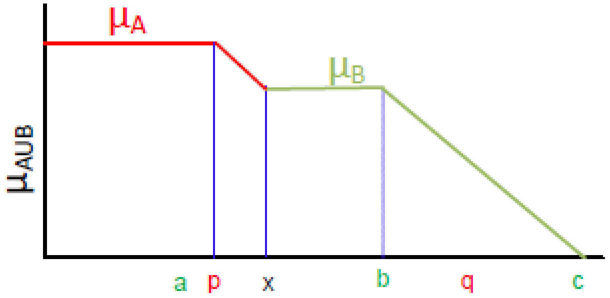
Operations

Union ($A \cup B$):

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$$

Example:
$A = \{(x_1, 0.5), (x_2, 0.1), (x_3, 0.4)\}$ and
$B = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5)\}$;
$C = A \cup B = \{(x_1, 0.5), (x_2, 0.3), (x_3, 0.5)\}$



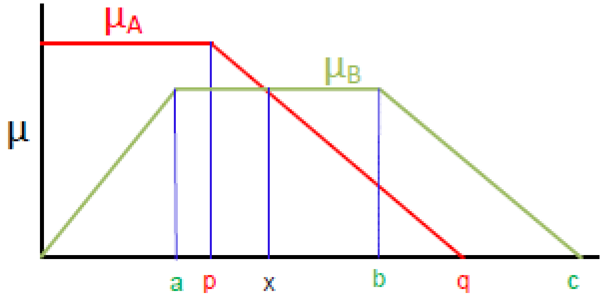


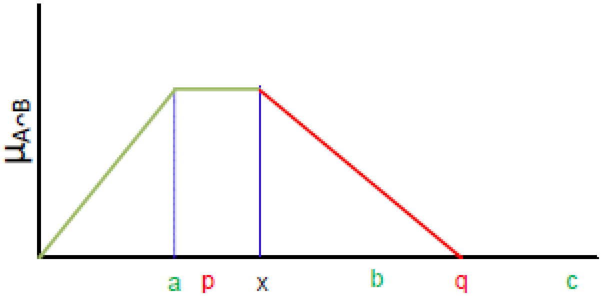
Operations

Intersection ($A \cap B$):

$$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$$

Example:
$A = \{(x_1, 0.5), (x_2, 0.1), (x_3, 0.4)\}$ and
$B = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5)\}$;
$C = A \cap B = \{(x_1, 0.2), (x_2, 0.1), (x_3, 0.4)\}$





Operations

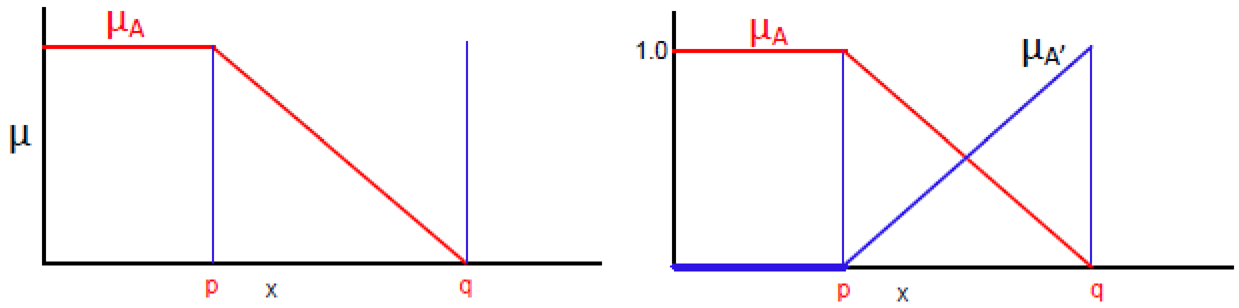
Complement (A^C):

$$\mu_{A_{A^C}}(x) = 1 - \mu_A(x)$$

Example:

$A = \{(x_1, 0.5), (x_2, 0.1), (x_3, 0.4)\}$

$C = A^C = \{(x_1, 0.5), (x_2, 0.9), (x_3, 0.6)\}$



Operations

Algebric product or Vector product (A•B):

$$\mu_{A \bullet B}(x) = \mu_A(x) \bullet \mu_B(x)$$

Scalar product $(\alpha \times A)$:

$$\mu_{\alpha A}(x) = \alpha \cdot \mu_A(x)$$

Operations

Sum ($A + B$):

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$$

Difference ($A - B = A \cap B^C$):

$$\mu_{A-B}(x) = \mu_{A \cap B^C}(x)$$

Disjunctive sum: $A \oplus B = (A^C \cap B) \cup (A \cap B^C))$

Bounded Sum: $|\ \mathrm{A(x)} \oplus \mathrm{B(x)}\ |$

$$\mu_{|A(x) \oplus B(x)|} = \min\{1, \mu_A(x) + \mu_B(x)\}$$

Bounded Difference: $|\ \mathrm{A(x)} \ominus \mathrm{B(x)}\ |$

$$\mu_{|A(x) \ominus B(x)|} = \max\{0, \mu_A(x) + \mu_B(x) - 1\}$$

Operations

Equality ($A = B$):

$$\mu_A(x) = \mu_B(x)$$

Power of a fuzzy set A^α:

$$\mu_{A^\alpha}(x) = \{\mu_A(x)\}^\alpha$$

- If $\alpha < 1$, then it is called *dilation*
- If $\alpha > 1$, then it is called *concentration*

Operations

Cartesian Product ($A \times B$):

$$\mu_{A\times B}(x, y) = min\{\mu_A(x), \mu_B(y)\}$$

Example 3:

$A(x) = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5), (x_4, 0.6)\}$

$B(y) = \{(y_1, 0.8), (y_2, 0.6), (y_3, 0.3)\}$

$$A \times B = \min\{\mu_A(x), \mu_B(y)\} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 \\ 0.5 & 0.5 & 0.3 \\ 0.6 & 0.6 & 0.3 \end{bmatrix} \end{array}$$

Operations

Commutativity :

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Associativity :

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

Distributivity :

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Operations

Idempotence :

$$A \cup A = A$$
$$A \cap A = \emptyset$$
$$A \cup \emptyset = A$$
$$A \cap \emptyset = \emptyset$$

Transitivity :

$$\text{If } A \subseteq B, B \subseteq C \text{ then } A \subseteq C$$

Involution :

$$(A^c)^c = A$$

De Morgan's law :

$$(A \cap B)^c = A^c \cup B^c$$
$$(A \cup B)^c = A^c \cap B^c$$

Operations

Given a membership function of a fuzzy set representing a linguistic hedge, we can derive many more MFs representing several other linguistic hedges using the concept of Concentration and Dilation.

- **Concentration:**

$$A^k = [\mu_A(x)]^k \; ; k > 1$$

- **Dilation:**

$$A^k = [\mu_A(x)]^k \; ; k < 1$$

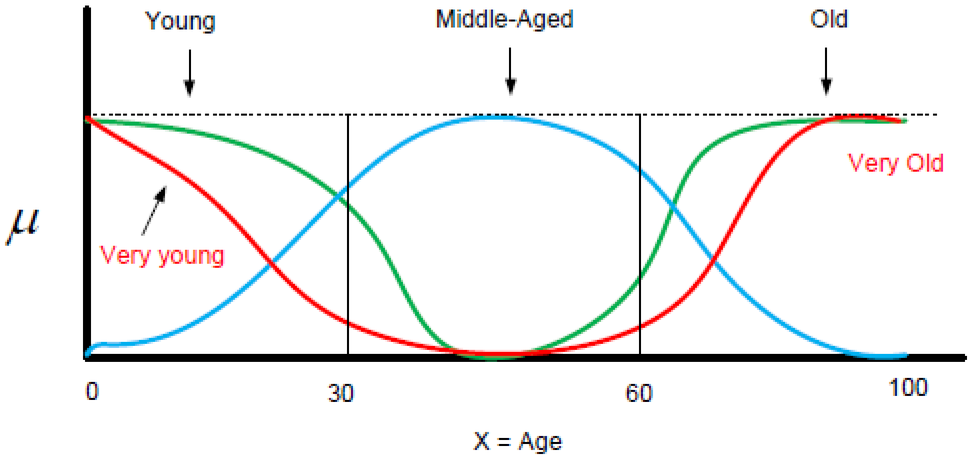
Example : Age = { Young, Middle-aged, Old }

Thus, corresponding to Young, we have : Not young, Very young, Not very young and so on.

Similarly, with Old we can have : old, very old, very very old, extremely old etc.
Thus, Extremely old = $(((old)^2)^2)^2$ and so on

Or, More or less old = $A^{0.5} = (old)^{0.5}$

Operations



$$\mu_{young}(x) = bell(x, 20, 2, 0) = \frac{1}{1+(\frac{x}{20})^4}$$

$$\mu_{old}(x) = bell(x, 30, 3, 100) = \frac{1}{1+(\frac{x-100}{30})^6}$$

$$\mu_{middle-aged} = bell(x, 30, 60, 50)$$

Not young = $\overline{\mu_{young}(x)} = 1 - \mu_{young}(x)$

Young but not too young = $\mu_{young}(x) \cap \overline{\mu_{young}(x)}$

Types

- **Ebrahim Mamdani Fuzzy Models**
- **Sugeno Fuzzy Models**
- **Tsukamoto Fuzzy Models**

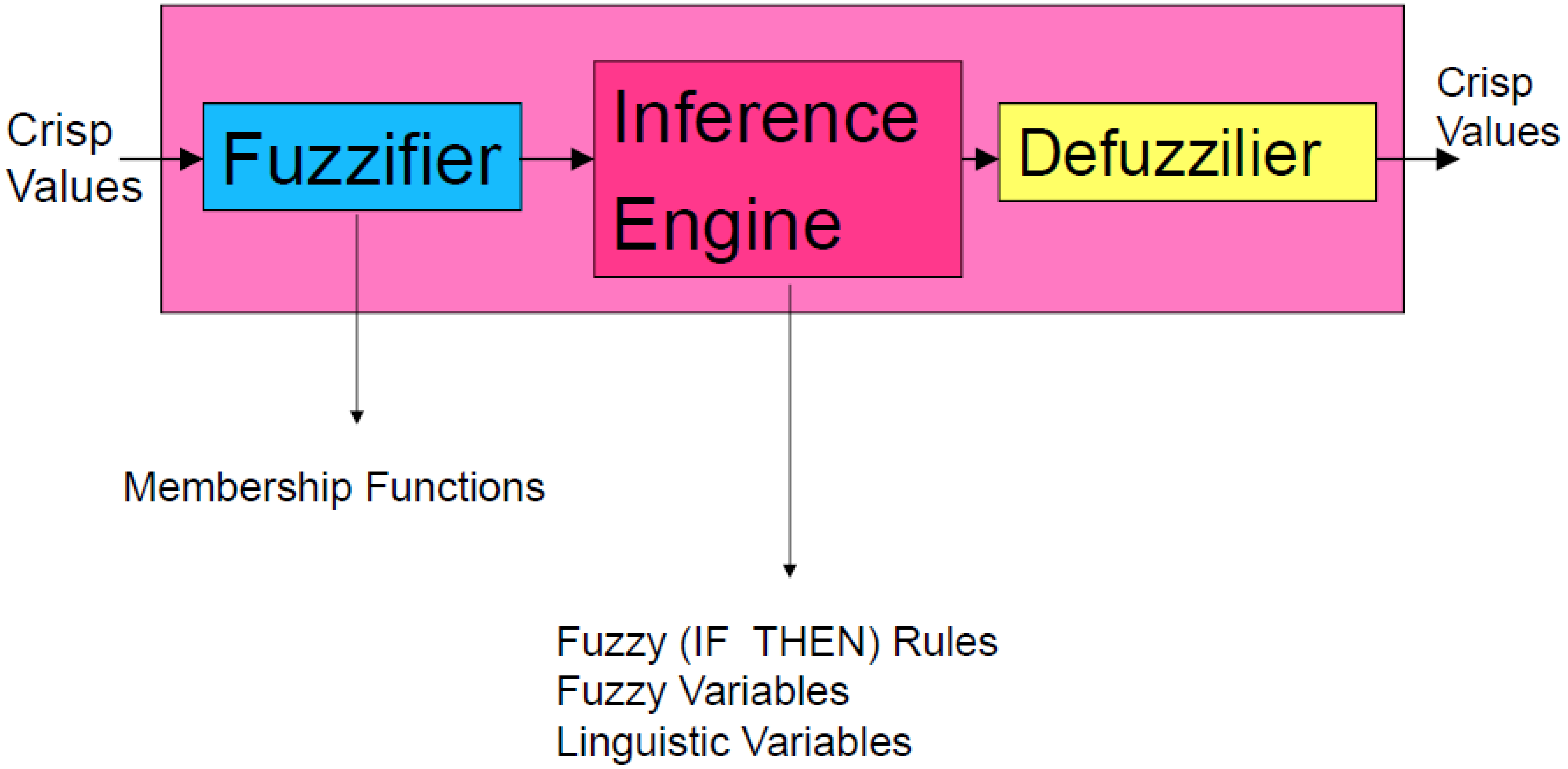
- The differences between these three FISs lie in the consequents of their fuzzy rules, and thus their aggregation and defuzzification procedures differ accordingly.

Mamdani Fuzzy Model

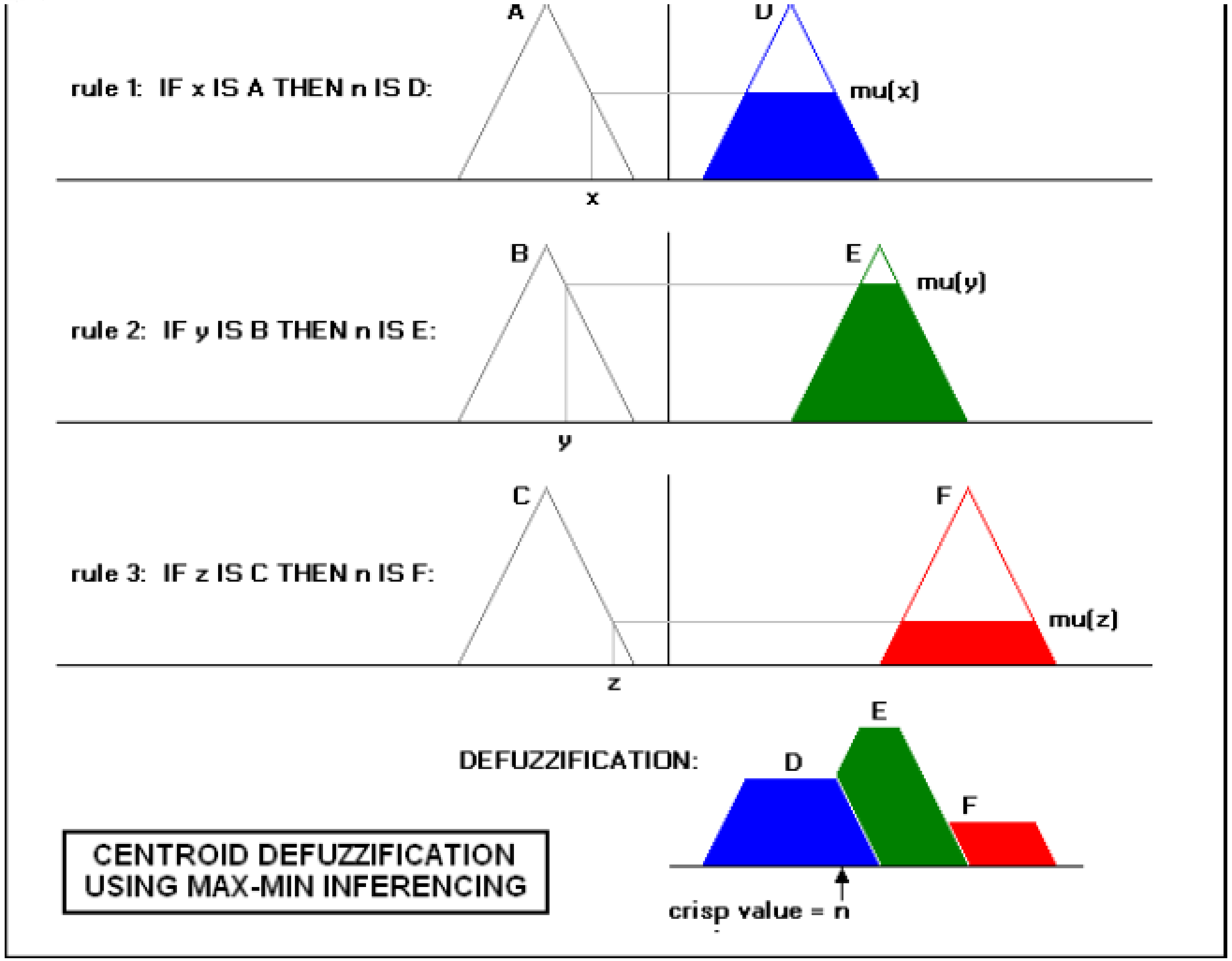
- The most commonly used fuzzy inference technique is the so-called **Mamdani** method.
- In 1975, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination. He applied a set of fuzzy rules supplied by experienced human operators.
- The Mamdani-style fuzzy inference process is performed in four steps:

1. Fuzzification of the input variables
2. Rule evaluation (inference)
3. Aggregation of the rule outputs (composition)
4. Defuzzification

Mamdani Fuzzy Model



Mamdani Fuzzy Model

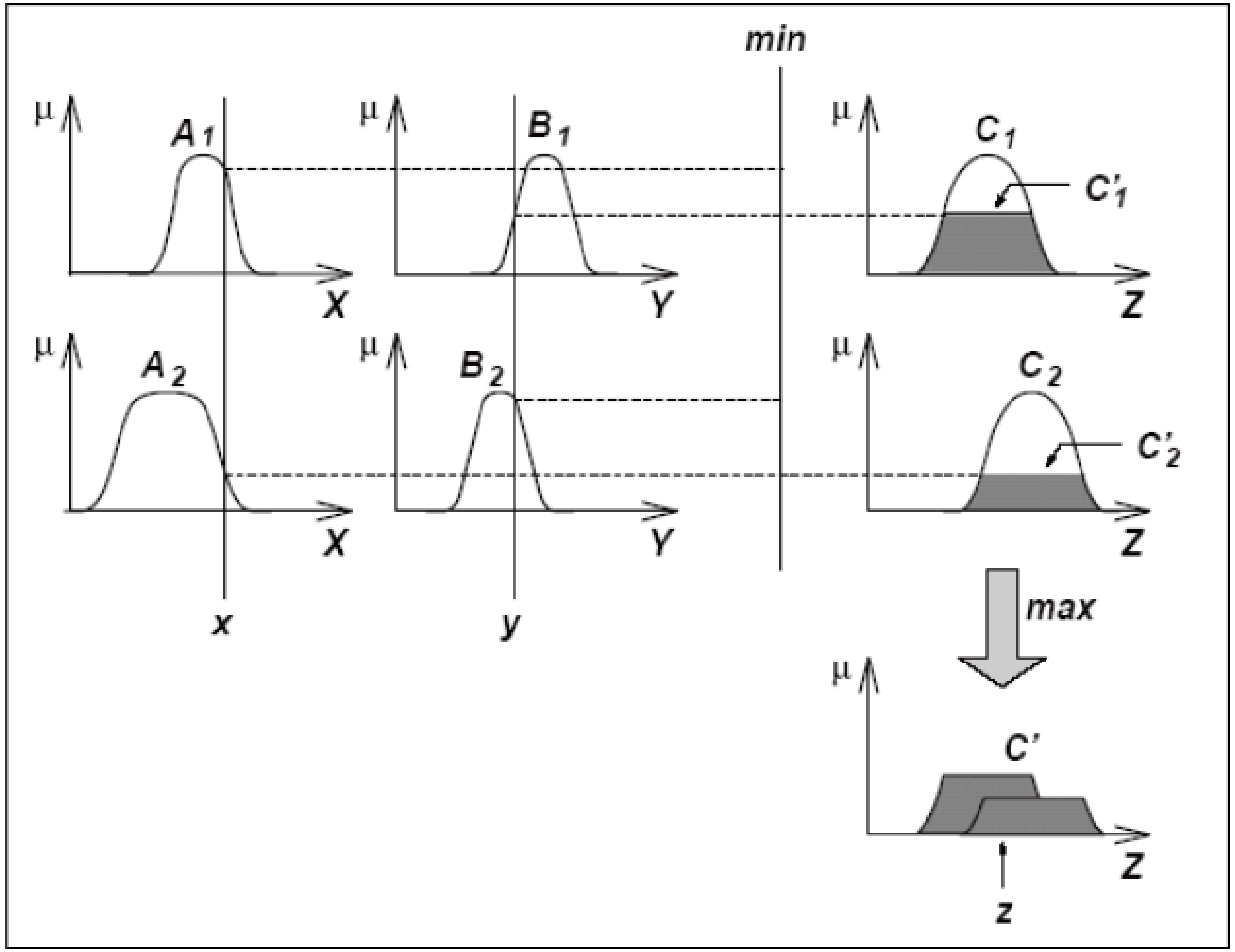


Mamdani composition of three SISO fuzzy outputs

http://en.wikipedia.org/wiki/Fuzzy_control_system

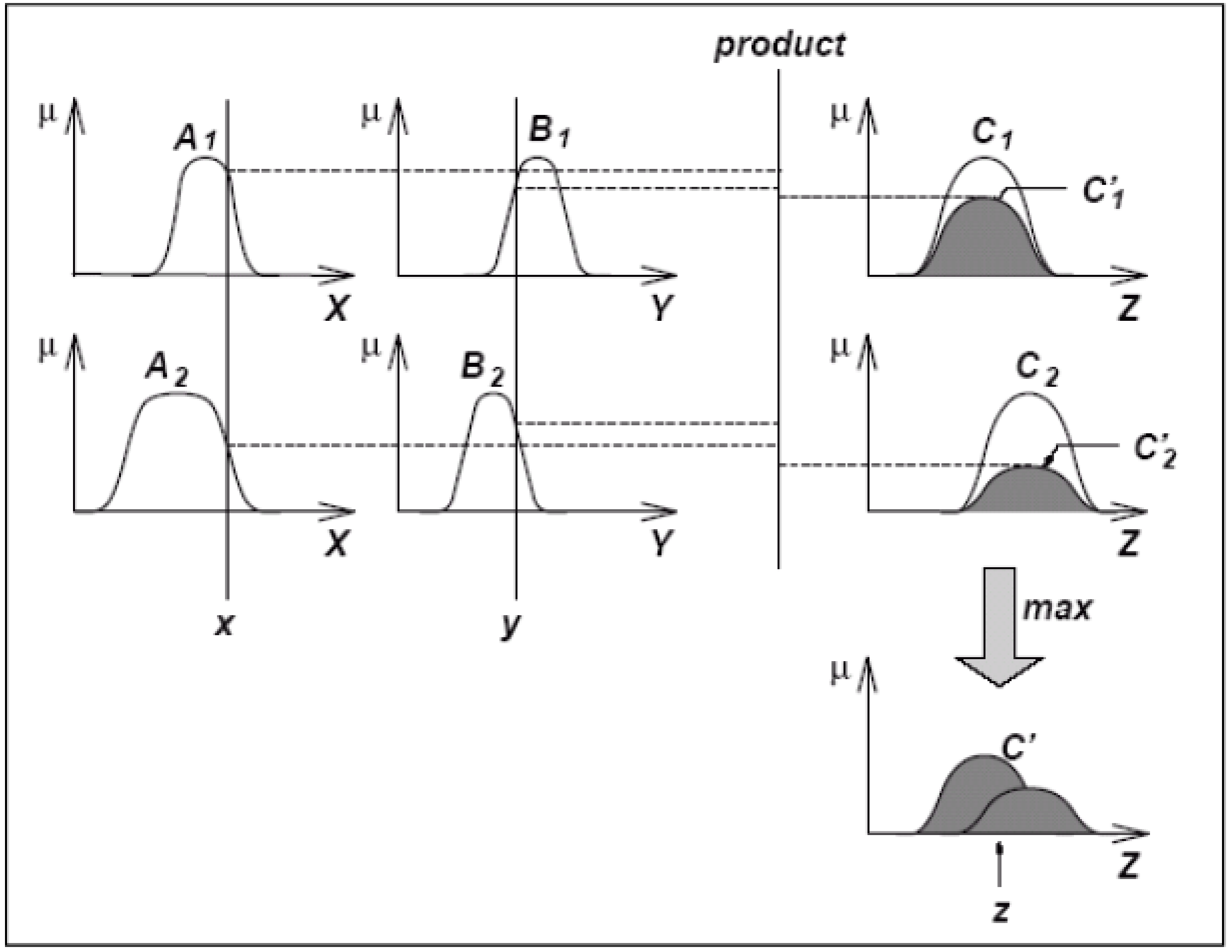
Mamdani Fuzzy Model

The mamdani FIS using **min** and **max** for **T-norm** and **S-norm** and subject to two crisp inputs x and y

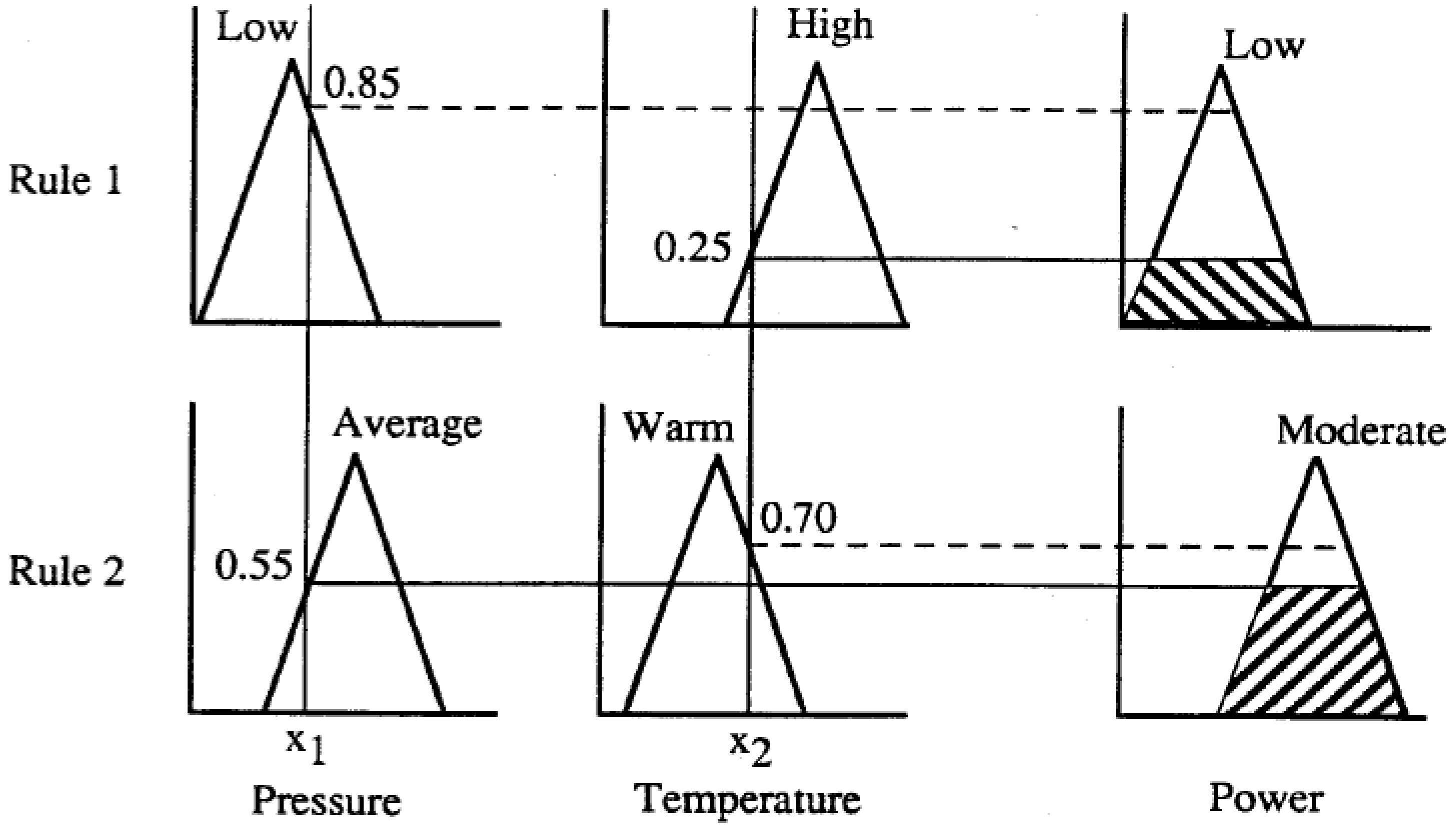


Mamdani Fuzzy Model

The mamdani FIS using **product** and **max** for **T-norm** and **S-norm** and subject to two crisp inputs x and y



Mamdani Fuzzy Model



Rule 1: If pressure is low and temperature is high then power is low
Rule 2: If pressure is average and temperature is warm then power is moderate

Mamdani Fuzzy Model

Two-input, one-ouput example:

If x is A_i and y is B_k then z is $C_{m(i,k)}$

y

	B_1	B_2	B_3	B_4
A_1	\	C_2	C_3	C_4
A_2	C_5	C_6	C_7	C_8
A_3	C_9	C_{10}	C_{11}	C_{12}

x

z

Y

	L	M	H	VH
L	Z	L	M	H
M	L	M	H	VH
H	M	H	VH	VH

x

Playing time

Mamdani Fuzzy Model

- In many applications we have to use crisp values as inputs for controlling of machines and systems.

- So, we have to use a defuzzifier to convert a fuzzy set to a crisp value.

Mamdani Fuzzy Model

- Defuzzification refers to the way a crisp value is extracted from a fuzzy set as a representative value.
- Defuzzification Methods:
  - Centroid of Area
  - Bisector of Area
  - Mean of Max
  - Smallest of Max
  - Largest of Max

Mamdani Fuzzy Model

$$z_{\mathrm{COA}} = \frac{\int_Z \mu_A(z) z \, dz}{\int_Z \mu_A(z) \, dz},$$

- where μ_A is aggregated output MF.
- This is the most widely adopted defuzzification strategy, which is reminiscent of the calculation of expected values of probability distributions.

Mamdani Fuzzy Model

- z_{BOA} satisfies

$$\int_{\alpha}^{z_{\text{BOA}}} \mu_A(z)\, dz = \int_{z_{\text{BOA}}}^{\beta} \mu_A(z)\, dz,$$

$$\boldsymbol{\alpha = min\{z|z \in Z\}} \qquad \boldsymbol{\beta = max\{z|z \in Z\}}$$

- That is, the vertical line $z = z_{BOA}$ partitions the region between $z = \alpha$, $z = \beta$, $y = 0$ and $y = \mu_A(z)$ into two regions with the same area.

Mamdani Fuzzy Model

- z_{MOM} is the mean of maximizing *z* at which the MF reaches maximum μ*. In Symbols,

$$z_{MOM} = \frac{\int_{Z'} z\, dz}{\int_{Z'} dz},$$

$$Z' = \{z | \mu_A(z) \in \mu^*\}$$

- In particular, if $\mu_A(z)$ has a single maximum at $z = z^*$, then the $z_{MOM} = z^*$.
- Moreover, if $\mu_A(z)$ reaches its maximum whenever

$$z \in [z_{left}, z_{right}]$$

then

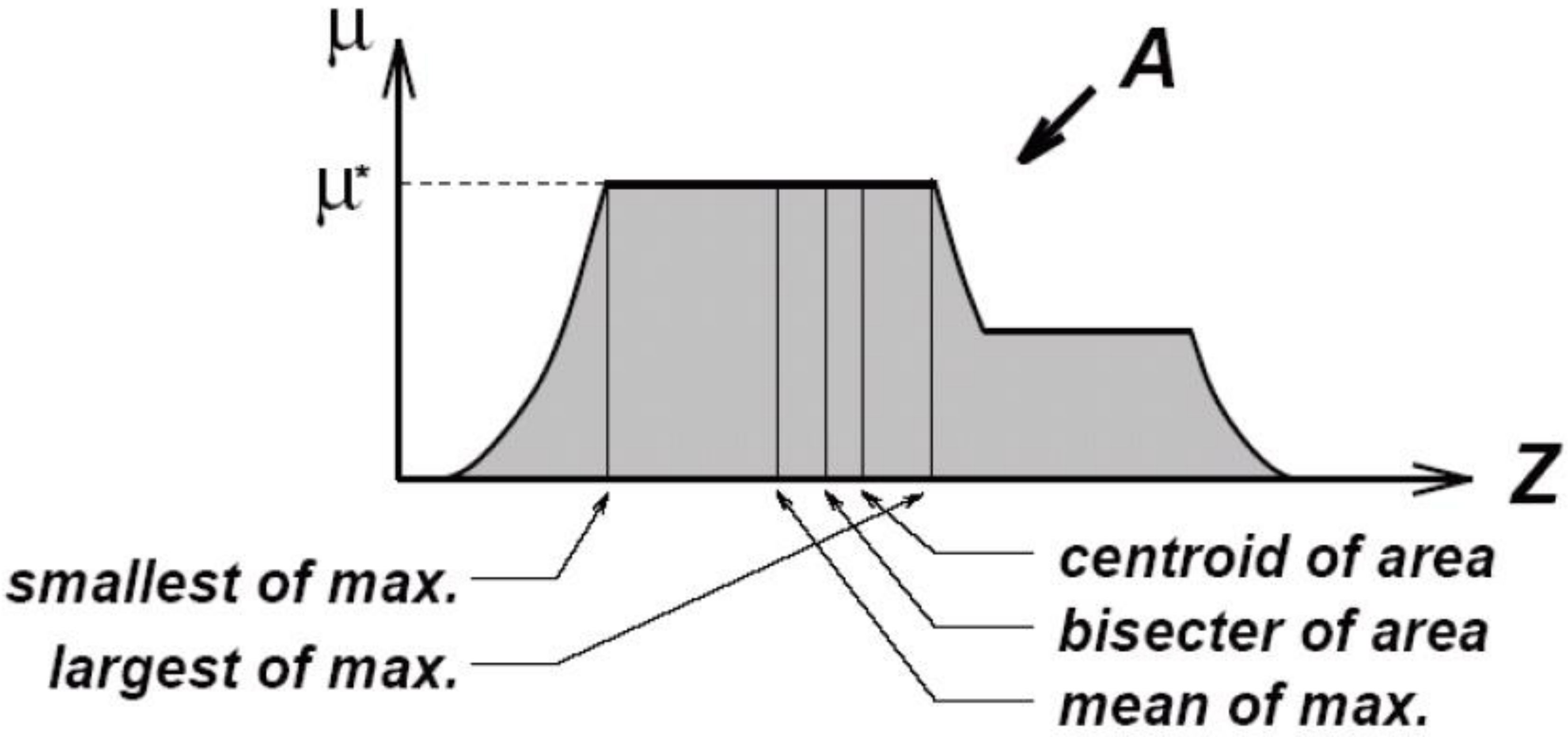
$$z_{MOM} = (z_{left} + z_{right})/2$$

Mamdani Fuzzy Model

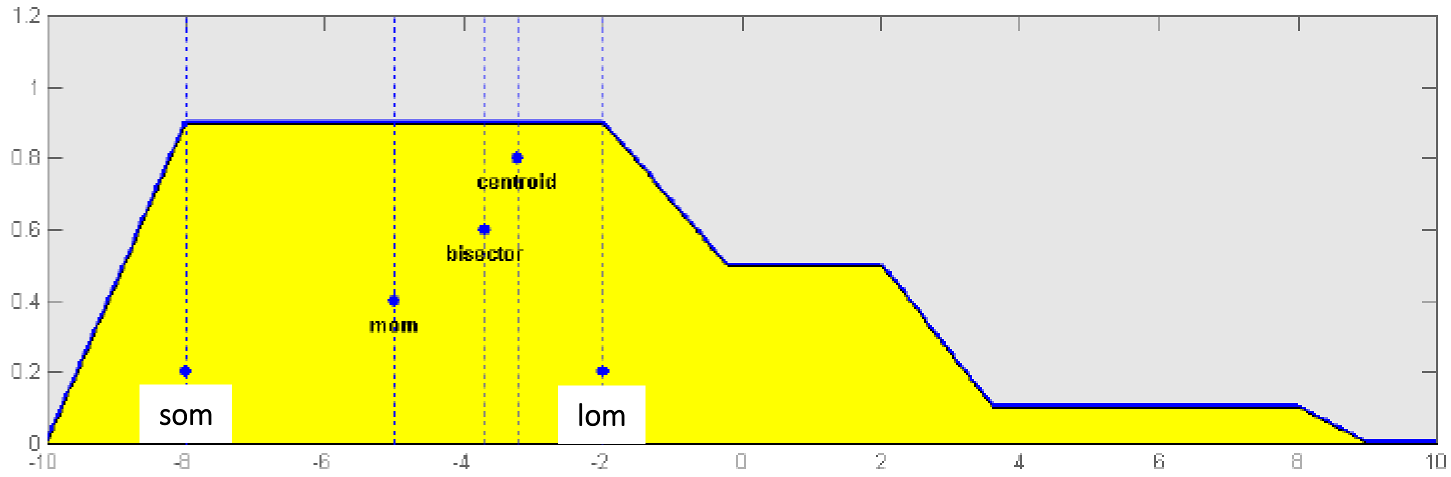
- z_{SOM} is the minimum (in terms of magnitude) of the maximizing z.
- z_{LOM} is the maximum (in terms of magnitude) of the maximizing z.

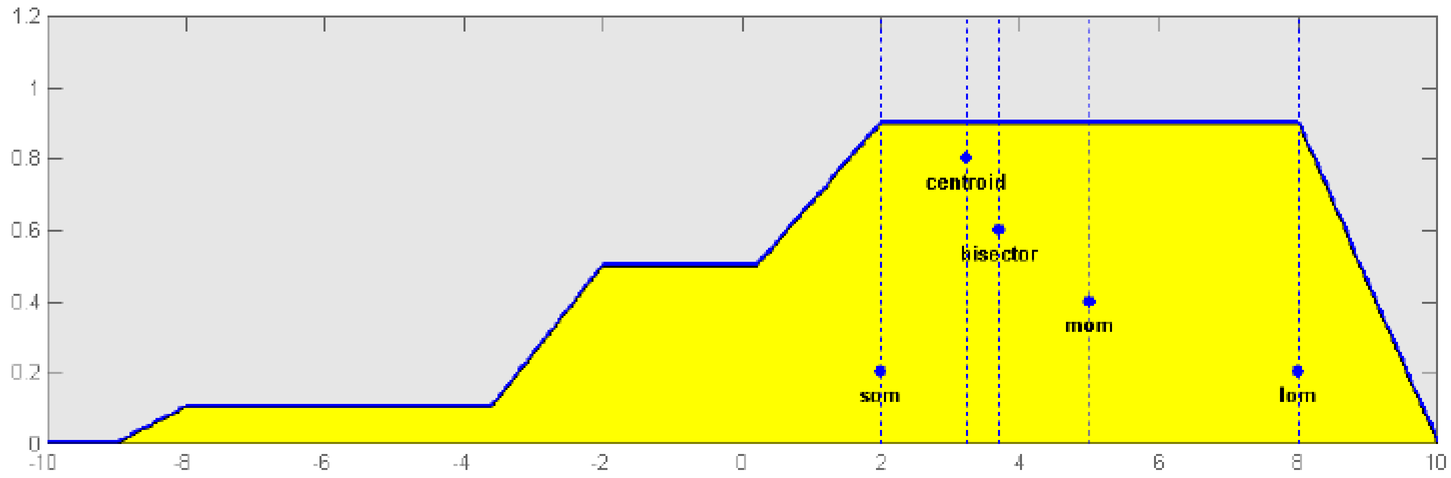
- Because of their obvious bias, z_{SOM} and z_{LOM} are not used as often as the other three defuzzification methods.

Mamdani Fuzzy Model

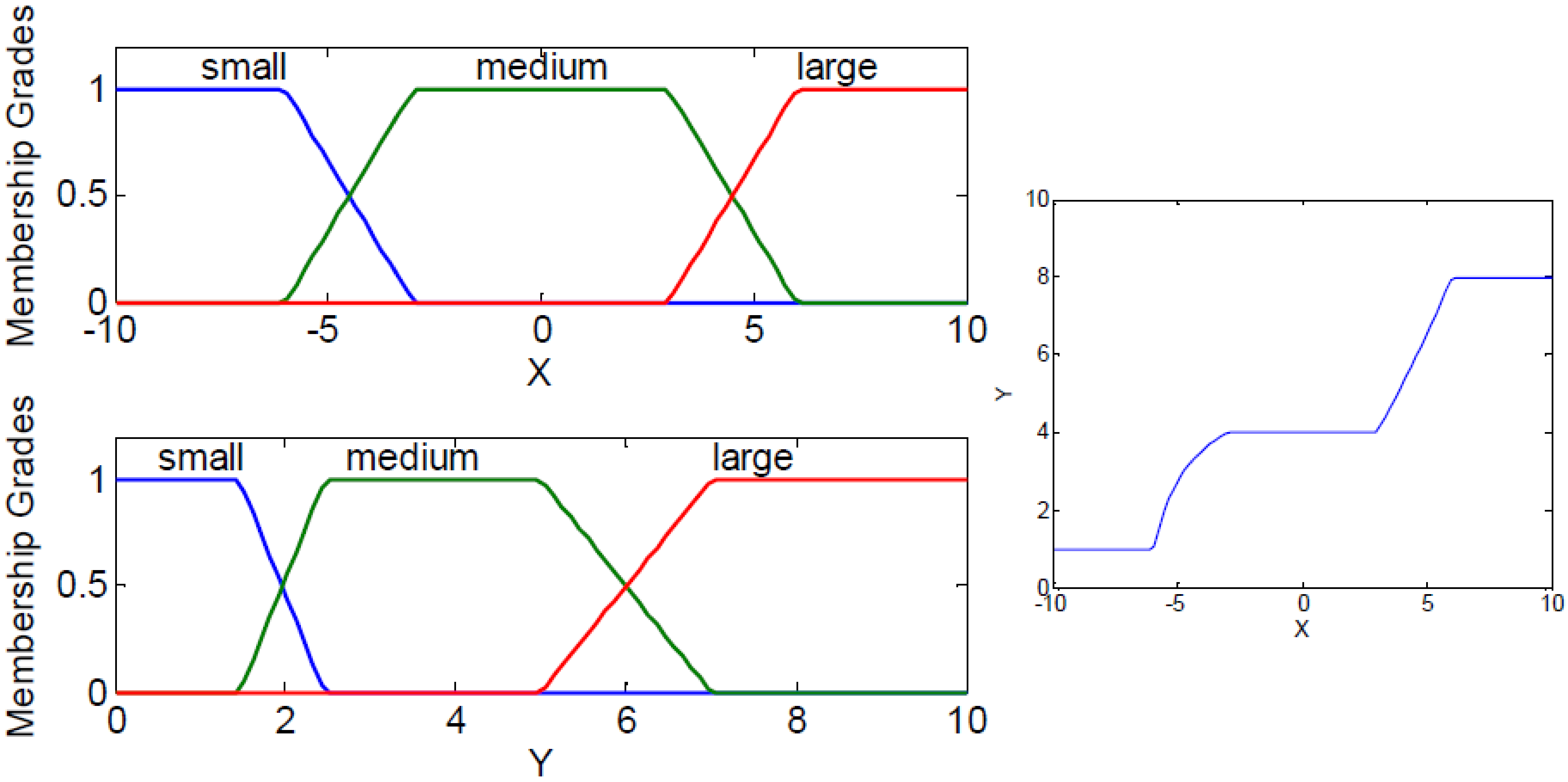


Mamdani Fuzzy Model



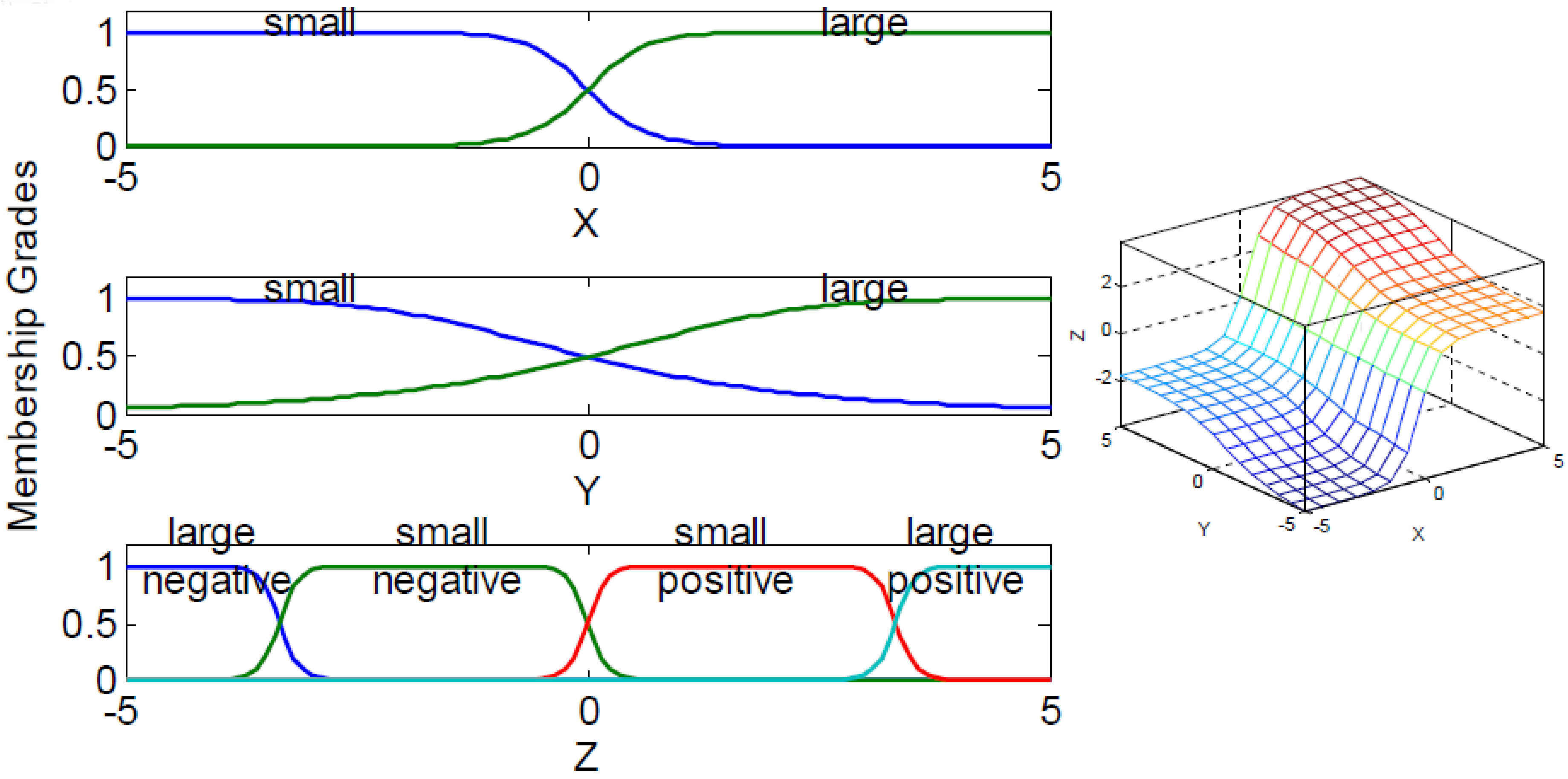


Mamdani Fuzzy Model



a) MFs of the input and output
b) Overall input-output curve

Mamdani Fuzzy Model



a) MFs of the inputs and output
b) Overall input-output curve

Computational Intelligence & Machine Learning

Mamdani Fuzzy Model

We examine a simple two-input one-output problem that includes three rules:

Rule: 1	IF x is A3	OR	y is B1	THEN	z is C1
Rule: 2	IF x is A2	AND	y is B2	THEN	z is C2
Rule: 3	IF x is A1			THEN	z is C3

Real-life example for these kinds of rules:

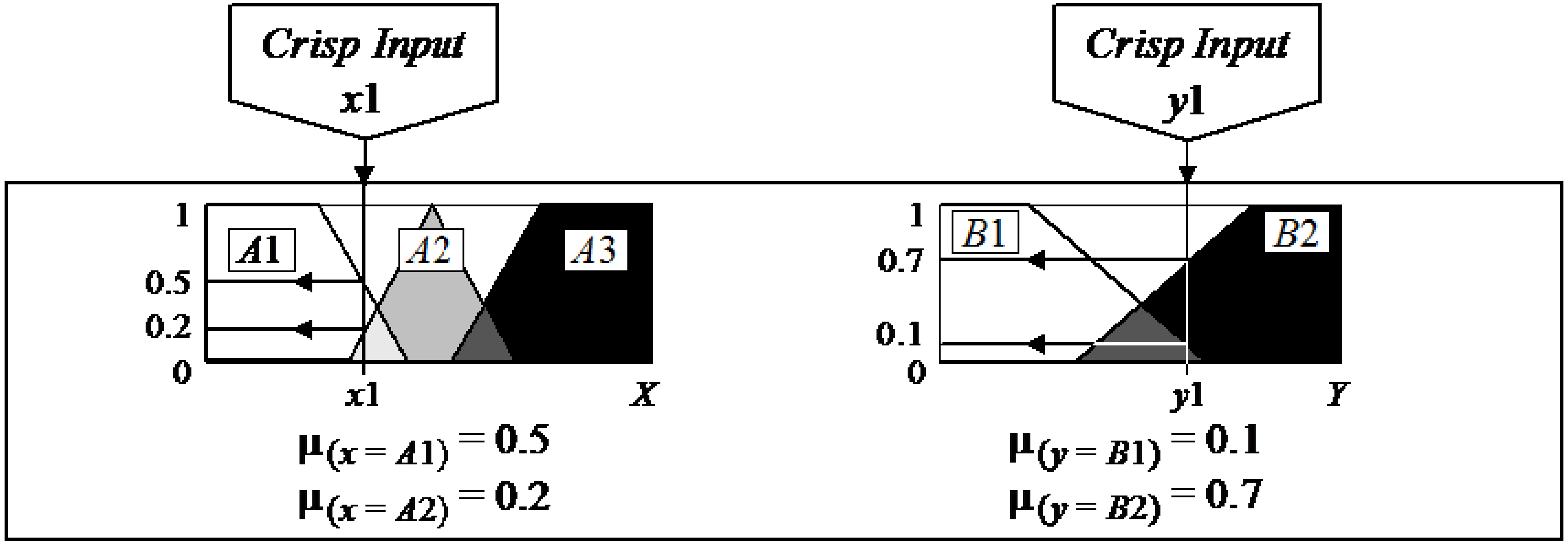
Rule: 1 IF project_funding is adequate OR project_staffing is small THEN risk is low

Rule: 2 IF project_funding is marginal AND project_staffing is large THEN risk is normal

Rule: 3 IF project_funding is inadequate THEN risk is high

Mamdani Fuzzy Model

- The first step is to take the crisp inputs, x1 and y1 (*project funding* and *project staffing*), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.



Mamdani Fuzzy Model

- The second step is to take the fuzzified inputs, $\mu_{(x=A1)} = 0.5$, $\mu_{(x=A2)} = 0.2$, $\mu_{(y=B1)} = 0.1$ and $\mu_{(y=B2)} = 0.7$, and apply them to the antecedents of the fuzzy rules.
- If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation.

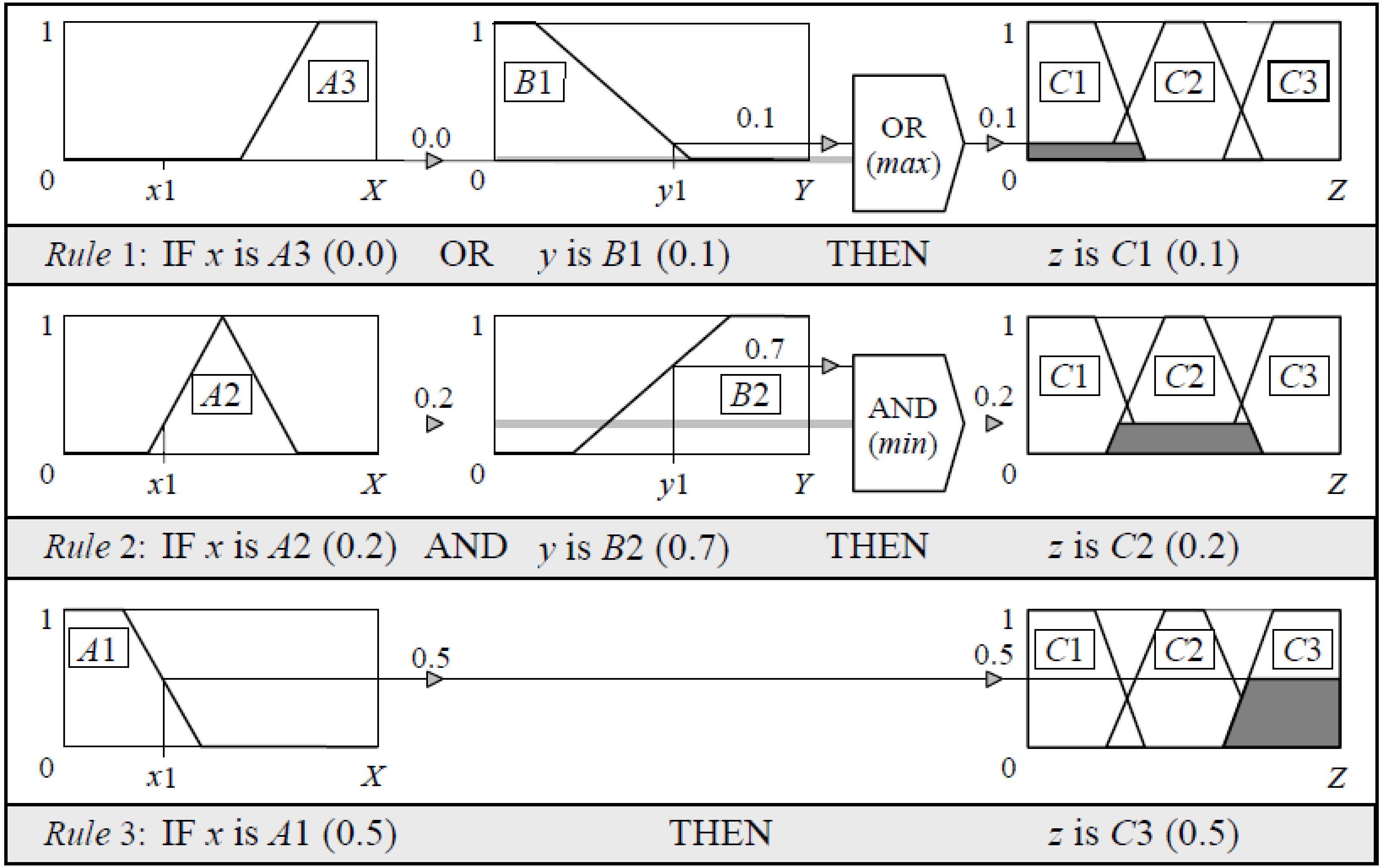
RECALL: To evaluate the disjunction of the rule antecedents, we use the **OR** fuzzy operation. Typically, fuzzy expert systems make use of the classical fuzzy operation union:

$$\mu_{A \cup B}(x) = \max [\mu_A(x), \mu_B(x)]$$

Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the **AND** fuzzy operation intersection:

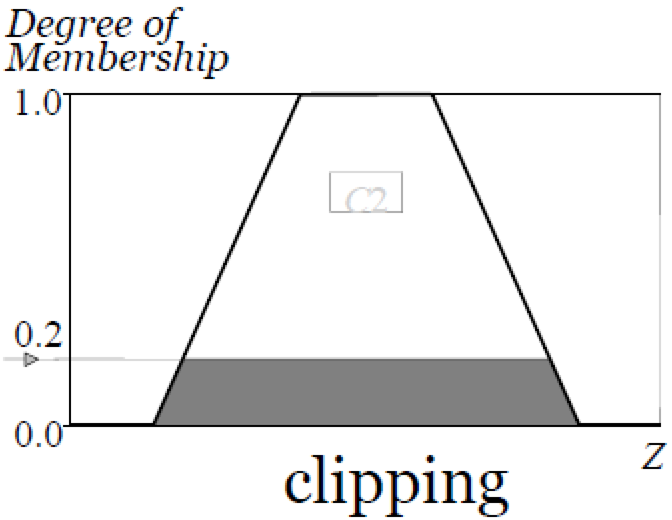
$$\mu_{A \cap B}(x) = \min [\mu_A(x), \mu_B(x)]$$

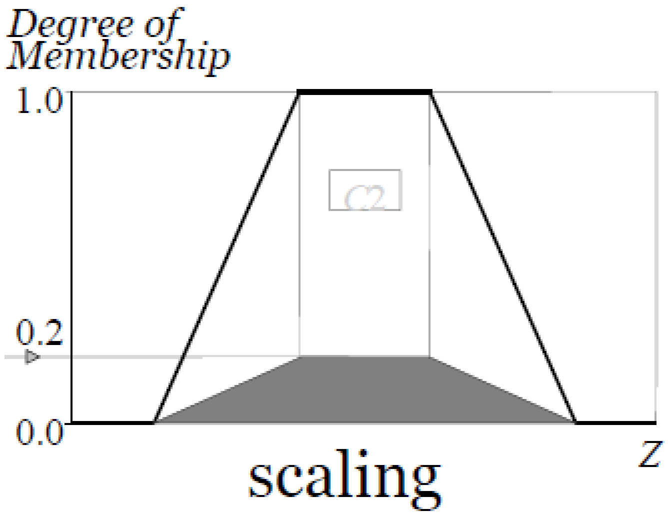
Mamdani Fuzzy Model



Mamdani Fuzzy Model

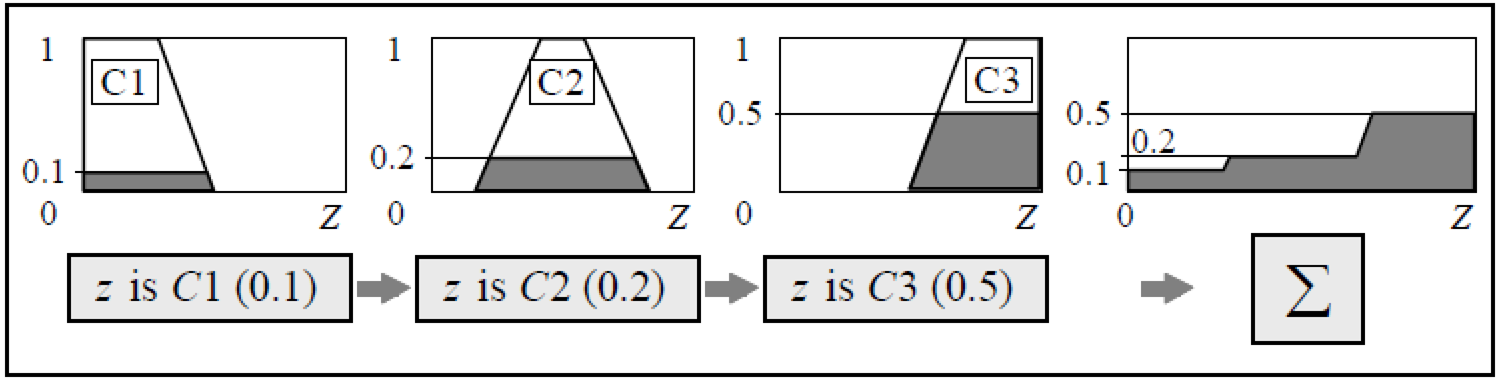
- Now the result of the antecedent evaluation can be applied to the membership function of the consequent.
- The most common method is to cut the consequent membership function at the level of the antecedent truth. This method is called **clipping** (alpha-cut).
  - Since the top of the membership function is sliced, the clipped fuzzy set loses some information.
  - However, clipping is still often preferred because it involves less complex and faster mathematics, and generates an aggregated output surface that is easier to defuzzify.
- While clipping is a frequently used method, **scaling** offers a better approach for preserving the original shape of the fuzzy set.
  - The original membership function of the rule consequent is adjusted by multiplying all its membership degrees by the truth value of the rule antecedent.
  - This method, which generally loses less information, can be very useful in fuzzy expert systems.





Mamdani Fuzzy Model

- Aggregation is the process of unification of the outputs of all rules.
- We take the membership functions of all rule consequents previously clipped or scaled and combine them into a single fuzzy set.
- The input of the aggregation process is the list of clipped or scaled consequent membership functions, and the output is one fuzzy set for each output variable.



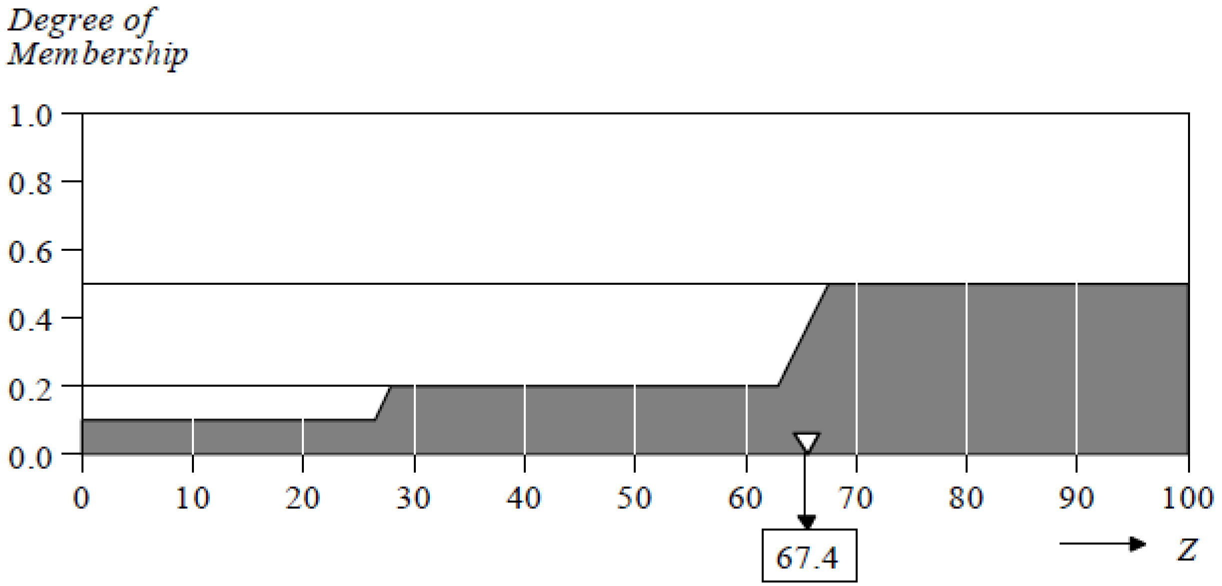
Mamdani Fuzzy Model

- The last step in the fuzzy inference process is defuzzification.
- Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.
- The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number.
- There are several defuzzification methods, but probably the most popular one is the **centroid technique**. It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this **centre of gravity** (**COG**) can be expressed as:

$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}$$

Mamdani Fuzzy Model

- Centroid defuzzification method finds a point representing the centre of gravity of the aggregated fuzzy set *A*, on the interval [*a*, *b*].
- A reasonable estimate can be obtained by calculating it over a sample of points.



$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$

Sugeno Fuzzy Inference

- Mamdani-style inference, as we have just seen, requires us to find the centroid of a two-dimensional shape by integrating across a continuously varying function. In general, this process is not computationally efficient.
- Michio Sugeno suggested to use a single spike, a singleton, as the membership function of the rule consequent.
- A singleton, or more precisely a fuzzy singleton, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

Sugeno Fuzzy Inference

- Also known as the TSK fuzzy model (proposed by Takagi, Sugeno, and Kang)
- For developing a systematic approach to generating fuzzy rules from a given input-output data set
- A typical fuzzy rule in a Sugeno fuzzy model:

  if *x is A and y is B then* $z = f(x, y)$
- A and B: fuzzy sets
- $z = f(x, y)$: a crisp function (usually polynomial in the input variables *x and y*)

Sugeno Fuzzy Inference

- Sugeno-style fuzzy inference is very similar to the Mamdani method.
- Sugeno changed only a rule consequent: instead of a fuzzy set, he used a mathematical function of the input variable.
- The format of the **Sugeno-style fuzzy rule** is

  IF ***x* is *A*** **AND** ***y* is *B*** **THEN** ***z* is *f*(*x*, *y*)**

  where:
  - *x*, *y* and *z* are linguistic variables;
  - A and *B* are fuzzy sets on universe of discourses *X* and *Y*, respectively;
  - *f* (*x*, *y*) is a mathematical function.

- The most commonly used **zero-order Sugeno fuzzy model** applies fuzzy rules in the following form:

  IF ***x* is *A*** **AND** ***y* is *B*** **THEN** ***z* is *k***
- where *k* is a constant.
- In this case, the output of each fuzzy rule is constant and all consequent **membership functions** are represented by **singleton spikes.**

Sugeno Fuzzy Inference

- **First-order Sugeno fuzzy model**: *f*(*x*, *y*) *is a first-order polynomial*
- **zero-order Sugeno fuzzy model**: *f* is a constant
  - a special case of the Mamdani fuzzy inference system, in which each rule's consequent is specified by a fuzzy singleton;
  - or a special case of the Tsukamoto fuzzy model (to be introduced next) in which each rule's consequent is specified by an MF of a step function center at the constant

Sugeno Fuzzy Inference

The output is a weighted average:

$$z = \frac{\sum \mu_{A_i,B_k}(x,y) f_{m(i,k)}(x,y)}{\sum \mu_{A_i,B_k}(x,y)}$$

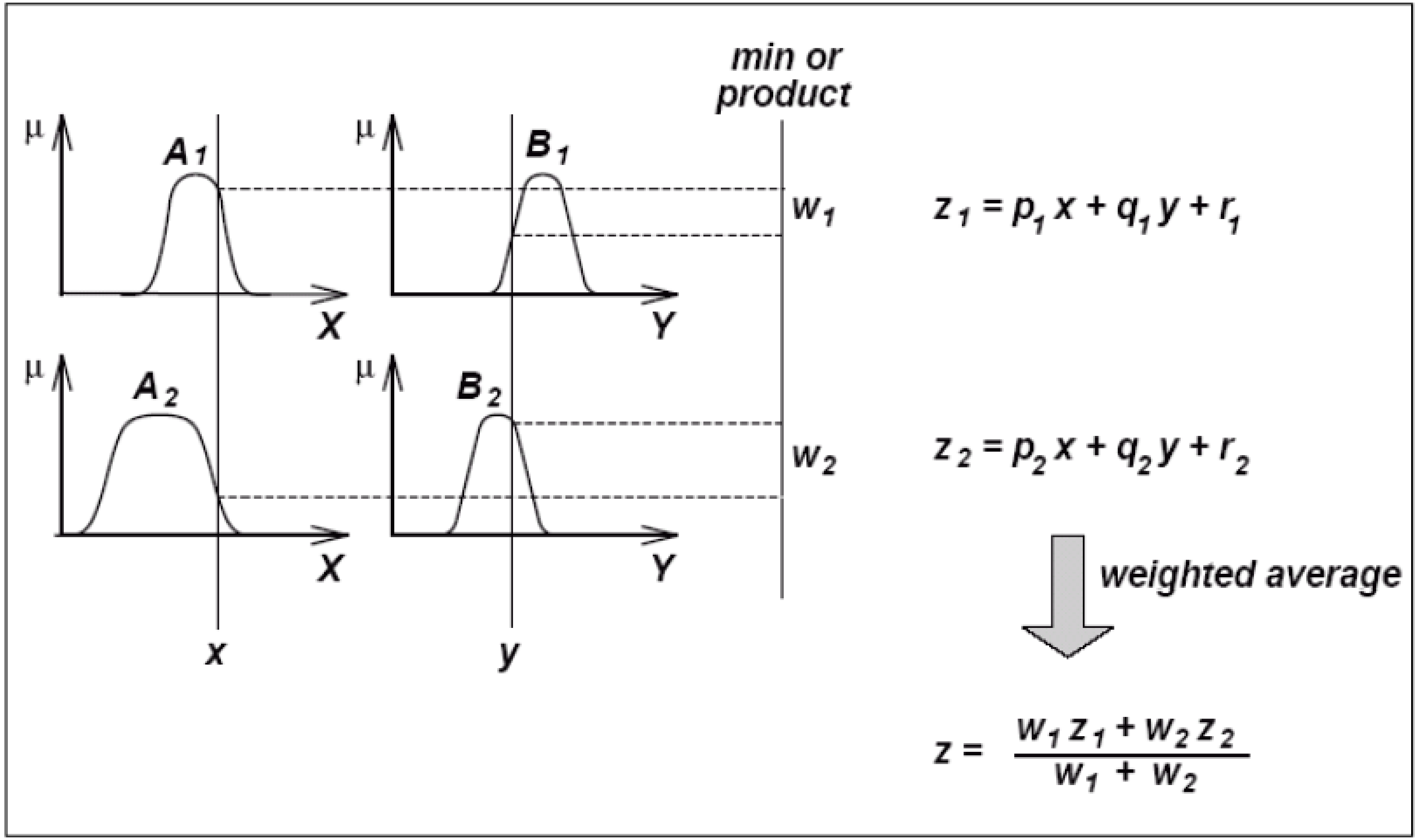
Double summation over all *i* (x MFs) and all *k* (y MFs)

$$= \frac{\sum w_i f_i(x,y)}{\sum w_i}$$

Summation over all *i* (fuzzy rules)

where w_i is the firing strength of the *i*-th output

Sugeno Fuzzy Inference

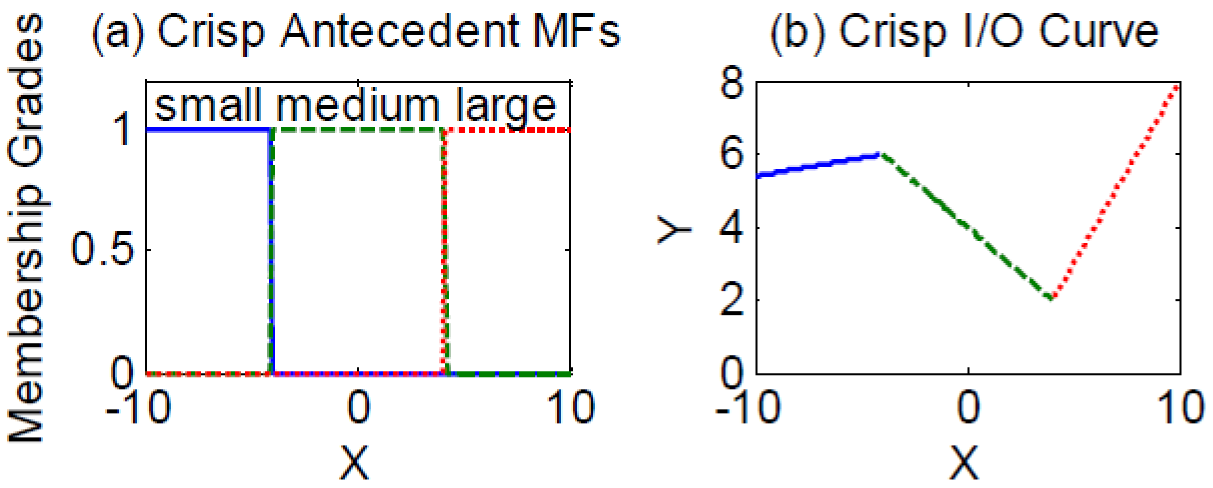


Sugeno Fuzzy Inference

- An example of a single-input Sugeno fuzzy model:
  - If *X is small then* $Y = 0.1X + 6.4$.
  - If *X is medium then* $Y = -0.5X + 4$.
  - If *X is large then* $Y = X - 2$.

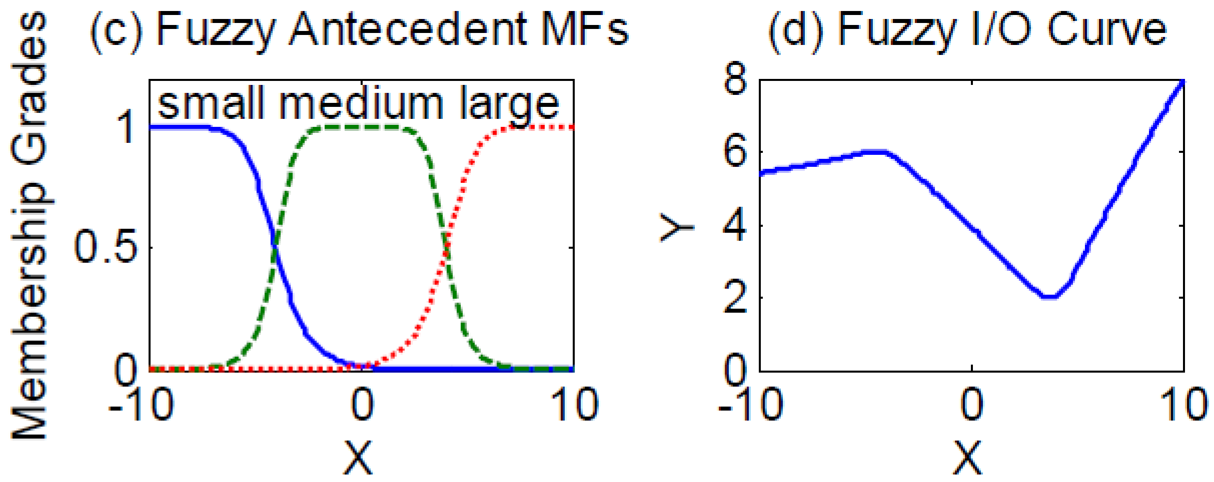
Sugeno Fuzzy Inference

- If "small," "medium," and "large" are nonfuzzy sets with membership functions shown in figure (a), then the overall input-output curve is piecewise linear, as shown in figure (b):



Sugeno Fuzzy Inference

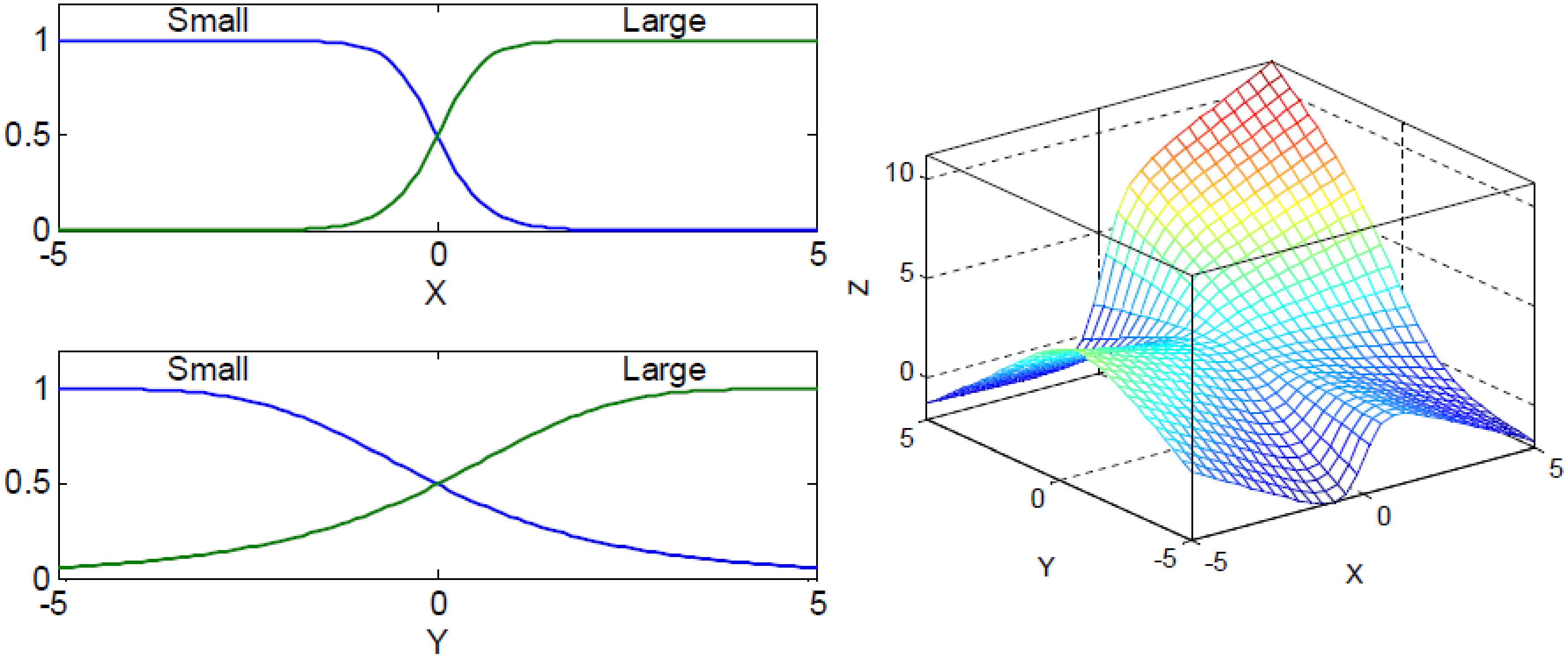
- If we have smooth membership functions [figure (c)] instead, the overall input-output curve [figure (d)] becomes a smoother one:



Sugeno Fuzzy Inference

- An example of a two-input single-output Sugeno fuzzy model with four rules:
  - If *X is small and Y is small then* $z = -x + y + 1$.
  - If *X is small and Y is large then* $z = -y + 3$.
  - If *X is large and Y is small then* $z = -x + 3$.
  - If *X is large and Y is large then* $z = x + y + 2$.

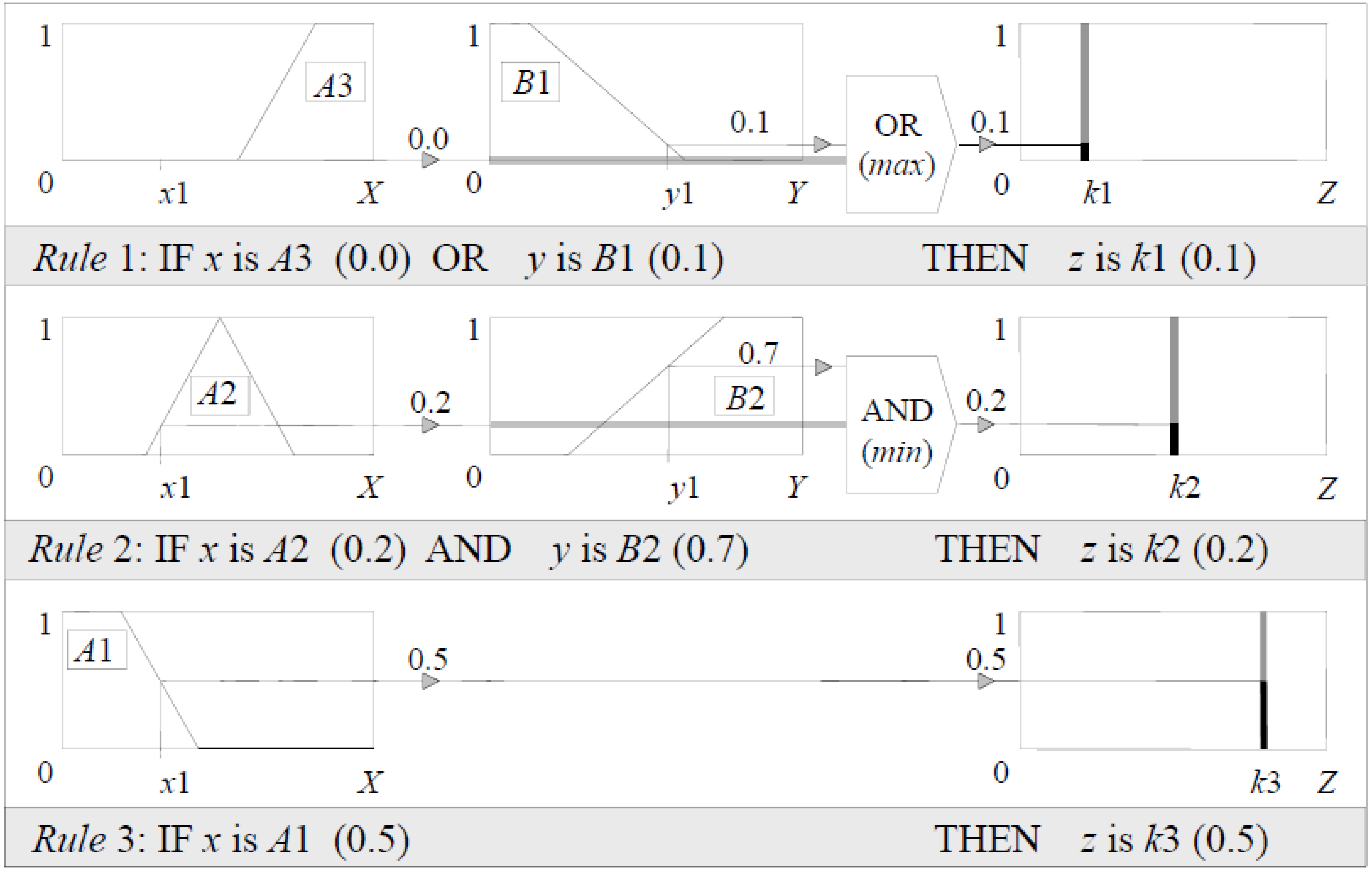
Sugeno Fuzzy Inference



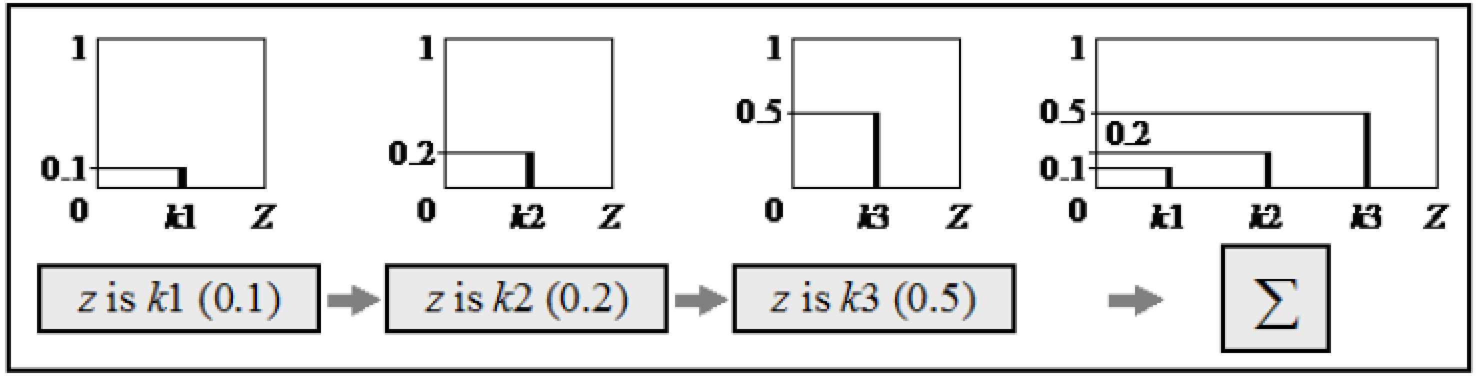
a) MFs of the inputs and output
b) Overall input-output curve

- The surface is composed of four planes, each of which is specified by the output equation of a fuzzy rule.

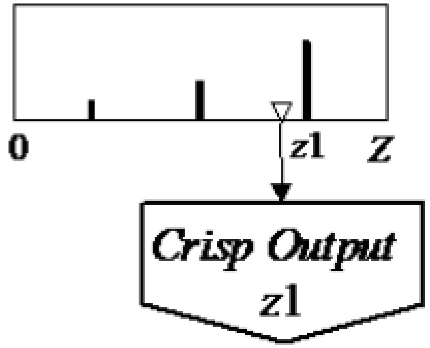
Sugeno Fuzzy Inference



Sugeno Fuzzy Inference



COG becomes Weighted Average (WA)



$$WA = \frac{\mu(k1)\times k1 + \mu(k2)\times k2 + \mu(k3)\times k3}{\mu(k1)+\mu(k2)+\mu(k3)} = \frac{0.1\times 20 + 0.2\times 50 + 0.5\times 80}{0.1+0.2+0.5} = 65$$

Sugeno Fuzzy Inference

- Unlike the Mamdani fuzzy model, the Sugeno fuzzy model cannot follow the compositional rule of inference strictly in its fuzzy reasoning mechanism
- Without the time-consuming and mathematically intractable defuzzification operation, the Sugeno fuzzy model is by far the most popular candidate for sample data-based fuzzy modeling (we will see an application in ANFIS)

Sugeno Fuzzy Inference

- Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, Mamdani-type fuzzy inference entails a substantial computational burden.
- On the other hand, Sugeno method is computationally effective and works well with optimisation and adaptive techniques, which makes it very attractive in adaptive problems, particularly for dynamic nonlinear systems.

Building a Fuzzy System

- A service centre keeps spare parts and repairs parts.
- A customer brings a failed item and receives a spare of the same type.
- Failed parts are repaired by **servers**, placed on the shelf, and thus become spares.
- The objective here is to advise a manager of the service centre on certain decision policies to keep the customers satisfied.
- Advise on the initial number of spares to keep **delay** reasonable

From: http://www2.cs.siu.edu/~rahimi

Building a Fuzzy System

There are four main linguistic variables: average waiting time (mean delay) *m*, repair utilisation factor of the service centre ρ, number of servers *s*, and initial number of spare parts *n*.

$$\rho = \frac{CustomerArrivalRate}{CustomerDepartureRate}$$

The system must advise management on the number of spares to keep as well as the number of servers. Increasing either will increase cost and decrease waiting time in some proportion.

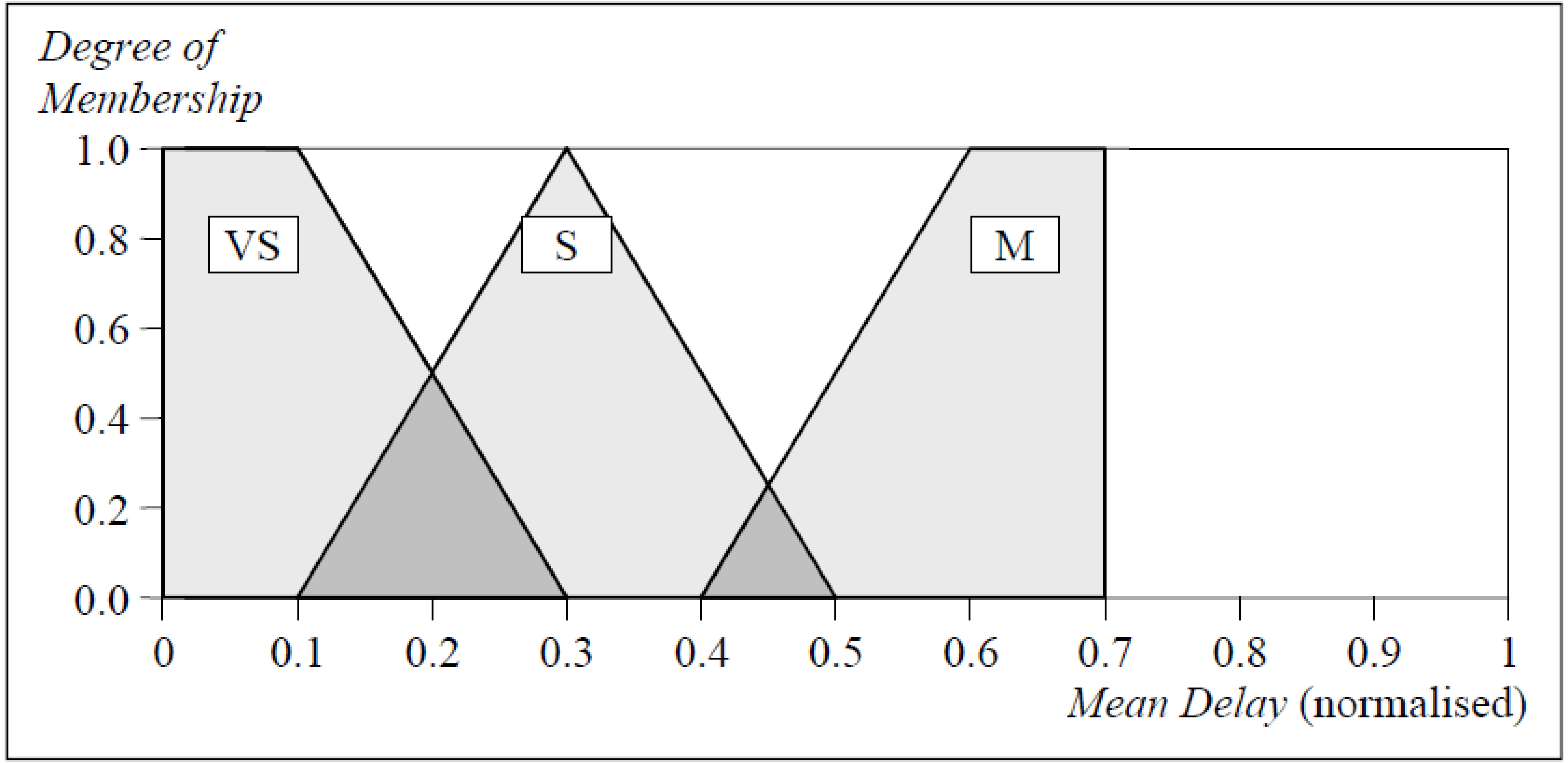
Building a Fuzzy System

Linguistic Variable: *Mean Delay, m*		
Linguistic Value	Notation	Numerical Range (normalised)
Very Short	VS	[0, 0.3]
Short	S	[0.1, 0.5]
Medium	M	[0.4, 0.7]
Linguistic Variable: *Number of Servers, s*		
Linguistic Value	Notation	Numerical Range (normalised)
Small	S	[0, 0.35]
Medium	M	[0.30, 0.70]
Large	L	[0.60, 1]
Linguistic Variable: *Repair Utilisation Factor*, ρ		
Linguistic Value	Notation	Numerical Range
Low	L	[0, 0.6]
Medium	M	[0.4, 0.8]
High	H	[0.6, 1]
Linguistic Variable: *Number of Spares, n*		
Linguistic Value	Notation	Numerical Range (normalised)
Very Small	VS	[0, 0.30]
Small	S	[0, 0.40]
Rather Small	RS	[0.25, 0.45]
Medium	M	[0.30, 0.70]
Rather Large	RL	[0.55, 0.75]
Large	L	[0.60, 1]
Very Large	VL	[0.70, 1]

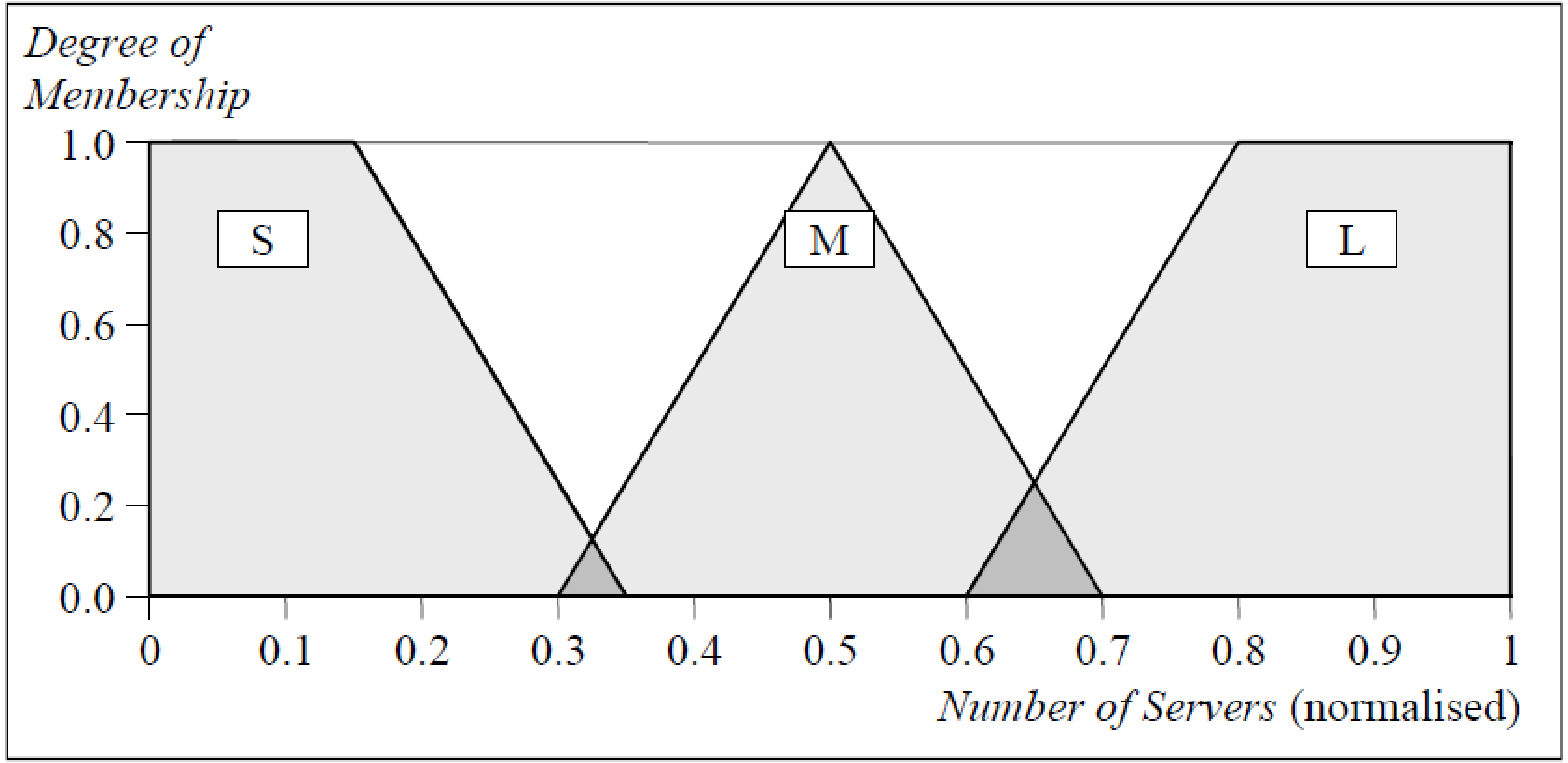
Building a Fuzzy System

Fuzzy sets can have a variety of shapes. However, a triangle or a trapezoid can often provide an adequate representation of the expert knowledge, and at the same time, significantly simplifies the process of computation.

Building a Fuzzy System



Building a Fuzzy System



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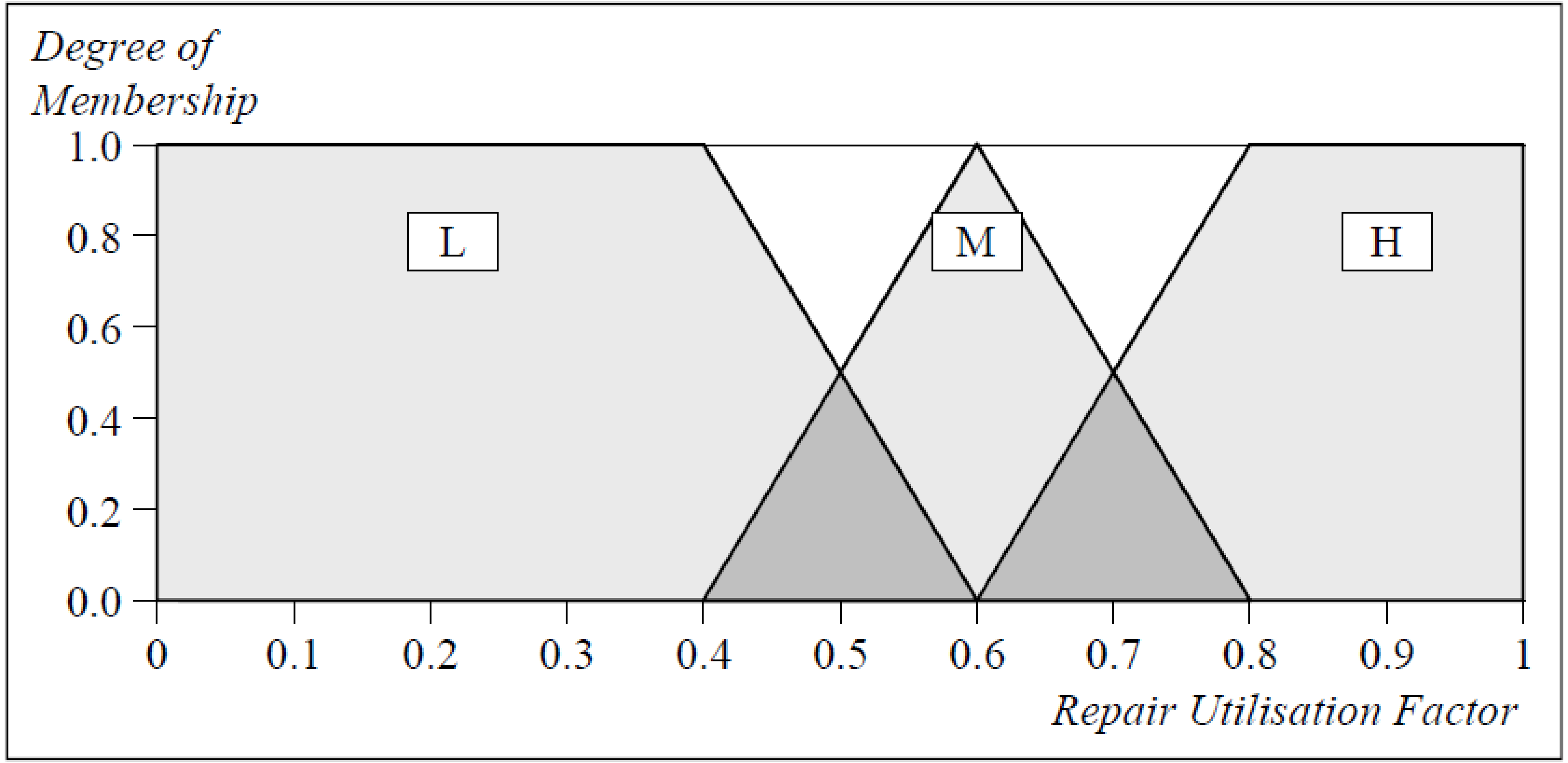

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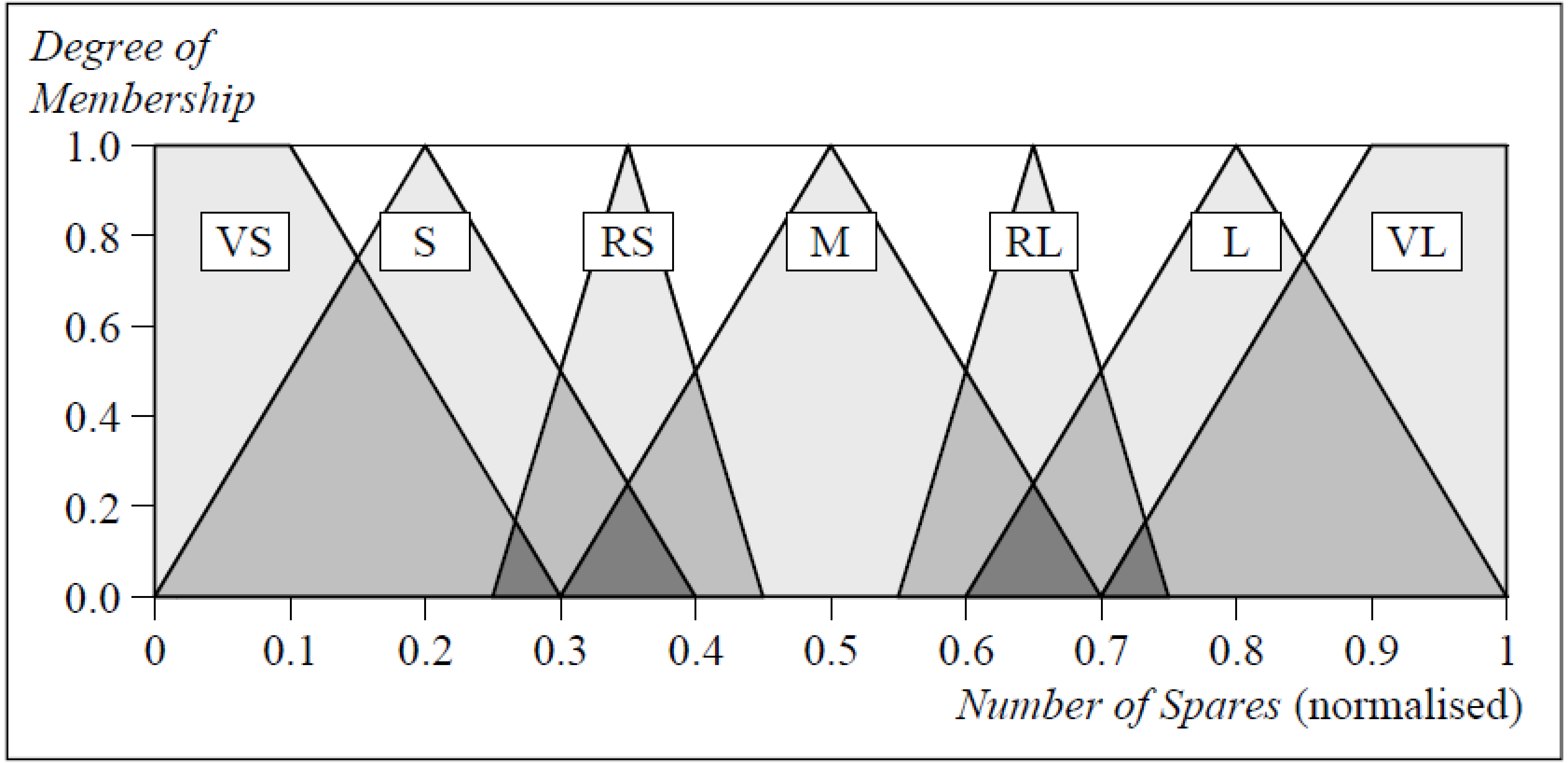

Create Fuzzy Rules

To accomplish this task, we might ask the expert to describe how the problem can be solved using the fuzzy linguistic variables defined previously.

Required knowledge also can be collected from other sources such as books, computer databases, flow diagrams and observed human behaviour.

Create Fuzzy Rules

1. If (utilisation_factor is L) then (number_of_spares is S)
2. If (utilisation_factor is M) then (number_of_spares is M)
3. If (utilisation_factor is H) then (number_of_spares is L)
4. If (mean_delay is VS) and (number_of_servers is S) then (number_of_spares is VL)
5. If (mean_delay is S) and (number_of_servers is S) then (number_of_spares is L)
6. If (mean_delay is M) and (number_of_servers is S) then (number_of_spares is M)
7. If (mean_delay is VS) and (number_of_servers is M) then (number_of_spares is RL)
8. If (mean_delay is S) and (number_of_servers is M) then (number_of_spares is RS)
9. If (mean_delay is M) and (number_of_servers is M) then (number_of_spares is S)
10. If (mean_delay is VS) and (number_of_servers is L) then (number_of_spares is M)
11. If (mean_delay is S) and (number_of_servers is L) then (number_of_spares is S)
12. If (mean_delay is M) and (number_of_servers is L) then (number_of_spares is VS)

Create Fuzzy Rules

Rule	m	s	ρ	n	Rule	m	s	ρ	n	Rule	m	s	ρ	n
1	VS	S	L	VS	10	VS	S	M	S	19	VS	S	H	VL
2	S	S	L	VS	11	S	S	M	VS	20	S	S	H	L
3	M	S	L	VS	12	M	S	M	VS	21	M	S	H	M
4	VS	M	L	VS	13	VS	M	M	RS	22	VS	M	H	M
5	S	M	L	VS	14	S	M	M	S	23	S	M	H	M
6	M	M	L	VS	15	M	M	M	VS	24	M	M	H	S
7	VS	L	L	S	16	VS	L	M	M	25	VS	L	H	RL
8	S	L	L	S	17	S	L	M	RS	26	S	L	H	M
9	M	L	L	VS	18	M	L	M	S	27	M	L	H	RS

if mean_delay is VS
and number_servers is S
and utilization is Low

Evaluation and Tuning

- The last and the most laborious task is to evaluate and tune the system. We want to see whether our fuzzy system meets the requirements specified at the beginning.
- Several test situations depend on the mean delay, number of servers and repair utilisation factor.
- The MatLab's Fuzzy Logic Toolbox can generate surface to help us analyse the system's performance.
- However, the expert might not be satisfied with the system performance.
- To improve the system performance, we may use additional sets – *Rather Small* and *Rather Large* – on the universe of discourse *Number of Servers*, and then extend the rule base.

Evaluation and Tuning

1. Review model input and output variables, and if required redefine their ranges.
2. Review the fuzzy sets, and if required define additional sets on the universe of discourse.
3. Provide sufficient overlap between neighbouring sets. It is suggested that triangle-to-triangle and trapezoid-to-triangle fuzzy sets should overlap between 25% to 50% of their bases.

Evaluation and Tuning

4. Review the existing rules, and if required add new rules to the rule base.
5. Examine the rule-base for opportunities to write hedge rules to capture the pathological behaviour of the system.
6. Adjust the rule execution weights. Most fuzzy logic tools allow control of the importance of rules by changing a weight multiplier
7. Revise shapes of the fuzzy sets. In most cases, fuzzy systems are highly tolerant of a shape approximation.

Evaluation and Tuning

- certain common issues concerning all these three fuzzy inference systems
  - how to partition an input space
  - how to construct a fuzzy inference system for a particular application

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Fuzzy Systems

Adopted from Fei-Fei Li, Justin Johnson, Serena Yeung

Calculations

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Calculations

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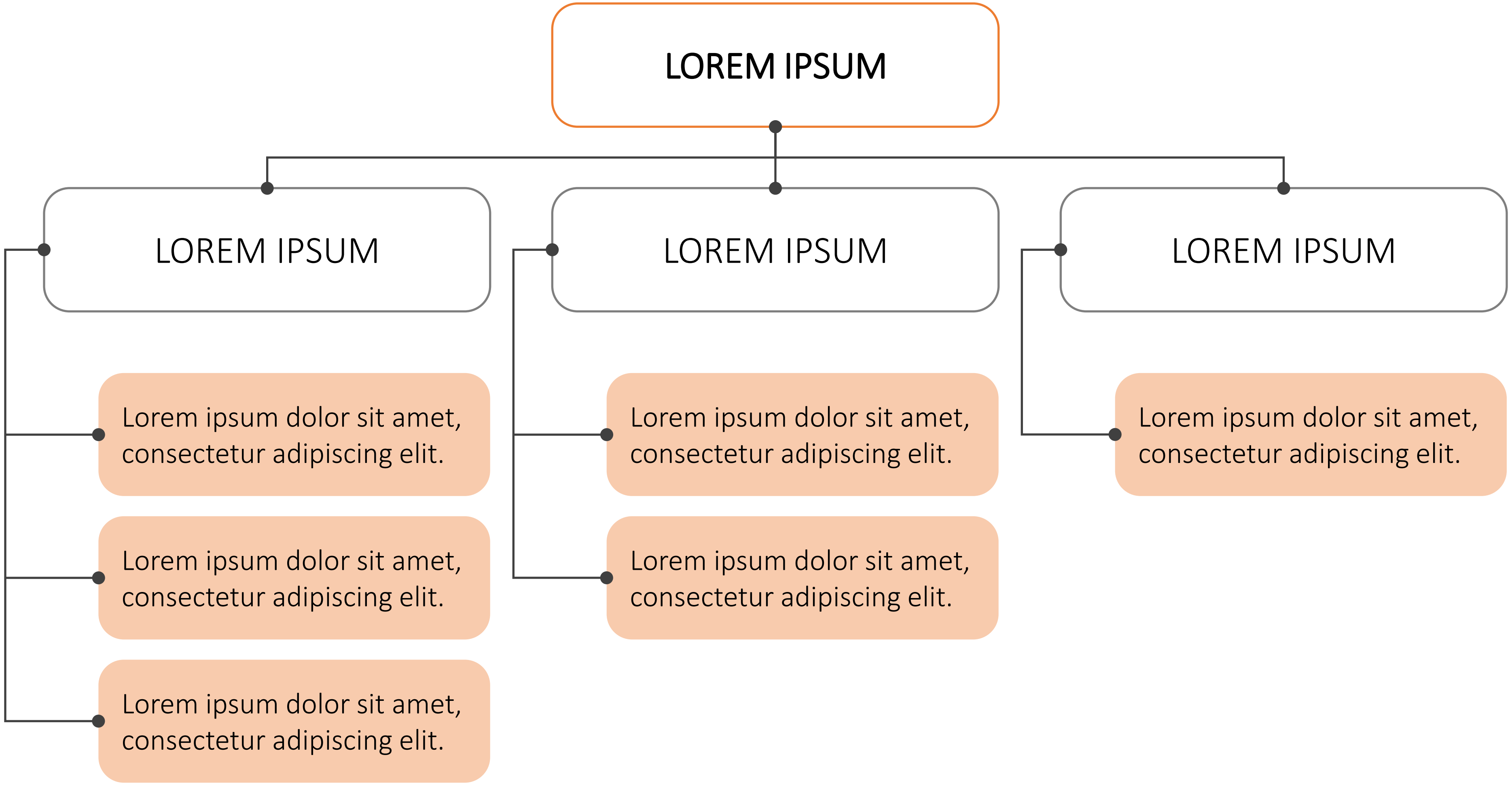

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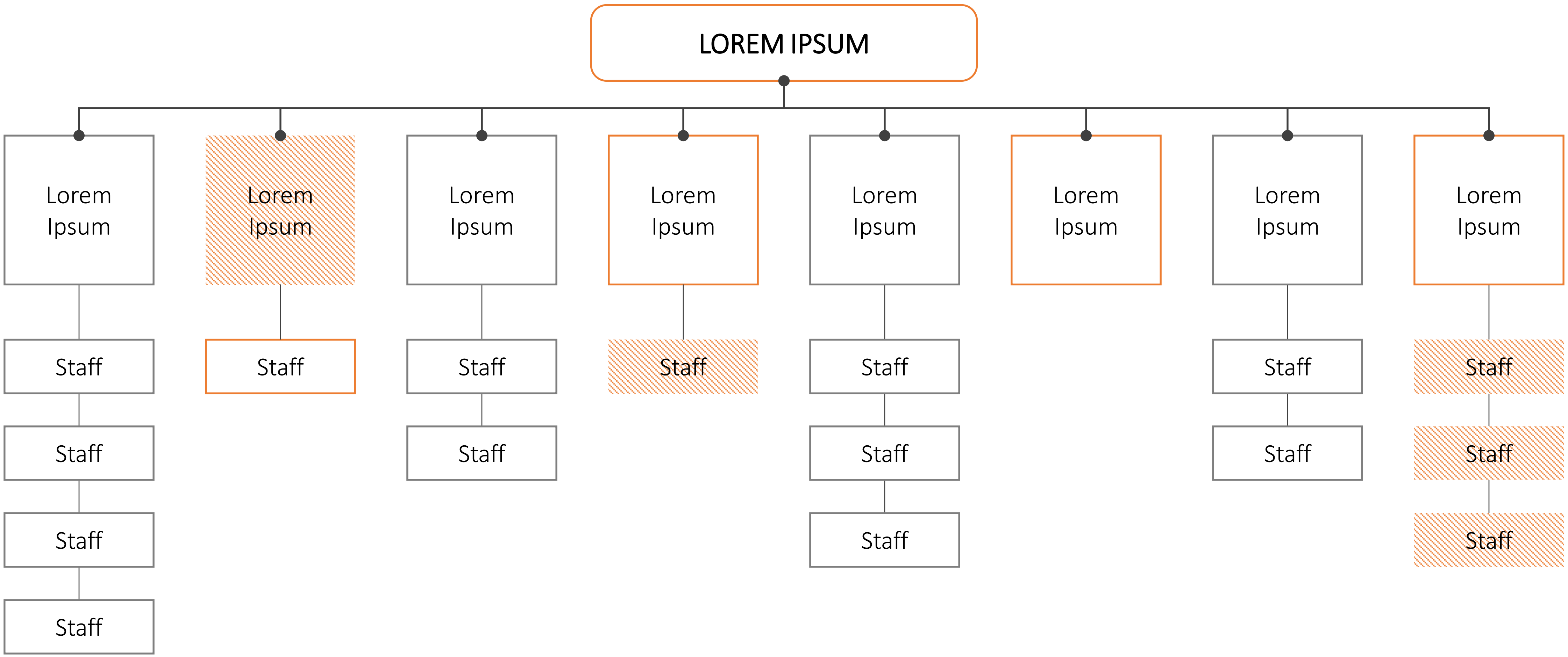

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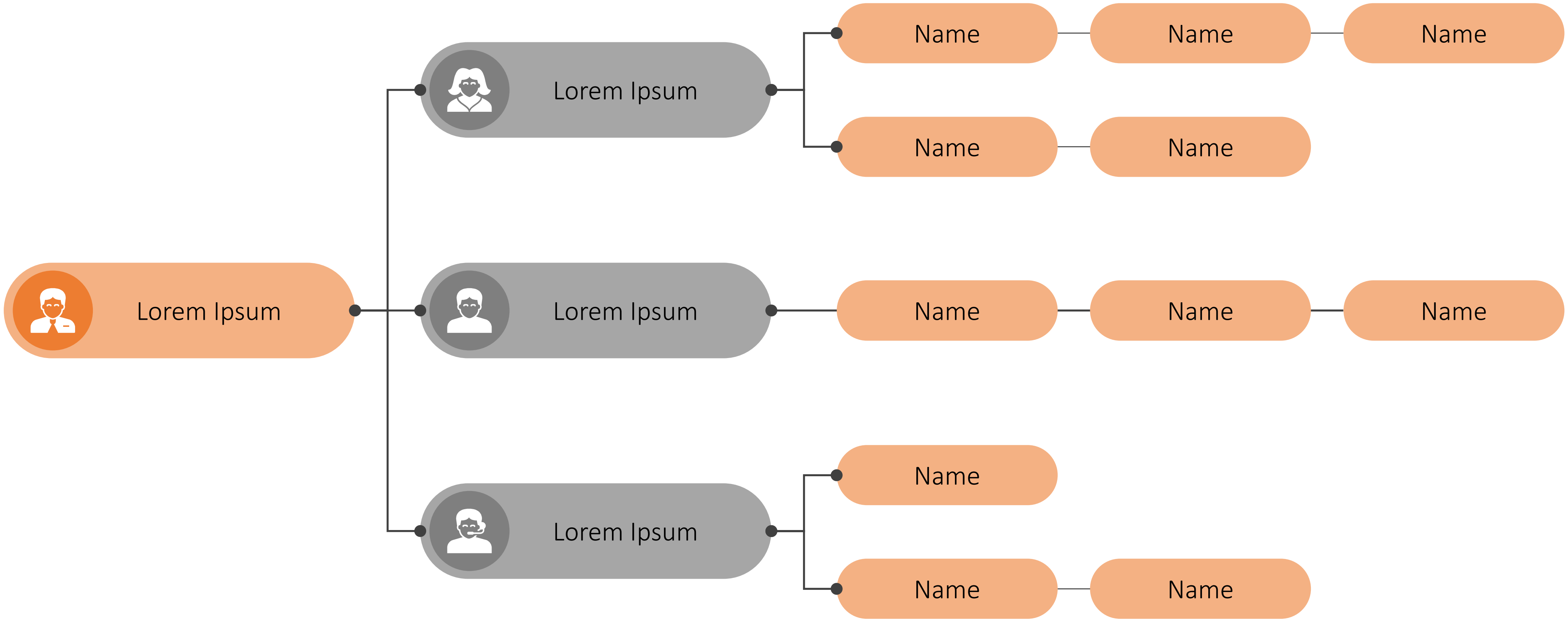

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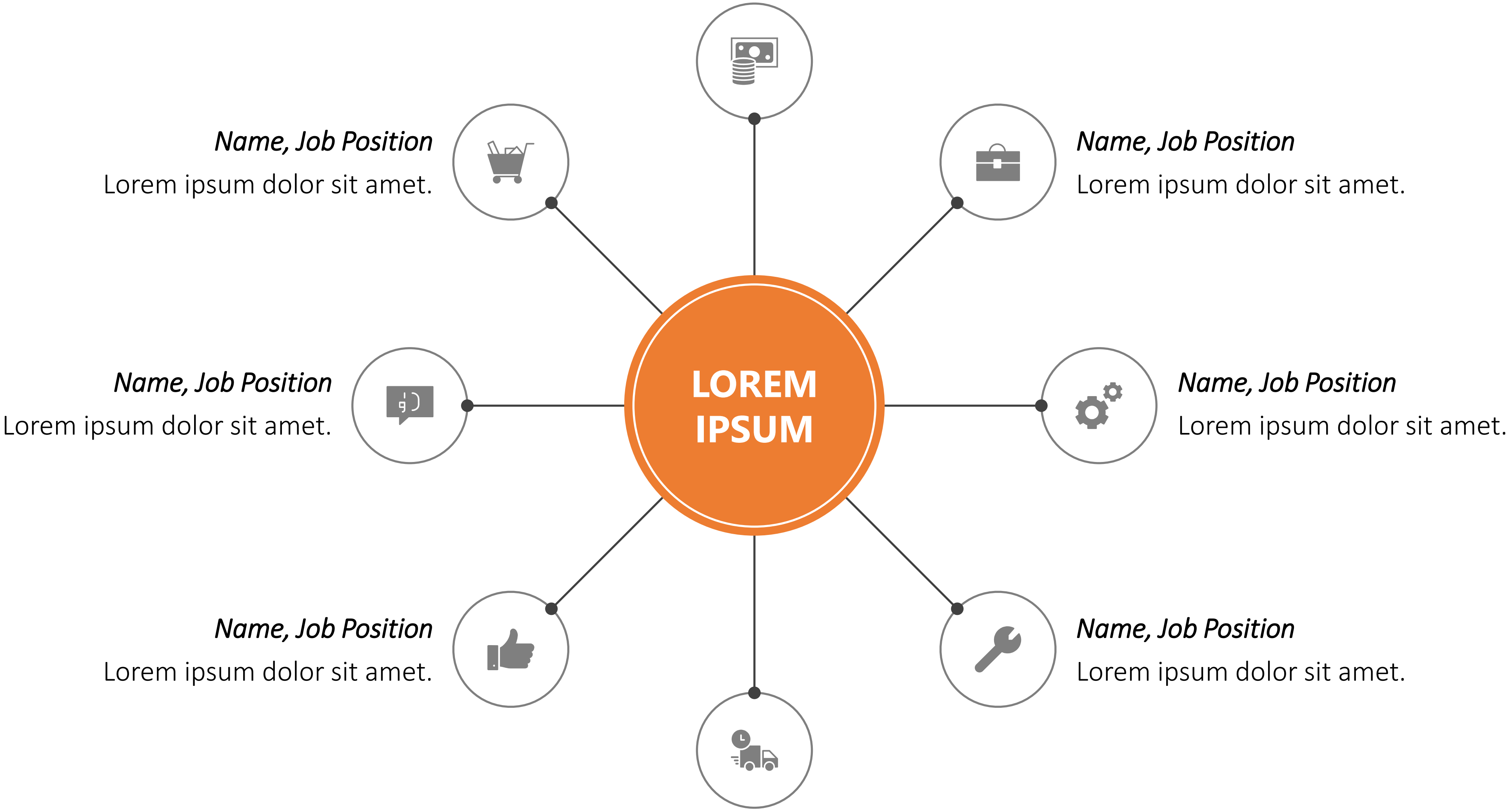

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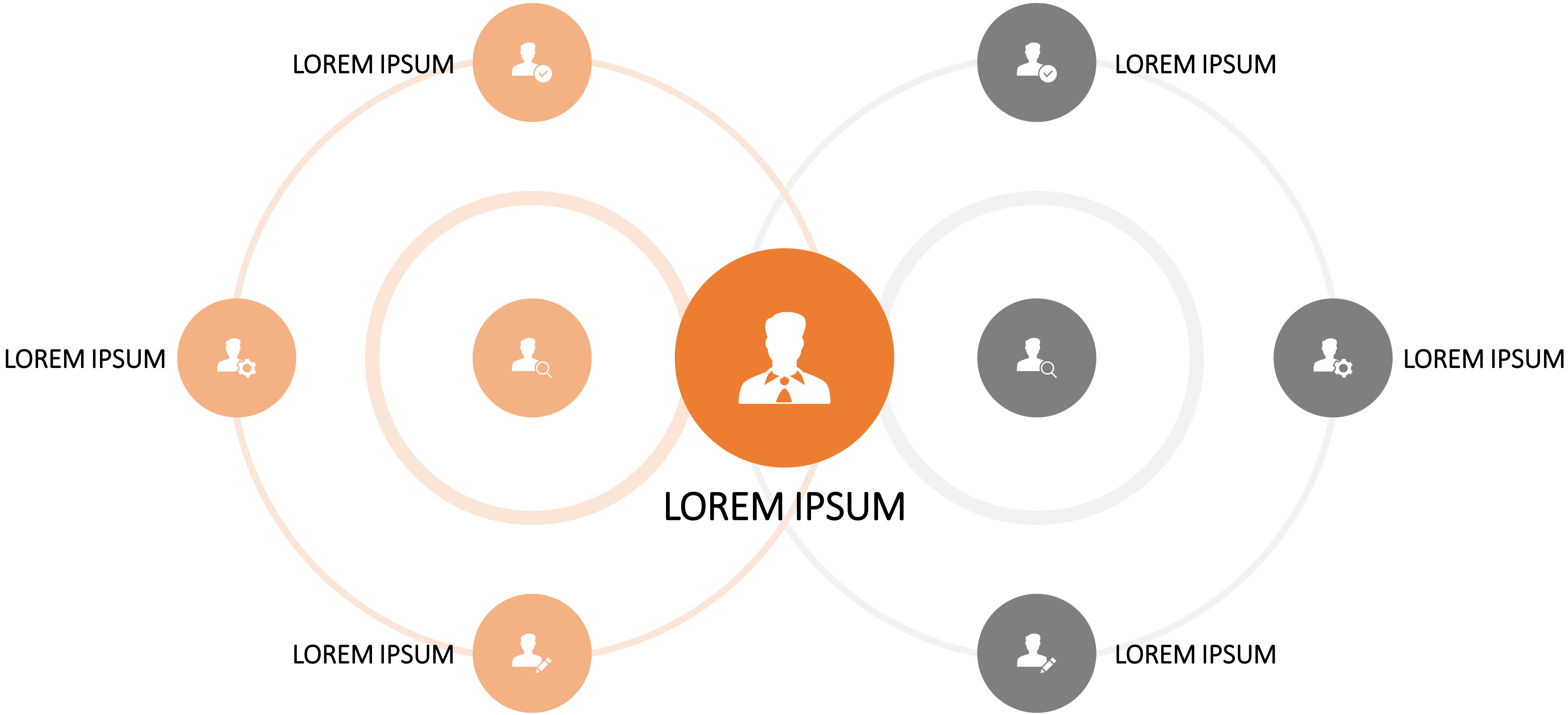

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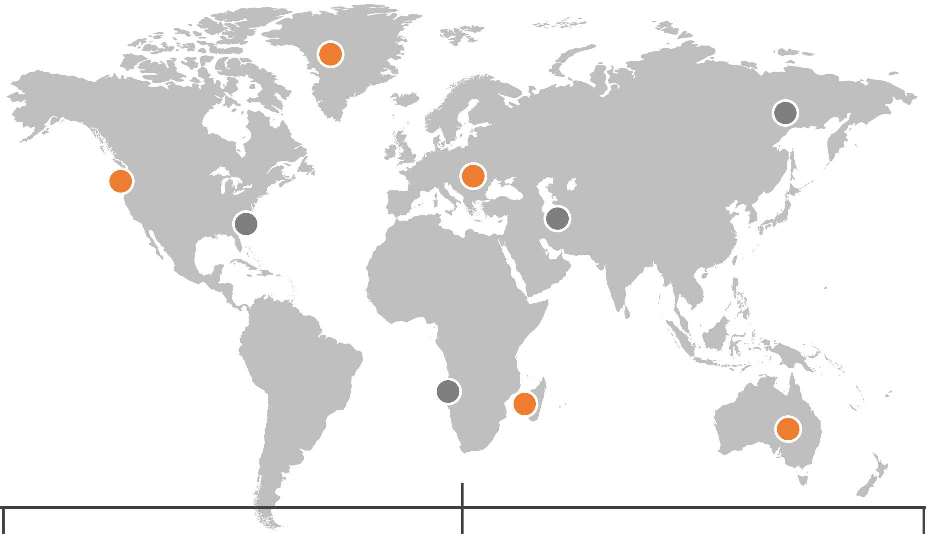

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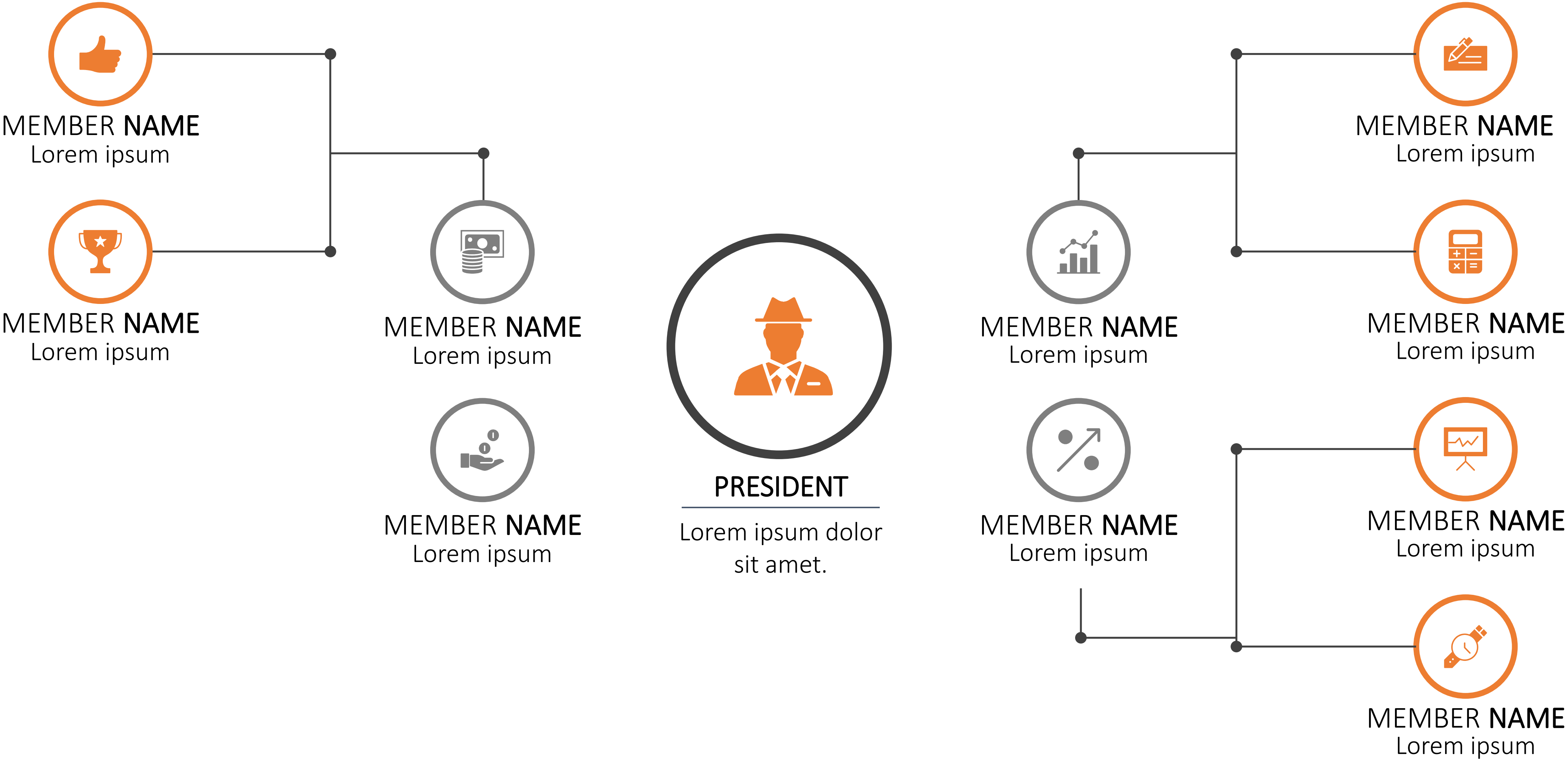

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