CHAPTER 13

Mechanical Design of Process Equipment

13.1. INTRODUCTION

This chapter covers those aspects of the mechanical design of chemical plant that are of particular interest to chemical engineers. The main topic considered is the design of pressure vessels. The design of storage tanks, centrifuges and heat-exchanger tube sheets are also discussed briefly.

The chemical engineer will not usually be called on to undertake the detailed mechanical design of a pressure vessel. Vessel design is a specialised subject, and will be carried out by mechanical engineers who are conversant with the current design codes and practices, and methods of stress analysis. However, the chemical engineer will be responsible for developing and specifying the basic design information for a particular vessel, and needs to have a general appreciation of pressure vessel design to work effectively with the specialist designer.

The basic data needed by the specialist designer will be:

1. Vessel function.
2. Process materials and services.
3. Operating and design temperature and pressure.
4. Materials of construction.
5. Vessel dimensions and orientation.
6. Type of vessel heads to be used.
7. Openings and connections required.
8. Specification of heating and cooling jackets or coils.
9. Type of agitator.
10. Specification of internal fittings.

A data sheet for pressure vessel design is given in Appendix G.

There is no strict definition of what constitutes a pressure vessel, but it is generally accepted that any closed vessel over 150 mm diameter subject to a pressure difference of more than 0.5 bar should be designed as a pressure vessel.

It is not possible to give a completely comprehensive account of vessel design in one chapter. The design methods and data given should be sufficient for the preliminary design of conventional vessels. Sufficient for the chemical engineer to check the feasibility of a proposed equipment design; to estimate the vessel cost for an economic analysis; and to determine the vessel's general proportions and weight for plant layout purposes. For a more detailed account of pressure vessel design the reader should refer to the books by Singh and Soler (1992), Escoe (1994) and Moss (1987). Other useful books on the mechanical design of process equipment are listed in the bibliography at the end of this chapter.

An elementary understanding of the principles of the "Strength of Materials" (Mechanics of Solids) will be needed to follow this chapter. Readers who are not familiar with the subject should consult one of the many textbooks available; such as those by Case *et al.* (1999), Mott, R. L. (2001), Seed (2001) and Gere and Timoshenko (2000).

13.1.1. Classification of pressure vessels

For the purposes of design and analysis, pressure vessels are sub-divided into two classes depending on the ratio of the wall thickness to vessel diameter: thin-walled vessels, with a thickness ratio of less than 1 : 10; and thick-walled above this ratio.

The principal stresses (see Section 13.3.1) acting at a point in the wall of a vessel, due to a pressure load, are shown in Figure 13.1. If the wall is thin, the radial stress σ_3 will be small and can be neglected in comparison with the other stresses, and the longitudinal and circumferential stresses σ_1 and σ_2 can be taken as constant over the wall thickness. In a thick wall, the magnitude of the radial stress will be significant, and the circumferential stress will vary across the wall. The majority of the vessels used in the chemical and allied industries are classified as thin-walled vessels. Thick-walled vessels are used for high pressures, and are discussed in Section 13.15.

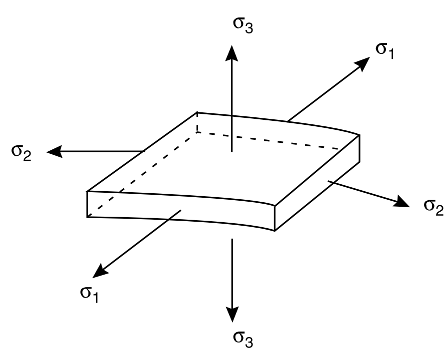


Figure 13.1. Principal stresses in pressure-vessel wall

13.2. PRESSURE VESSEL CODES AND STANDARDS

In all the major industrialised countries the design and fabrication of thin-walled pressure vessels is covered by national standards and codes of practice. In most countries the standards and codes are legally enforceable.

In the United Kingdom all conventional pressure vessels for use in the chemical and allied industries will invariably be designed and fabricated according to the British Standard PD 5500 or the European Standard EN 13445; or an equivalent code such as the American Society of Mechanical Engineers code Section VIII (the ASME code). The codes and standards cover design, materials of construction, fabrication (manufacture and workmanship), and inspection and testing. They form a basis of agreement between the manufacturer and customer, and the customer's insurance company.

In the European Union the design, manufacture and use of pressure systems is also covered by the Pressure Equipment Directive (Council Directive 97/23/EC) whose use became mandatory in May 2002.

The current (2003) edition of PD 5500 covers vessels fabricated in carbon and alloy steels, and aluminium. The design of vessels constructed from reinforced plastics is covered by BS 4994. The ASME code covers steels, non-ferrous metals, and fibre-reinforced plastics.

Where national codes are not available, the British, European or American codes would be used.

Information and guidance on the pressure vessel codes can be found on the Internet; www.bsi-global.com.

A comprehensive review of the ASME code is given by Chuse and Carson (1992) and Yokell (1986); see also Perry *et al.* (1997).

The national codes and standards dictate the minimum requirements, and give general guidance for design and construction; any extension beyond the minimum code requirement will be determined by agreement between the manufacturer and customer.

The codes and standards are drawn up by committees of engineers experienced in vessel design and manufacturing techniques, and are a blend of theory, experiment and experience. They are periodically reviewed, and revisions issued to keep abreast of developments in design, stress analysis, fabrication and testing. The latest version of the appropriate national code or standard should always be consulted before undertaking the design of any pressure vessel.

Computer programs to aid in the design of vessels to PD 5500 and the ASME code are available from several commercial organisations and can be found by making a search of the World Wide Web.

13.3. FUNDAMENTAL PRINCIPLES AND EQUATIONS

This section has been included to provide a basic understanding of the fundamental principles that underlie the design equations given in the sections that follow. The derivation of the equations is given in outline only. A full discussion of the topics covered can be found in any text on the "Strength of Materials" (Mechanics of Solids).

13.3.1. Principal stresses

The state of stress at a point in a structural member under a complex system of loading is described by the magnitude and direction of the principal stresses. The principal stresses are the maximum values of the normal stresses at the point; which act on planes on which the shear stress is zero. In a two-dimensional stress system, Figure 13.2, the principal stresses at any point are related to the normal stresses in the x and y directions σ_x and σ_y and the shear stress τ_{xy} at the point by the following equation:

$$\text{Principal stresses, } \sigma_1, \sigma_2 = \frac{1}{2}(\sigma_y + \sigma_x) \pm \frac{1}{2}\sqrt{[(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2]} \tag{13.1}$$

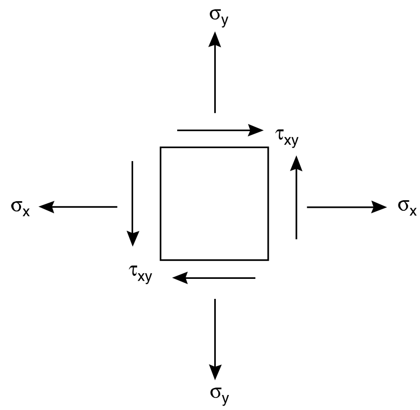


Figure 13.2. Two-dimensional stress system

The maximum shear stress at the point is equal to half the algebraic difference between the principal stresses:

$$\text{Maximum shear stress } = \tfrac{1}{2}(\sigma_1 - \sigma_2) \tag{13.2}$$

Compressive stresses are conventionally taken as negative; tensile as positive.

13.3.2. Theories of failure

The failure of a simple structural element under unidirectional stress (tensile or compressive) is easy to relate to the tensile strength of the material, as determined in a standard tensile test, but for components subjected to combined stresses (normal and shear stress) the position is not so simple, and several theories of failure have been proposed. The three theories most commonly used are described below:

Maximum principal stress theory: which postulates that a member will fail when one of the principal stresses reaches the failure value in simple tension, σ'_e. The failure point in a simple tension is taken as the yield-point stress, or the tensile strength of the material, divided by a suitable factor of safety.

Maximum shear stress theory: which postulates that failure will occur in a complex stress system when the maximum shear stress reaches the value of the shear stress at failure in simple tension.

For a system of combined stresses there are three shear stresses maxima:

$$\tau_1 = \frac{\sigma_1 - \sigma_2}{2} \tag{13.3a}$$

$$\tau_2 = \frac{\sigma_2 - \sigma_3}{2} \tag{13.3b}$$

$$\tau_3 = \frac{\sigma_3 - \sigma_1}{2} \tag{13.3c}$$

In the tensile test,

$$\tau_e = \frac{\sigma'_e}{2} \tag{13.4}$$

The maximum shear stress will depend on the sign of the principal stresses as well as their magnitude, and in a two-dimensional stress system, such as that in the wall of a thin-walled pressure vessel, the maximum value of the shear stress may be that given by putting $\sigma_3 = 0$ in equations 13.3*b* and *c*.

The maximum shear stress theory is often called Tresca's, or Guest's, theory.

Maximum strain energy theory: which postulates that failure will occur in a complex stress system when the total strain energy per unit volume reaches the value at which failure occurs in simple tension.

The maximum shear-stress theory has been found to be suitable for predicting the failure of ductile materials under complex loading and is the criterion normally used in the pressure-vessel design.

13.3.3. Elastic stability

Under certain loading conditions failure of a structure can occur not through gross yielding or plastic failure, but by buckling, or wrinkling. Buckling results in a gross and sudden change of shape of the structure; unlike failure by plastic yielding, where the structure retains the same basic shape. This mode of failure will occur when the structure is not elastically stable: when it lacks sufficient stiffness, or rigidity, to withstand the load. The stiffness of a structural member is dependent not on the basic strength of the material but on its elastic properties (E and v) and the cross-sectional shape of the member.

The classic example of failure due to elastic instability is the buckling of tall thin columns (struts), which is described in any elementary text on the "Strength of Materials".

For a structure that is likely to fail by buckling there will be a certain critical value of load below which the structure is stable; if this value is exceeded catastrophic failure through buckling can occur.

The walls of pressure vessels are usually relatively thin compared with the other dimensions and can fail by buckling under compressive loads.

Elastic buckling is the decisive criterion in the design of thin-walled vessels under external pressure.

13.3.4. Membrane stresses in shells of revolution

A shell of revolution is the form swept out by a line or curve rotated about an axis. (A solid of revolution is formed by rotating an area about an axis.) Most process vessels are made up from shells of revolution: cylindrical and conical sections; and hemispherical, ellipsoidal and torispherical heads; Figure 13.3.

The walls of thin vessels can be considered to be "membranes"; supporting loads without significant bending or shear stresses; similar to the walls of a balloon.

The analysis of the membrane stresses induced in shells of revolution by internal pressure gives a basis for determining the minimum wall thickness required for vessel shells. The actual thickness required will also depend on the stresses arising from the other loads to which the vessel is subjected.

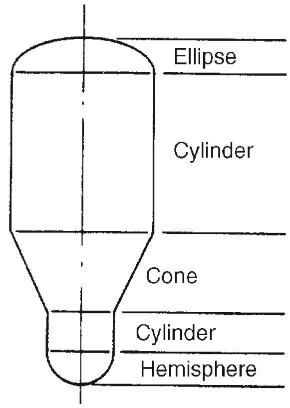


Figure 13.3. Typical vessel shapes

Consider the shell of revolution of general shape shown in Figure 13.4, under a loading that is rotationally symmetric; that is, the load per unit area (pressure) on the shell is constant round the circumference, but not necessarily the same from top to bottom.

Let P = pressure,
t = thickness of shell,
σ_1 = the meridional (longitudinal) stress, the stress acting along a meridian,
σ_2 = the circumferential or tangential stress, the stress acting along parallel circles (often called the hoop stress),
r_1 = the meridional radius of curvature,
r_2 = circumferential radius of curvature.

Note: the vessel has a double curvature; the values of r_1 and r_2 are determined by the shape.

Consider the forces acting on the element defined by the points a, b, c, d. Then the normal component (component acting at right angles to the surface) of the pressure force on the element

$$= P\left[2r_1 \sin\left(\frac{d\theta_1}{2}\right)\right]\left[2r_2 \sin\left(\frac{d\theta_2}{2}\right)\right]$$

This force is resisted by the normal component of the forces associated with the membrane stresses in the walls of the vessel (given by, force = stress × area)

$$= 2\sigma_2 t \mathrm{d}S_1 \sin\left(\frac{\mathrm{d}\theta_2}{2}\right) + 2\sigma_1 t\, \mathrm{d}S_2 \sin\left(\frac{\mathrm{d}\theta_1}{2}\right)$$

Equating these forces and simplifying, and noting that in the limit $\mathrm{d}\theta/2 \to \mathrm{d}S/2r$, and $\sin \mathrm{d}\theta \to \mathrm{d}\theta$, gives:

$$\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = \frac{P}{t} \quad (13.5)$$

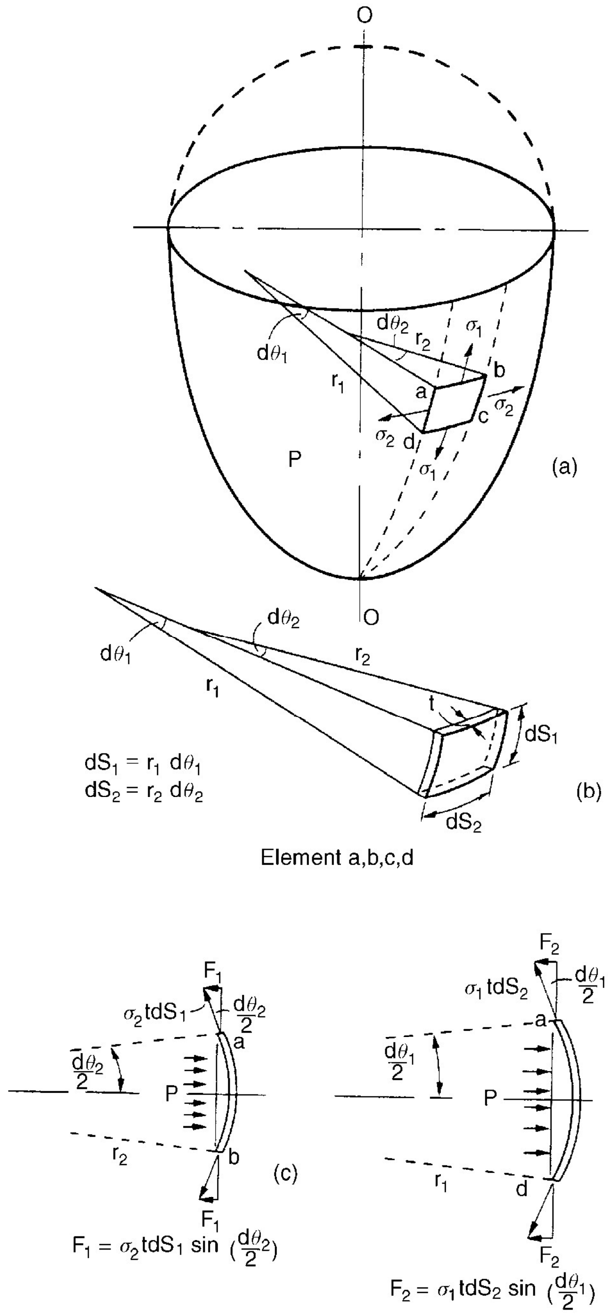


Figure 13.4(*a*)(*b*). Stress in a shell of revolution (*c*)(*d*). Forces acting on sides of element *abcd*

An expression for the meridional stress σ_1 can be obtained by considering the equilibrium of the forces acting about any circumferential line, Figure 13.5. The vertical component of the pressure force

$$= P\pi(r_2 \sin\theta)^2$$

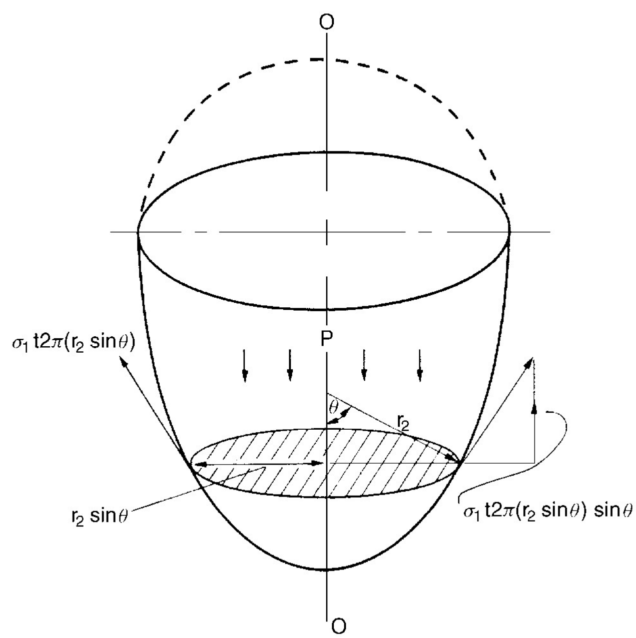


Figure 13.5. Meridional stress, force acting at a horizontal plane

This is balanced by the vertical component of the force due to the meridional stress acting in the ring of the wall of the vessel

$$= 2\sigma_1 t\pi(r_2 \sin\theta)\sin\theta$$

Equating these forces gives:

$$\sigma_1 = \frac{Pr_2}{2t} \tag{13.6}$$

Equations 13.5 and 13.6 are completely general for any shell of revolution.

Cylinder (Figure 13.6a)

A cylinder is swept out by the rotation of a line parallel to the axis of revolution, so:

$$r_1 = \infty$$

$$r_2 = \frac{D}{2}$$

where D is the cylinder diameter.

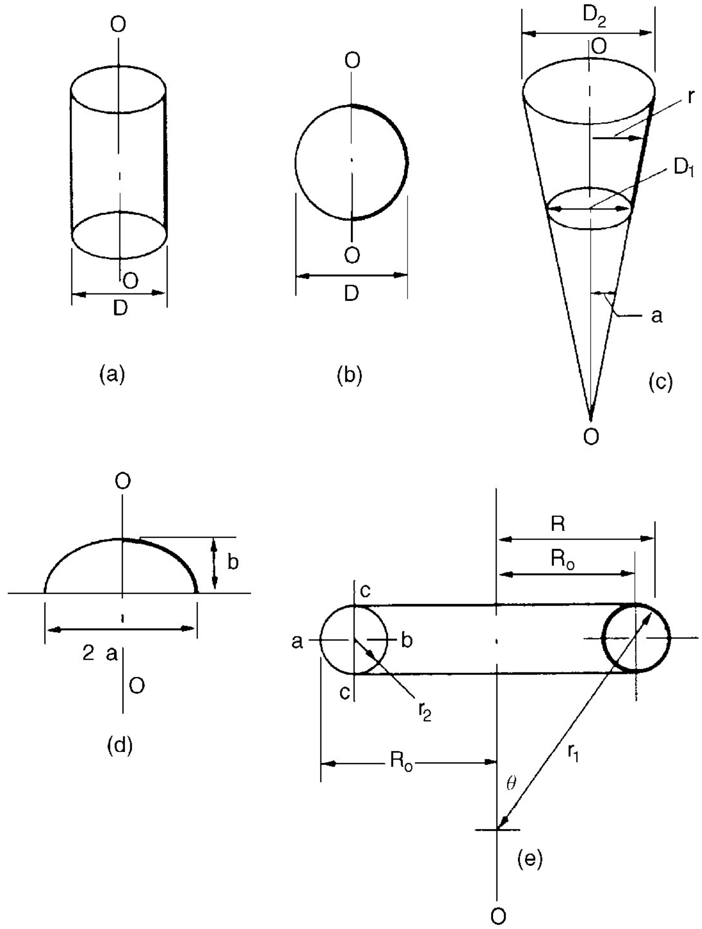


Figure 13.6. Shells of revolution

Substitution in equations 13.5 and 13.6 gives:

$$\sigma_2 = \frac{PD}{2t} \tag{13.7}$$

$$\sigma_1 = \frac{PD}{4t} \tag{13.8}$$

Sphere (Figure 13.6b)

$$r_1 = r_2 = \frac{D}{2}$$

hence:

$$\sigma_1 = \sigma_2 = \frac{PD}{4t} \tag{13.9}$$

Cone (Figure 13.6c)

A cone is swept out by a straight line inclined at an angle α to the axis.

$$r_1 = \infty$$

$$r_2 = \frac{r}{\cos\alpha}$$

substitution in equations 13.5 and 13.6 gives:

$$\sigma_2 = \frac{Pr}{t\cos\alpha} \tag{13.10}$$

$$\sigma_1 = \frac{Pr}{2t\cos\alpha} \tag{13.11}$$

The maximum values will occur at $r = D_2/2$.

Ellipsoid (Figure 13.6d)

For an ellipse with major axis $2a$ and minor axis $2b$, it can be shown that (see any standard geometry text):

$$r_1 = \frac{r_2^3 b^2}{a^4}$$

From equations 13.5 and 13.6

$$\sigma_1 = \frac{Pr_2}{2t} \qquad \text{(equation 13.6)}$$

$$\sigma_2 = \frac{P}{t}\left[r_2 - \frac{r_2^2}{2r_1}\right] \tag{13.12}$$

At the crown (top)

$$r_1 = r_2 = \frac{a^2}{b}$$

$$\sigma_1 = \sigma_2 = \frac{Pa^2}{2tb} \tag{13.13}$$

At the equator (bottom) $r_2 = a$, so $r_1 = b^2/a$

so
$$\sigma_1 = \frac{Pa}{2t} \tag{13.13}$$

$$\sigma_2 = \frac{P}{t}\left[a - \frac{a^2}{2b^2/a}\right] = \frac{Pa}{t}\left[1 - \frac{1}{2}\frac{a^2}{b^2}\right] \tag{13.14}$$

It should be noted that if $\frac{1}{2}(a/b)^2 > 1$, σ_2 will be negative (compressive) and the shell could fail by buckling. This consideration places a limit on the practical proportions of ellipsoidal heads.

Torus (Figure 13.6e)

A torus is formed by rotating a circle, radius r_2, about an axis.

$$\sigma_1 = \frac{Pr_2}{2t} \qquad \text{(equation 13.6)}$$

$$r_1 = \frac{R}{\sin\theta} = \frac{R_0 + r_2 \sin\theta}{\sin\theta}$$

and

$$\sigma_2 = \frac{Pr_2}{t}\left[1 - \frac{r_2 \sin\theta}{2(R_0 + r_2 \sin\theta)}\right] \tag{13.15}$$

On the centre line of the torus, point c, $\theta = 0$ and

$$\sigma_2 = \frac{Pr_2}{t} \tag{13.16}$$

At the outer edge, point a, $\theta = \pi/2$, $\sin\theta = 1$ and

$$\sigma_2 = \frac{Pr_2}{2t}\left[\frac{2R_0 + r_2}{R_0 + r_2}\right] \tag{13.17}$$

the minimum value.

At the inner edge, point b, $\theta = 3\pi/2$, $\sin\theta = -1$ and

$$\sigma_2 = \frac{Pr_2}{2t}\left[\frac{2R_0 - r_2}{R_0 - r_2}\right] \tag{13.18}$$

the maximum value.

So σ_2 varies from a maximum at the inner edge to a minimum at the outer edge.

Torispherical heads

A torispherical shape, which is often used as the end closure of cylindrical vessels, is formed from part of a torus and part of a sphere, Figure 13.7. The shape is close to that of an ellipse but is easier and cheaper to fabricate.

In Figure 13.7 R_k is the knuckle radius (the radius of the torus) and R_c the crown radius (the radius of the sphere). For the spherical portion:

$$\sigma_1 = \sigma_2 = \frac{PR_c}{2t} \tag{13.19}$$

For the torus:

$$\sigma_1 = \frac{PR_k}{2t} \tag{13.20}$$

σ_2 depends on the location, and is a function of R_c and R_k; it can be calculated from equations 13.15 and 13.9.

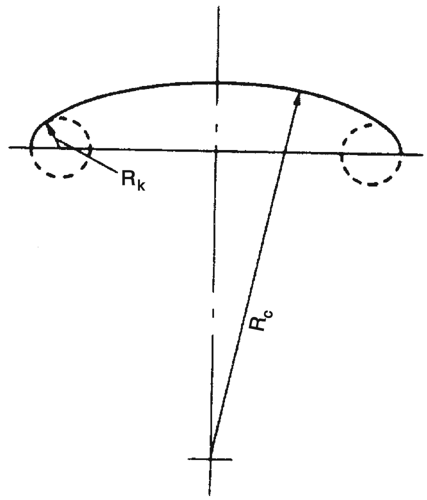


Figure 13.7. Torisphere

The ratio of the knuckle radius to crown radius should be made not less than 6/100 to avoid buckling. The stress will be higher in the torus section than the spherical section.

13.3.5. Flat plates

Flat plates are used as covers for manholes, as blind flanges, and for the ends of small diameter and low pressure vessels.

For a uniformly loaded circular plate supported at its edges, the slope ϕ at any radius x is given by:

$$\phi = -\frac{\mathrm{d}w}{\mathrm{d}x} = -\frac{1}{\mathbf{D}}\frac{Px^3}{16} + \frac{C_1 x}{2} + \frac{C_2}{x} \tag{13.21}$$

(The derivation of this equation can be found in any text on the strength of materials.)

Integration gives the deflection w:

$$w = \frac{Px^4}{64\mathbf{D}} - C_1\frac{x^2}{4} - C_2 \ln x + C_3 \tag{13.22}$$

where P = intensity of loading (pressure),
x = radial distance to point of interest,
$\mathbf{D}$ = flexual rigidity of plate = $(Et^3)/(12(1 - \nu^2))$,
t = plate thickness,
ν = Poisson's ratio for the material,
E = modulus of elasticity of the material (Young's modulus).

C_1, C_2, C_3 are constants of integration which can be obtained from the boundary conditions at the edge of the plate.

Two limiting situations are possible:

1. When the edge of the plate is rigidly clamped, not free to rotate; which corresponds to a heavy flange, or a strong joint.
2. When the edge is free to rotate (simply supported); corresponding to a weak joint, or light flange.

1. Clamped edges (Figure 13.8a)

The edge (boundary) conditions are:

$$\phi = 0 \text{ at } x = 0$$

$$\phi = 0 \text{ at } x = a$$

$$w = 0 \text{ at } x = a$$

where a is the radius of the plate.
Which gives:

$$C_2 = 0, \quad C_1 = \frac{Pa^2}{8\mathbf{D}}, \quad \text{and} \quad C_3 = \frac{Pa^4}{64\mathbf{D}}$$

hence

$$\phi = \frac{Px}{16\mathbf{D}}(a^2 - x^2) \tag{13.23}$$

and

$$w = \frac{P}{64\mathbf{D}}(x^2 - a^2)^2 \tag{13.24}$$

The maximum deflection will occur at the centre of the plate at $x = 0$

$$\hat{w} = \frac{Pa^4}{64\mathbf{D}} \tag{13.25}$$

The bending moments per unit length due to the pressure load are related to the slope and deflection by:

$$M_1 = \mathbf{D}\left[\frac{d\phi}{dx} + \nu\frac{\phi}{x}\right] \tag{13.26}$$

$$M_2 = \mathbf{D}\left[\frac{\phi}{x} + \nu\frac{d\phi}{dx}\right] \tag{13.27}$$

Where M_1 is the moment acting along cylindrical sections, and M_2 that acting along diametrical sections.

Substituting for ϕ and $d\phi/dx$ in equations 13.26 and 13.27 gives:

$$M_1 = \frac{P}{16}[a^2(1 + \nu) - x^2(3 + \nu)] \tag{13.28}$$

$$M_2 = \frac{P}{16}[a^2(1 + \nu) - x^2(1 + 3\nu)] \tag{13.29}$$

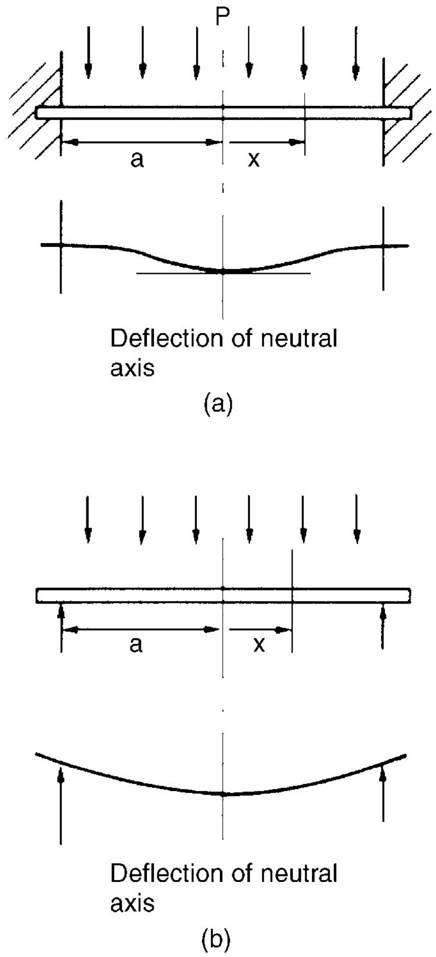


Figure 13.8. Flat circular plates (*a*) Clamped edges (*b*) Simply supported

The maximum values will occur at the edge of the plate, $x = a$.

$$\hat{M}_1 = -\frac{Pa^2}{8}, \quad \hat{M}_2 = -\nu\frac{Pa^2}{8}$$

The bending stress is given by:

$$\sigma_b = \frac{M_1}{I'} \times \frac{t}{2}$$

where I' = second moment of area per unit length $= t^3/12$, hence

$$\hat{\sigma}_b = \frac{6\hat{M}_1}{t^2} = \frac{3}{4}\frac{Pa^2}{t^2} \tag{13.30}$$

2. Simply supported plate (Figure 13.8b)

The edge (boundary) conditions are:

$$\phi = 0 \text{ at } x = 0$$

$$w = 0 \text{ at } x = a$$

$$M_1 = 0 \text{ at } x = a \text{ (free to rotate)}$$

which gives C_2 and $C_3 = 0$.
Hence

$$\phi = -\frac{1}{\mathbf{D}}\frac{Px^3}{16} + \frac{C_1 x}{2}$$

and

$$\frac{d\phi}{dx} = -\frac{1}{\mathbf{D}}\left[\frac{3Px^2}{16}\right] + \frac{C_1}{2}$$

Substituting these values in equation 13.26, and equating to zero at $x = a$, gives:

$$C_1 = \frac{Pa^2}{8\mathbf{D}}\frac{(3+\nu)}{(1+\nu)}$$

and hence

$$M_1 = \frac{P}{16}(3+\nu)(a^2 - x^2) \qquad (13.31)$$

The maximum bending moment will occur at the centre, where $M_1 = M_2$

so
$$\hat{M}_1 = \hat{M}_2 = \frac{P(3+\nu)a^2}{16} \qquad (13.32)$$

and
$$\hat{\sigma}_b = \frac{6\hat{M}_1}{t^2} = \tfrac{3}{8}(3+\nu)\frac{Pa^2}{t^2} \qquad (13.33)$$

General equation for flat plates

A general equation for the thickness of a flat plate required to resist a given pressure load can be written in the form:

$$t = CD\sqrt{\frac{P}{f}} \qquad (13.34)$$

where f = the maximum allowable stress (the design stress),
D = the effective plate diameter,
C = a constant, which depends on the edge support.

The limiting value of C can be obtained from equations 13.30 and 13.33. Taking Poisson's ratio as 0.3, a typical value for steels, then if the edge can be taken as completely rigid $C = 0.43$, and if it is essentially free to rotate $C = 0.56$.

13.3.6. Dilation of vessels

Under internal pressure a vessel will expand slightly. The radial growth can be calculated from the elastic strain in the radial direction. The principal strains in a two-dimensional system are related to the principal stresses by:

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \tag{13.35}$$

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \tag{13.36}$$

The radial (diametrical strain) will be the same as the circumferential strain ε_2. For any shell of revolution the dilation can be found by substituting the appropriate expressions for the circumferential and meridional stresses in equation 13.36.

The diametrical dilation $\Delta = D\varepsilon_1$.

For a cylinder

$$\sigma_1 = \frac{PD}{4t}$$

$$\sigma_2 = \frac{PD}{2t}$$

substitution in equation 13.36 gives:

$$\Delta_c = \frac{PD^2}{4tE}(2 - \nu) \tag{13.37}$$

For a sphere (or hemisphere)

$$\sigma_1 = \sigma_2 = \frac{PD}{4t}$$

and

$$\Delta_s = \frac{PD^2}{4tE}(1 - \nu) \tag{13.38}$$

So for a cylinder closed by a hemispherical head of the same thickness the difference in dilation of the two sections, if they were free to expand separately, would be:

$$\Delta_c - \Delta_s = \frac{PD^2}{4tE}$$

13.3.7. Secondary stresses

In the stress analysis of pressure vessels and pressure vessel components stresses are classified as primary or secondary. Primary stresses can be defined as those stresses that are necessary to satisfy the conditions of static equilibrium. The membrane stresses induced by the applied pressure and the bending stresses due to wind loads are examples of primary stresses. Primary stresses are not self-limiting; if they exceed the yield point of the material, gross distortion, and in the extreme situation, failure of the vessel will occur.

Secondary stresses are those stresses that arise from the constraint of adjacent parts of the vessel. Secondary stresses are self-limiting; local yielding or slight distortion will satisfy the conditions causing the stress, and failure would not be expected to occur in one application of the loading. The "thermal stress" set up by the differential expansion of parts of the vessel, due to different temperatures or the use of different materials, is an example of a secondary stress. The discontinuity that occurs between the head and the cylindrical section of a vessel is a major source of secondary stress. If free, the dilation of the head would be different from that of the cylindrical section (see Section 13.3.6); they are constrained to the same dilation by the welded joint between the two parts. The induced bending moment and shear force due to the constraint give rise to secondary bending and shear stresses at the junction. The magnitude of these discontinuity stresses can be estimated by analogy with the behaviour of beams on elastic foundations; see Hetenyi (1958) and Harvey (1974). The estimation of the stresses arising from discontinuities is covered in the books by Bednar (1990), and Jawad and Farr (1989).

Other sources of secondary stresses are the constraints arising at flanges, supports, and the change of section due to reinforcement at a nozzle or opening (see Section 13.6).

Though secondary stresses do not affect the "bursting strength" of the vessel, they are an important consideration when the vessel is subject to repeated pressure loading. If local yielding has occurred, residual stress will remain when the pressure load is removed, and repeated pressure cycling can lead to fatigue failure.

13.4. GENERAL DESIGN CONSIDERATIONS: PRESSURE VESSELS

13.4.1. Design pressure

A vessel must be designed to withstand the maximum pressure to which it is likely to be subjected in operation.

For vessels under internal pressure, the design pressure is normally taken as the pressure at which the relief device is set. This will normally be 5 to 10 per cent above the normal working pressure, to avoid spurious operation during minor process upsets. When deciding the design pressure, the hydrostatic pressure in the base of the column should be added to the operating pressure, if significant.

Vessels subject to external pressure should be designed to resist the maximum differential pressure that is likely to occur in service. Vessels likely to be subjected to vacuum should be designed for a full negative pressure of 1 bar, unless fitted with an effective, and reliable, vacuum breaker.

13.4.2. Design temperature

The strength of metals decreases with increasing temperature (see Chapter 7) so the maximum allowable design stress will depend on the material temperature. The design temperature at which the design stress is evaluated should be taken as the maximum working temperature of the material, with due allowance for any uncertainty involved in predicting vessel wall temperatures.

13.4.3. Materials

Pressure vessels are constructed from plain carbon steels, low and high alloy steels, other alloys, clad plate, and reinforced plastics.

Selection of a suitable material must take into account the suitability of the material for fabrication (particularly welding) as well as the compatibility of the material with the process environment.

The pressure vessel design codes and standards include lists of acceptable materials; in accordance with the appropriate material standards.

13.4.4. Design stress (nominal design strength)

For design purposes it is necessary to decide a value for the maximum allowable stress (nominal design strength) that can be accepted in the material of construction.

This is determined by applying a suitable "design stress factor" (factor of safety) to the maximum stress that the material could be expected to withstand without failure under standard test conditions. The design stress factor allows for any uncertainty in the design methods, the loading, the quality of the materials, and the workmanship.

For materials not subject to high temperatures the design stress is based on the yield stress (or proof stress), or the tensile strength (ultimate tensile stress) of the material at the design temperature.

For materials subject to conditions at which the creep is likely to be a consideration, the design stress is based on the creep characteristics of the material: the average stress to produce rupture after 10^5 hours, or the average stress to produce a 1 per cent strain after 10^5 hours, at the design temperature. Typical design stress factors for pressure components are shown in Table 13.1.

Table 13.1. Design stress factors

Property	Material		
	Carbon Carbon-manganese, low alloy steels	Austenitic stainless steels	Non-ferrous metals
Minimum yield stress or 0.2 per cent proof stress, at the design temperature	1.5	1.5	1.5
Minimum tensile strength, at room temperature	2.35	2.5	4.0
Mean stress to produce rupture at 10^5 h at the design temperature	1.5	1.5	1.0

In the British Standard, PD 5500, the nominal design strengths (allowable design stresses), for use with the design methods given, are listed in the standard, for the range of materials covered by the standard. The standard should be consulted for the principles and design stress factors used in determining the nominal design strengths.

Typical design stress values for some common materials are shown in Table 13.2. These may be used for preliminary designs. The standards and codes should be consulted for the values to be used for detailed vessel design.

Table 13.2. Typical design stresses for plate
(The appropriate material standards should be consulted for particular grades and plate thicknesses)

Material	Tensile strength (N/mm^2)	Design stress at temperature °C (N/mm^2)									
		0 to 50	100	150	200	250	300	350	400	450	500
Carbon steel (semi-killed or silicon killed)	360	135	125	115	105	95	85	80	70		
Carbon-manganese steel (semi-killed or silicon killed)	460	180	170	150	140	130	115	105	100		
Carbon-molybdenum steel, 0.5 per cent Mo	450	180	170	145	140	130	120	110	110		
Low alloy steel (Ni, Cr, Mo, V)	550	240	240	240	240	240	235	230	220	190	170
Stainless steel 18Cr/8Ni unstabilised (304)	510	165	145	130	115	110	105	100	100	95	90
Stainless steel 18Cr/8Ni Ti stabilised (321)	540	165	150	140	135	130	130	125	120	120	115
Stainless steel 18Cr/8Ni Mo $2\frac{1}{2}$ per cent (316)	520	175	150	135	120	115	110	105	105	100	95

13.4.5. Welded joint efficiency, and construction categories

The strength of a welded joint will depend on the type of joint and the quality of the welding.

The soundness of welds is checked by visual inspection and by non-destructive testing (radiography).

The possible lower strength of a welded joint compared with the virgin plate is usually allowed for in design by multiplying the allowable design stress for the material by a "welded joint factor" J. The value of the joint factor used in design will depend on the type of joint and amount of radiography required by the design code. Typical values are shown in Table 13.3. Taking the factor as 1.0 implies that the joint is equally as strong as the virgin plate; this is achieved by radiographing the complete weld length, and cutting out and remaking any defects. The use of lower joint factors in design, though saving costs on radiography, will result in a thicker, heavier, vessel, and the designer must balance any cost savings on inspection and fabrication against the increased cost of materials.

Table 13.3. Maximum allowable joint efficiency

Type of joint	Degree of radiography		
	100 per cent	spot	none
Double-welded butt or equivalent	1.0	0.85	0.7
Single-weld butt joint with bonding strips	0.9	0.80	0.65

The national codes and standards divide vessel construction into different categories, depending on the amount of non-destructive testing required. The higher categories require 100 per cent radiography of the welds, and allow the use of highest values for the weld-joint factors. The lower-quality categories require less radiography, but allow only lower joint-efficiency factors, and place restrictions on the plate thickness and type of materials that can be used. The highest category will invariably be specified for process-plant pressure vessels.

The standards should be consulted to determine the limitations and requirements of the construction categories specified. Welded joint efficiency factors are not used, as such, in the design equations given in BS PD 5500; instead limitations are placed on the values of the nominal design strength (allowable design stress) for materials in the lower construction category. The standard specifies three construction categories:

Category 1: the highest class, requires 100 per cent non-destructive testing (NDT) of the welds; and allows the use of all materials covered by the standard, with no restriction on the plate thickness.

Category 2: requires less non-destructive testing but places some limitations on the materials which can be used and the maximum plate thickness.

Category 3: the lowest class, requires only visual inspection of the welds, but is restricted to carbon and carbon-manganese steels, and austenitic stainless steel; and limits are placed on the plate thickness and the nominal design stress. For carbon and carbon-manganese steels the plate thickness is restricted to less than 13 mm and the design stress is about half that allowed for categories 1 and 2. For stainless steel the thickness is restricted to less than 25 mm and the allowable design stress is around 80 per cent of that for the other categories.

13.4.6. Corrosion allowance

The "corrosion allowance" is the additional thickness of metal added to allow for material lost by corrosion and erosion, or scaling (see Chapter 7). The allowance to be used should be agreed between the customer and manufacturer. Corrosion is a complex phenomenon, and it is not possible to give specific rules for the estimation of the corrosion allowance required for all circumstances. The allowance should be based on experience with the material of construction under similar service conditions to those for the proposed design. For carbon and low-alloy steels, where severe corrosion is not expected, a minimum allowance of 2.0 mm should be used; where more severe conditions are anticipated this should be increased to 4.0 mm. Most design codes and standards specify a minimum allowance of 1.0 mm.

13.4.7. Design loads

A structure must be designed to resist gross plastic deformation and collapse under all the conditions of loading. The loads to which a process vessel will be subject in service are listed below. They can be classified as major loads, that must always be considered in vessel design, and subsidiary loads. Formal stress analysis to determine the effect of the subsidiary loads is only required in the codes and standards where it is not possible to demonstrate the adequacy of the proposed design by other means; such as by comparison with the known behaviour of existing vessels.

Major loads

1. Design pressure: including any significant static head of liquid.
2. Maximum weight of the vessel and contents, under operating conditions.
3. Maximum weight of the vessel and contents under the hydraulic test conditions.
4. Wind loads.
5. Earthquake (seismic) loads.
6. Loads supported by, or reacting on, the vessel.

Subsidiary loads

1. Local stresses caused by supports, internal structures and connecting pipes.
2. Shock loads caused by water hammer, or by surging of the vessel contents.
3. Bending moments caused by eccentricity of the centre of the working pressure relative to the neutral axis of the vessel.
4. Stresses due to temperature differences and differences in the coefficient expansion of materials.
5. Loads caused by fluctuations in temperature and pressure.

A vessel will not be subject to all these loads simultaneously. The designer must determine what combination of possible loads gives the worst situation, and design for that loading condition.

13.4.8. Minimum practical wall thickness

There will be a minimum wall thickness required to ensure that any vessel is sufficiently rigid to withstand its own weight, and any incidental loads. As a general guide the wall thickness of any vessel should not be less than the values given below; the values include a corrosion allowance of 2 mm:

Vessel diameter (m)	Minimum thickness (mm)
1	5
1 to 2	7
2 to 2.5	9
2.5 to 3.0	10
3.0 to 3.5	12

13.5. THE DESIGN OF THIN-WALLED VESSELS UNDER INTERNAL PRESSURE

13.5.1. Cylinders and spherical shells

For a cylindrical shell the minimum thickness required to resist internal pressure can be determined from equation 13.7; the cylindrical stress will be the greater of the two principal stresses.

If D_i is internal diameter and e the minimum thickness required, the mean diameter will be $(D_i + e)$; substituting this for D in equation 13.7 gives:

$$e = \frac{P_i(D_i + e)}{2f}$$

where f is the design stress and P_i the internal pressure. Rearranging gives:

$$e = \frac{P_i D_i}{2f - P_i} \tag{13.39}$$

This is the form of the equation given in the British Standard PD 5500.

An equation for the minimum thickness of a sphere can be obtained from equation 13.9:

$$e = \frac{P_i D_i}{4f - P_i} \tag{13.40}$$

The equation for a sphere given in BS 5500 is:

$$e = \frac{P_i D_i}{4f - 1.2P_i} \tag{13.41}$$

The equation given in the British Standard PD 5500 differs slightly from equation 13.40, as it is derived from the formula for thick-walled vessels; see Section 13.15.

If a welded joint factor is used equations 13.39 and 13.40 are written:

$$e = \frac{P_i D_i}{2Jf - P_i} \tag{13.39a}$$

and

$$e = \frac{P_i D_i}{4Jf - 1.2P_i} \tag{13.40b}$$

where J is the joint factor.

Any consistent set of units can be used for equations 13.39*a* to 13.40*b*.

13.5.2. Heads and closures

The ends of a cylindrical vessel are closed by heads of various shapes. The principal types used are:

1. Flat plates and formed flat heads; Figure 13.9.
2. Hemispherical heads; Figure 13.10*a*.
3. Ellipsoidal heads; Figure 13.10*b*.
4. Torispherical heads; Figure 13.10*c*.

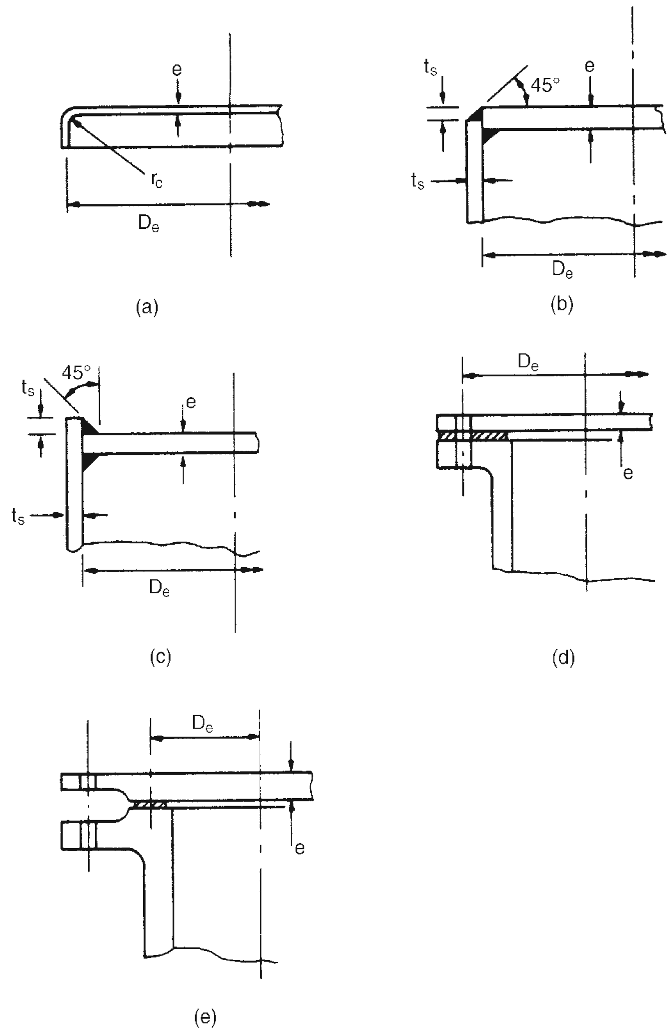


Figure 13.9. Flat-end closures (*a*) Flanged plate (*b*) Welded plate (*c*) Welded plate (*d*) Bolted cover (*e*) Bolted cover

Hemispherical, ellipsoidal and torispherical heads are collectively referred to as domed heads. They are formed by pressing or spinning; large diameters are fabricated from formed sections. Torispherical heads are often referred to as dished ends.

The preferred proportions of domed heads are given in the standards and codes.

Choice of closure

Flat plates are used as covers for manways, and as the channel covers of heat exchangers. Formed flat ends, known as "flange-only" ends, are manufactured by turning over a flange with a small radius on a flat plate, Figure 13.9*a*. The corner radius reduces the abrupt

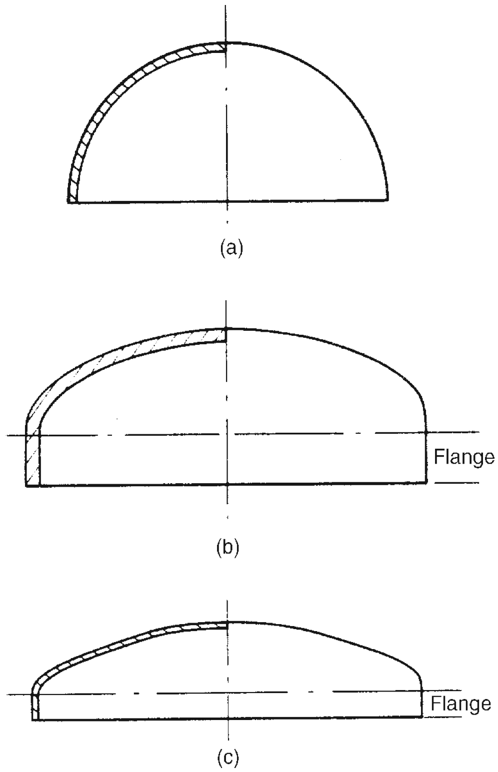


Figure 13.10. Domed heads (*a*) Hemispherical (*b*) Ellipsoidal (*c*) Torispherical

change of shape, at the junction with the cylindrical section; which reduces the local stresses to some extent: "Flange-only" heads are the cheapest type of formed head to manufacture, but their use is limited to low-pressure and small-diameter vessels.

Standard torispherical heads (dished ends) are the most commonly used end closure for vessels up to operating pressures of 15 bar. They can be used for higher pressures, but above 10 bar their cost should be compared with that of an equivalent ellipsoidal head. Above 15 bar an ellipsoidal head will usually prove to be the most economical closure to use.

A hemispherical head is the strongest shape; capable of resisting about twice the pressure of a torispherical head of the same thickness. The cost of forming a hemispherical head will, however, be higher than that for a shallow torispherical head. Hemispherical heads are used for high pressures.

13.5.3. Design of flat ends

Though the fabrication cost is low, flat ends are not a structurally efficient form, and very thick plates would be required for high pressures or large diameters.

The design equations used to determine the thickness of flat ends are based on the analysis of stresses in flat plates; Section 13.3.5.

The thickness required will depend on the degree of constraint at the plate periphery. The minimum thickness required is given by:

$$e = C_p D_e \sqrt{\frac{P_i}{f}} \tag{13.42}$$

where C_p = a design constant, dependent on the edge constraint,
D_e = nominal plate diameter,
f = design stress.

Any consistent set of units can be used.

Values for the design constant C_p and the nominal plate diameter D_e are given in the design codes and standards for various arrangements of flat end closures.

The values of the design constant and nominal diameter for the typical designs shown in Figure 13.9 are given below:

(*a*) Flanged-only end, for diameters less than 0.6 m and corner radii at least equal to $0.25e$, C_p can be taken as 0.45; D_e is equal to D_i.
(*b*, *c*) Plates welded to the end of the shell with a fillet weld, angle of fillet 45° and depth equal to the plate thickness, take C_p as 0.55 and $D_e = D_i$.
(*d*) Bolted cover with a full face gasket (see Section 13.10), take $C_p = 0.4$ and D_e equal to the bolt circle diameter.
(*e*) Bolted end cover with a narrow-face gasket, take $C_p = 0.55$ and D_e equal to the mean diameter of the gasket.

13.5.4. Design of domed ends

Design equations and charts for the various types of domed heads are given in the codes and standards and should be used for detailed design. The codes and standards cover both unpierced and pierced heads. Pierced heads are those with openings or connections. The head thickness must be increased to compensate for the weakening effect of the holes where the opening or branch is not locally reinforced (see Section 13.6).

For convenience, simplified design equations are given in this section. These are suitable for the preliminary sizing of unpierced heads and for heads with fully compensated openings or branches.

Hemispherical heads

It can be seen by examination of equations 13.7 and 13.9, that for equal stress in the cylindrical section and hemispherical head of a vessel the thickness of the head need only be half that of the cylinder. However, as the dilation of the two parts would then be different, discontinuity stresses would be set up at the head and cylinder junction. For no difference in dilation between the two parts (equal diametrical strain) it can be shown that for steels (Poisson's ratio = 0.3) the ratio of the hemispherical head thickness to cylinder thickness should be 7/17. However, the stress in the head would then be greater than that in the cylindrical section; and the optimum thickness ratio is normally taken as 0.6; see Brownell and Young (1959).

Ellipsoidal heads

Most standard ellipsoidal heads are manufactured with a major and minor axis ratio of 2 : 1. For this ratio, the following equation can be used to calculate the minimum thickness required:

$$e = \frac{P_i D_i}{2Jf - 0.2P_i} \tag{13.43}$$

Torispherical heads

There are two junctions in a torispherical end closure: that between the cylindrical section and the head, and that at the junction of the crown and the knuckle radii. The bending and shear stresses caused by the differential dilation that will occur at these points must be taken into account in the design of the heads. One approach taken is to use the basic equation for a hemisphere and to introduce a stress concentration, or shape, factor to allow for the increased stress due to the discontinuity. The stress concentration factor is a function of the knuckle and crown radii.

$$e = \frac{P_i R_c C_s}{2fJ + P_i(C_s - 0.2)} \tag{13.44}$$

where C_s = stress concentration factor for torispherical heads $= \frac{1}{4}(3 + \sqrt{R_c/R_k})$,
R_c = crown radius,
R_k = knuckle radius.

The ratio of the knuckle to crown radii should not be less than 0.06, to avoid buckling; and the crown radius should not be greater than the diameter of the cylindrical section. Any consistent set of units can be used with equations 13.43 and 13.44. For formed heads (no joints in the head) the joint factor J is taken as 1.0.

Flanges (skirts) on domed heads

Formed domed heads are made with a short straight cylindrical section, called a flange or skirt; Figure 13.10. This ensures that the weld line is away from the point of discontinuity between the head and the cylindrical section of the vessel.

13.5.5. Conical sections and end closures

Conical sections (reducers) are used to make a gradual reduction in diameter from one cylindrical section to another of smaller diameter.

Conical ends are used to facilitate the smooth flow and removal of solids from process equipment; such as, hoppers, spray-dryers and crystallisers.

From equation 13.10 it can be seen that the thickness required at any point on a cone is related to the diameter by the following expression:

$$e = \frac{P_i D_c}{2fJ - P_i} \cdot \frac{1}{\cos\alpha} \quad (13.45)$$

where D_c is the diameter of the cone at the point,
α = half the cone apex angle.

This equation will only apply at points away from the cone to cylinder junction. Bending and shear stresses will be caused by the different dilation of the conical and cylindrical sections. This can be allowed for by introducing a stress concentration factor, in a similar manner to the method used for torispherical heads,

$$e = \frac{C_c P_i D_c}{2fJ - P_i} \quad (13.46)$$

The design factor C_c is a function of the half apex angle α:

α	20°	30°	45°	60°
C_c	1.00	1.35	2.05	3.20

A formed section would normally be used for the transition between a cylindrical section and conical section; except for vessels operating at low pressures, or under hydrostatic pressure only. The transition section would be made thicker than the conical or cylindrical section and formed with a knuckle radius to reduce the stress concentration at the transition, Figure 13.11. The thickness at the knuckle can be calculated using equation 13.46, and that for the conical section away from the transition from equation 13.45.

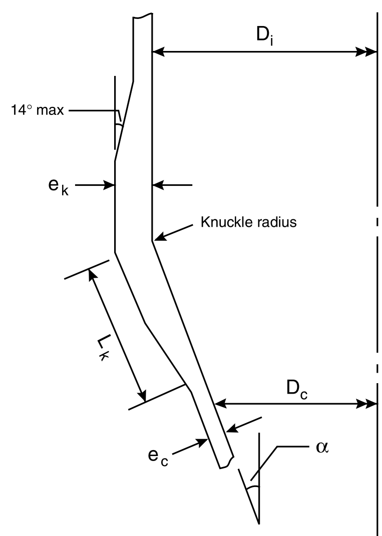


Figure 13.11. Conical transition section

The length of the thicker section L_k depends on the cone angle and is given by:

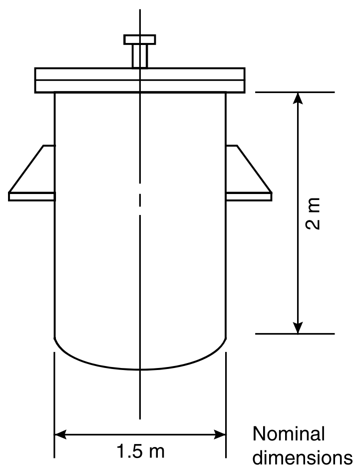
$$L_k = \sqrt{\frac{D_i e_k}{4 \cos \alpha}} \tag{13.47}$$

where e_k is the thickness at the knuckle.

Design procedures for conical sections are given in the codes and standards.

Example 13.1

Estimate the thickness required for the component parts of the vessel shown in the diagram. The vessel is to operate at a pressure of 14 bar (absolute) and temperature of 300°C. The material of construction will be plain carbon steel. Welds will be fully radiographed. A corrosion allowance of 2 mm should be used.



Solution

Design pressure, take as 10 per cent above operating pressure,

$$= (14 - 1) \times 1.1$$
$$= 14.3 \text{ bar}$$
$$= 1.43 \text{ N/mm}^2$$

Design temperature 300°C.
From Table 13.2, typical design stress = 85 N/mm^2.

Cylindrical section

$$e = \frac{1.43 \times 1.5 \times 10^3}{2 \times 85 - 1.43} = 12.7 \text{ mm} \tag{13.39}$$

add corrosion allowance 12.7 + 2 = 14.7
say <u>15 mm plate</u>

Domed head

(i) Try a standard dished head (torisphere):

$$\text{crown radius } R_c = D_i = 1.5 \text{ m}$$

$$\text{knuckle radius} = 6 \text{ per cent } R_c = 0.09 \text{ m}$$

A head of this size would be formed by pressing: no joints, so $J = 1$.

$$C_s = \tfrac{1}{4}\left(3 + \sqrt{\frac{R_c}{R_k}}\right) = \tfrac{1}{4}\left(3 + \sqrt{\frac{1.5}{0.09}}\right) = 1.77 \tag{13.44}$$

$$e = \frac{1.43 \times 1.5 \times 10^3 \times 1.77}{2 \times 85 + 1.43(1.77 - 0.2)} = \underline{\underline{22.0 \text{ mm}}} \tag{13.44}$$

(ii) Try a "standard" ellipsoidal head, ratio major : minor axes = 2 : 1

$$e = \frac{1.43 \times 1.5 \times 10^3}{2 \times 85 - 0.2 \times 1.43} \tag{13.43}$$

$$= \underline{\underline{12.7 \text{ mm}}}$$

So an ellipsoidal head would probably be the most economical. Take as same thickness as wall 15 mm.

Flat head

Use a full face gasket $C_p = 0.4$

D_e = bolt circle diameter, take as approx. 1.7 m.

$$e = 0.4 \times 1.7 \times 10^3 \sqrt{\frac{1.43}{85}} = \underline{88.4 \text{ mm}} \tag{13.42}$$

Add corrosion allowance and round-off to $\underline{\underline{90 \text{ mm}}}$.

This shows the inefficiency of a flat cover. It would be better to use a flanged domed head.

13.6. COMPENSATION FOR OPENINGS AND BRANCHES

All process vessels will have openings for connections, manways, and instrument fittings. The presence of an opening weakens the shell, and gives rise to stress concentrations. The stress at the edge of a hole will be considerably higher than the average stress in the surrounding plate. To compensate for the effect of an opening, the wall thickness is increased in the region adjacent to the opening. Sufficient reinforcement must be provided to compensate for the weakening effect of the opening without significantly altering the general dilation pattern of the vessel at the opening. Over-reinforcement will reduce the flexibility of the wall, causing a "hard spot", and giving rise to secondary stresses; typical arrangements are shown in Figure 13.12.

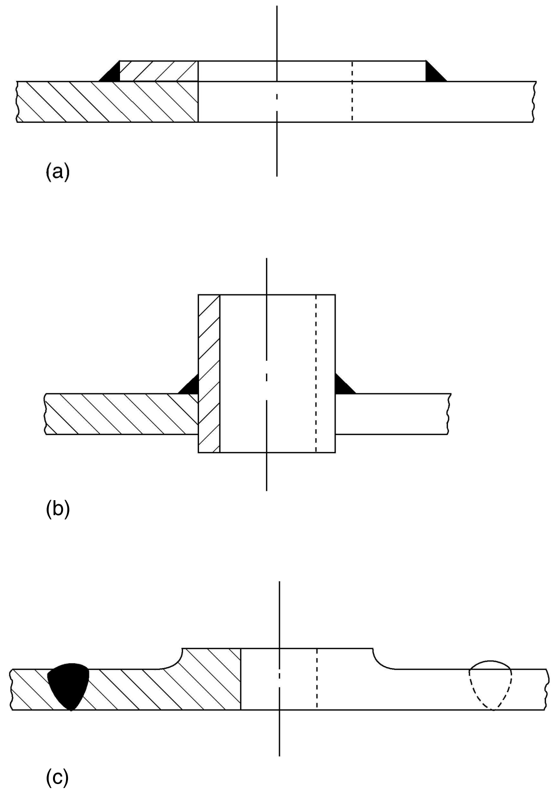


Figure 13.12. Types of compensation for openings (*a*) Welded pad (*b*) Inset nozzle (*c*) Forged ring

The simplest method of providing compensation is to weld a pad or collar around the opening, Figure 13.12*a*. The outer diameter of the pad is usually between $1\frac{1}{2}$ to 2 times the diameter of the hole or branch. This method, however, does not give the best disposition of the reinforcing material about the opening, and in some circumstances high thermal stress can arise due to the poor thermal conductivity of the pad to shell junction.

At a branch, the reinforcement required can be provided, with or without a pad, by allowing the branch, to protrude into the vessel, Figure 13.12*b*. This arrangement should be used with caution for process vessels, as the protrusion will act as a trap for crud, and local corrosion can occur. Forged reinforcing rings, Figure 13.12*c*, provide the most effective method of compensation, but are expensive. They would be used for any large openings and branches in vessels operating under severe conditions.

Calculation of reinforcement required

The "equal area method" is the simplest method used for calculating the amount of reinforcement required, and is allowed in most design codes and standards. The principle used is to provide reinforcement local to the opening, equal in cross-sectional area to the area removed in forming the opening, Figure 13.13. If the actual thickness of the vessel

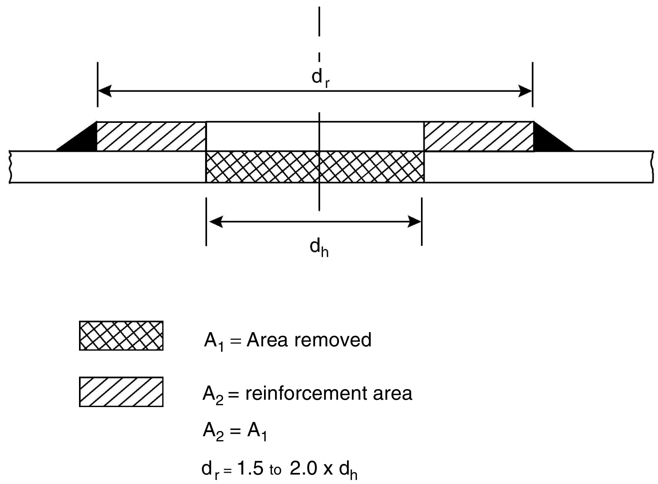
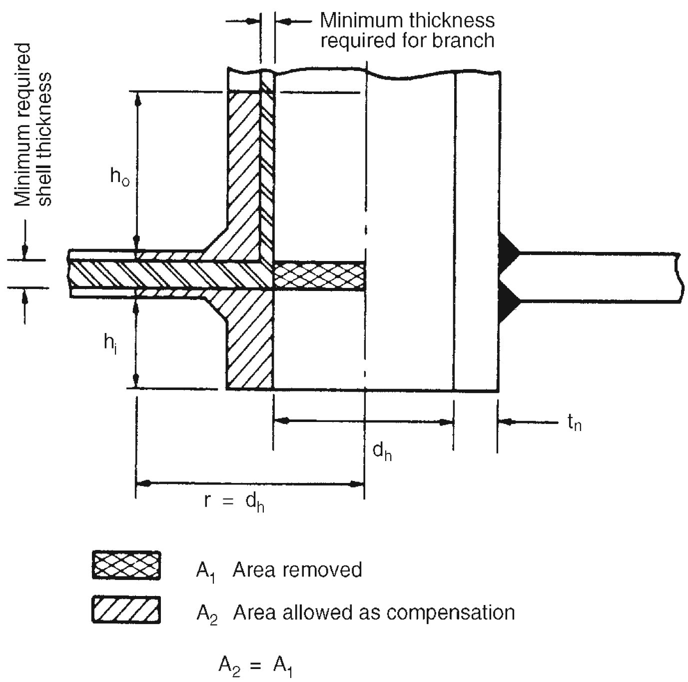


Figure 13.13. Equal-area method of compensation



Max. allowed h_0 and $h_i = 0.64\sqrt{(d_h + t_n)t_n}$

All dimensions shown are in the fully corroded condition (i.e. less corrosion allowance)

Figure 13.14. Branch compensation

wall is greater than the minimum required to resist the loading, the excess thickness can be taken into account when estimating the area of reinforcement required. Similarly with a branch connection, if the wall thickness of the branch or nozzle is greater than the minimum required, the excess material in the branch can be taken into account. Any corrosion allowance must be deducted when determining the excess thickness available as compensation. The standards and codes differ in the areas of the branch and shell considered to be effective for reinforcement, and should be consulted to determine the actual area allowed and the disposition of the various types of reinforcement. Figure 13.14 can be used for preliminary calculations. For branch connections of small diameter the reinforcement area can usually be provided by increasing the wall thickness of the branch pipe. Some design codes and standards do not require compensation for connections below 89 mm (3 in.) diameter.

If anything, the equal area method tends to over-estimate the compensation required and in some instances the additional material can reduce the fatigue life of the vessel. More sophisticated methods for determining the compensation required have been introduced into the latest editions of the codes and standards.

The equal-area method is generally used for estimating the increase in thickness required to compensate for multiple openings.

13.7. DESIGN OF VESSELS SUBJECT TO EXTERNAL PRESSURE

13.7.1. Cylindrical shells

Two types of process vessel are likely to be subjected to external pressure: those operated under vacuum, where the maximum pressure will be 1 bar (atm); and jacketed vessels, where the inner vessel will be under the jacket pressure. For jacketed vessels, the maximum pressure difference should be taken as the full jacket pressure, as a situation may arise in which the pressure in the inner vessel is lost. Thin-walled vessels subject to external pressure are liable to failure through elastic instability (buckling) and it is this mode of failure that determines the wall thickness required.

For an open-ended cylinder, the critical pressure to cause buckling P_c is given by the following expression; see Windenburg and Trilling (1934):

$$P_c = \frac{1}{3}\left[n^2 - 1 + \frac{2n^2 - 1 - v}{n^2\left(\dfrac{2L}{\pi D_0}\right)^2 - 1}\right]\frac{2E}{(1 - v^2)}\left(\frac{t}{D_0}\right)^3 + \frac{2Et/D_0}{(n^2 - 1)\left[n^2\left(\dfrac{2L}{\pi D_0}\right)^2 + 1\right]^2} \tag{13.48}$$

where L = the unsupported length of the vessel, the effective length,
D_0 = external diameter,
t = wall thickness,
E = Young's modulus,
v = Poisson's ratio,
n = the number of lobes formed at buckling.

For long tubes and cylindrical vessels this expression can be simplified by neglecting terms with the group $(2L/\pi D_0)^2$ in the denominator; the equation then becomes:

$$P_c = \tfrac{1}{3}\left[(n^2 - 1)\frac{2E}{(1 - v^2)}\right]\left(\frac{t}{D_0}\right)^3 \tag{13.49}$$

The minimum value of the critical pressure will occur when the number of lobes is 2, and substituting this value into equation 13.49 gives:

$$P_c = \frac{2E}{1 - v^2}\left(\frac{t}{D_0}\right)^3 \tag{13.50}$$

For most pressure-vessel materials Poisson's ratio can be taken as 0.3; substituting this in equation 13.50 gives:

$$P_c = 2.2E\left(\frac{t}{D_0}\right)^3 \tag{13.51}$$

For short closed vessels, and long vessels with stiffening rings, the critical buckling pressure will be higher than that predicted by equation 13.51. The effect of stiffening can be taken into account by introducing a "collapse coefficient", K_c, into equation 13.51.

$$P_c = K_c E\left(\frac{t}{D_0}\right)^3 \tag{13.52}$$

where K_c is a function of the diameter and thickness of the vessel, and the effective length L' between the ends or stiffening rings; and is obtained from Figure 13.16. The effective length for some typical arrangements is shown in Figure 13.15.

It can be shown (see Southwell, 1913) that the critical distance between stiffeners, L_c, beyond which stiffening will not be effective is given by:

$$L_c = \frac{4\pi\sqrt{6}D_0}{27}\left[(1 - v^2)^{1/4}\right]\left(\frac{D_0}{t}\right)^{1/2} \tag{13.53}$$

Substituting $v = 0.3$ gives:

$$L_c = 1.11D_0\left(\frac{D_0}{t}\right)^{1/2} \tag{13.54}$$

Any stiffening rings used must be spaced closer than L_c. Equation 13.52 can be used to determine the critical buckling pressure and hence the thickness required to resist a given external pressure; see Example 13.2. A factor of safety of at least 3 should be applied to the values predicted using equation 13.52.

The design methods and design curves given in the standards and codes should be used for the detailed design of vessels subject to external pressure.

Out of roundness

Any out-of-roundness in a shell after fabrication will significantly reduce the ability of the vessel to resist external pressure. A deviation from a true circular cross-section equal

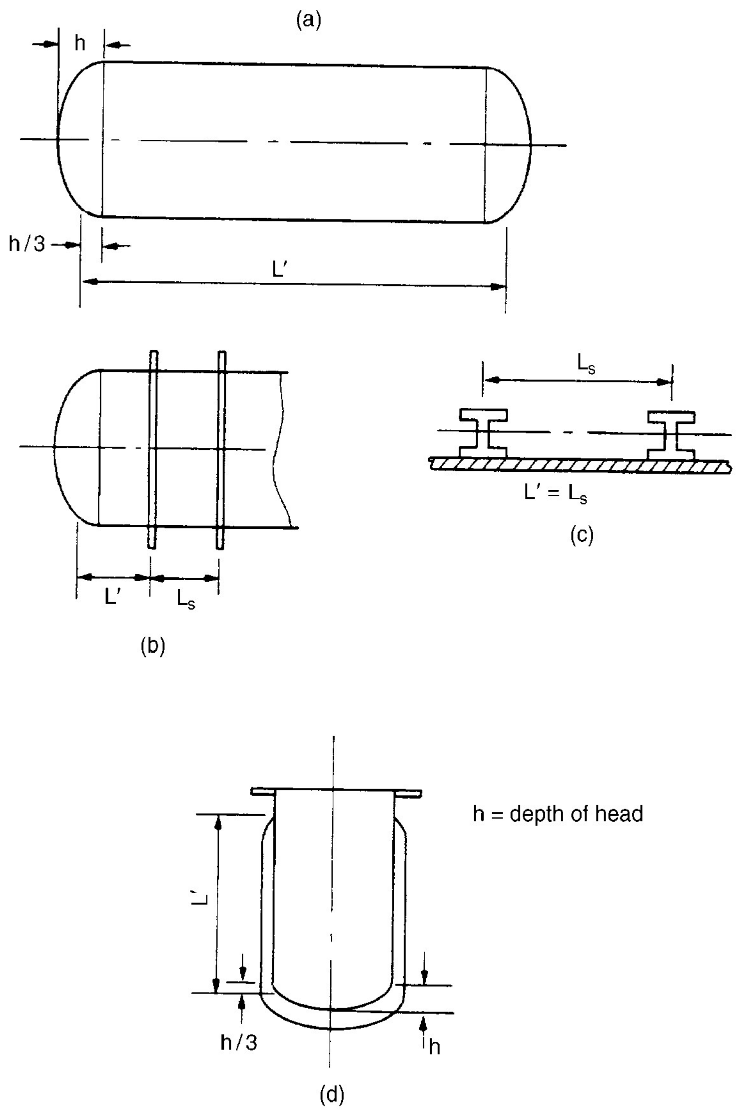


Figure 13.15. Effective length, vessel under external pressure (*a*) Plain vessel (*b*) With stiffeners (use smaller of L′ and L_s) (*c*) I—section stiffening rings (*d*) Jacketed vessel

to the shell thickness will reduce the critical buckling pressure by about 50 per cent. The ovality (out-of-roundness) of a cylinder is measured by:

$$\text{Ovality} = \frac{2(D_{max} - D_{min})}{(D_{max} + D_{min})} \times 100, \text{ per cent}$$

For vessels under external pressure this should not normally exceed 1.5 per cent.

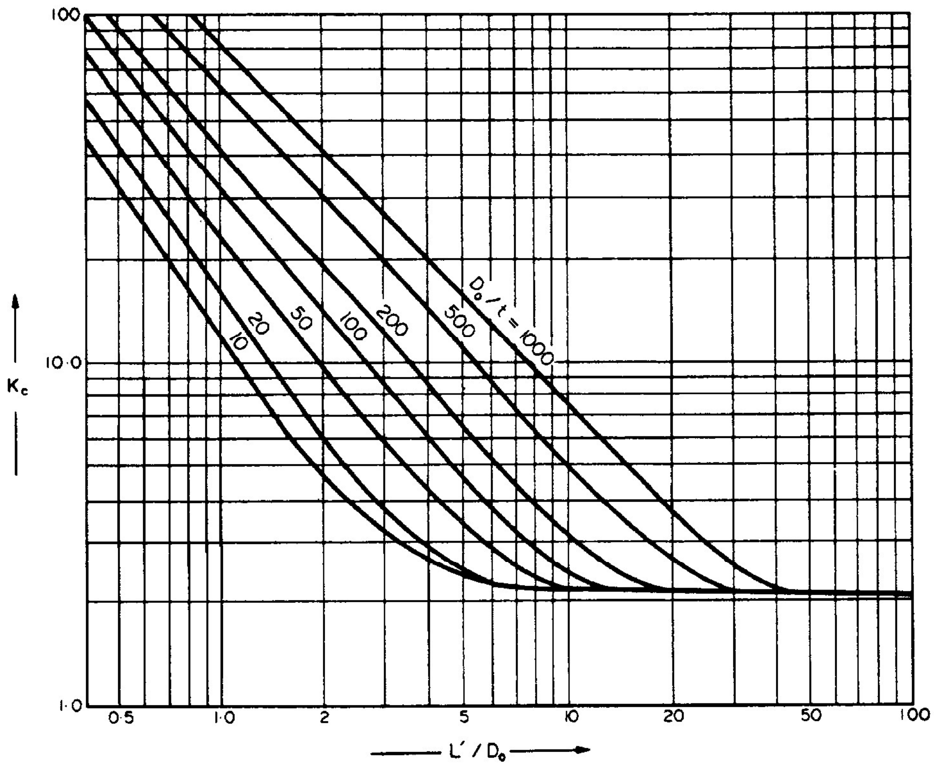


Figure 13.16. Collapse coefficients for cylindrical shells (after Brownell and Young, 1959)

13.7.2. Design of stiffness rings

The usual procedure is to design stiffening rings to carry the pressure load for a distance of $\frac{1}{2}L_s$ on each side of the ring, where L_s is the spacing between the rings. So, the load per unit length on a ring F_r will be given by:

$$F_r = P_e L_s \tag{13.55}$$

where P_e is the external pressure.

The critical load to cause buckling in a ring under a uniform radial load F_c is given by the following expression

$$F_c = \frac{24EI_r}{D_r^3} \tag{13.56}$$

where I_r = second moment of area of the ring cross-section,
D_r = diameter of the ring (approximately equal to the shell outside diameter).

Combining equations 13.55 and 13.56 will give an equation from which the required dimensions of the ring can be determined:

$$P_e L_s \not> \frac{24EI_r}{D_r^3} \div \text{(factor of safety)} \tag{13.57}$$

In calculating the second moment of area of the ring some allowance is normally made for the vessel wall; the use of I_r calculated for the ring alone will give an added factor of safety.

In vacuum distillation columns, the plate-support rings will act as stiffening rings and strengthen the vessel; see Example 13.2.

13.7.3. Vessel heads

The critical buckling pressure for a sphere subject to external pressure is given by (see Timoshenko, 1936):

$$P_c = \frac{2Et^2}{R_s^2\sqrt{3(1-v^2)}} \tag{13.58}$$

where R_s is the outside radius of the sphere. Taking Poisson's ratio as 0.3 gives:

$$P_c = 1.21E\left(\frac{t}{R_s}\right)^2 \tag{13.59}$$

This equation gives the critical pressure required to cause general buckling; local buckling can occur at a lower pressure. Karman and Tsien (1939) have shown that the pressure to cause a "dimple" to form is about one-quarter of that given by equation 13.59, and is given by:

$$P_c' = 0.365E\left(\frac{t}{R_s}\right)^2 \tag{13.60}$$

A generous factor of safety is needed when applying equation 13.60 to the design of heads under external pressure. A value of 6 is typically used, which gives the following equation for the minimum thickness:

$$e = 4R_s\sqrt{\left(\frac{P_e}{E}\right)} \tag{13.61}$$

Any consistent system of units can be used with equation 13.61.

Torispherical and ellipsoidal heads can be designed as equivalent hemispheres. For a torispherical head the radius R_s is taken as equivalent to the crown radius R_c. For an ellipsoidal head the radius can be taken as the maximum radius of curvature; that at the top, given by:

$$R_s = \frac{a^2}{b} \tag{13.62}$$

where $2a$ = major axis = D_0 (shell o.d.),
$2b$ = minor axis = $2h$,
h = height of the head from the tangent line.

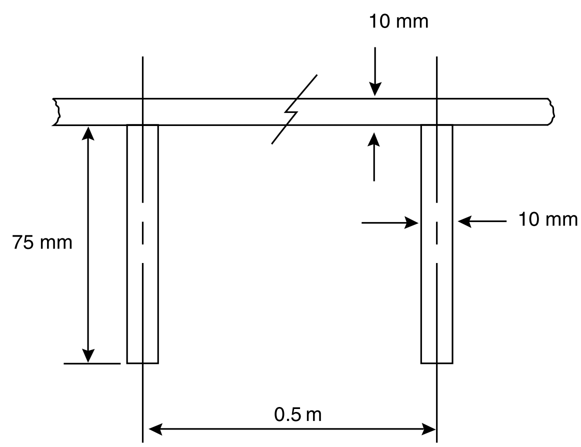
Because the radius of curvature of an ellipse is not constant the use of the maximum radius will over-size the thickness required.

Design methods for heads under external pressure are given in the standards and codes.

Example 13.2

A vacuum distillation column is to operate under a top pressure of 50 mmHg. The plates are supported on rings 75 mm wide, 10 mm deep. The column diameter is 1 m and the plate spacing 0.5 m. Check if the support rings will act as effective stiffening rings. The material of construction is carbon steel and the maximum operating temperature 50°C. If the vessel thickness is 10 mm, check if this is sufficient.

Solution



Take the design pressure as 1 bar external.

From equation 13.55 the load on each ring $= 0.5 \times 10^5$ N/m.

Taking E for steel at 50°C as 200,000 N/mm^2 $= 2 \times 10^{11}$ N/m^2, and using a factor of safety of 6, the second moment of area of the ring to avoid buckling is given by: equation 13.57

$$0.5 \times 10^5 = \frac{24 \times 2 \times 10^{11} \times I_r}{1^3 \times 6}$$

$$I_r = 6.25 \times 10^{-8} \text{ m}^4$$

For a rectangular section, the second moment of area is given by

$$I = \frac{\text{breadth} \times \text{depth}^3}{12}$$

$$\text{so } I_r \text{ for the support rings} = \frac{10 \times (75)^3 \times 10^{-12}}{12}$$

$$= 3.5 \times 10^{-7} \text{ m}^4$$

and the support ring is of an adequate size to be considered as a stiffening ring.

$$\frac{L'}{D_0} = \frac{0.5}{1} = 0.5$$

$$\frac{D_0}{t} = \frac{1000}{10} = 100$$

From Figure 13.16 $K_c = 75$
From equation 13.52

$$P_c = 75 \times 2 \times 10^{11} \left(\frac{1}{100}\right)^3 = \underline{\underline{15 \times 10^6\ \text{N/m}^2}}$$

which is well above the maximum design pressure of 10^5 N/m^2.

13.8. DESIGN OF VESSELS SUBJECT TO COMBINED LOADING

Pressure vessels are subjected to other loads in addition to pressure (see Section 13.4.7) and must be designed to withstand the worst combination of loading without failure It is not practical to give an explicit relationship for the vessel thickness to resist combined loads. A trial thickness must be assumed (based on that calculated for pressure alone) and the resultant stress from all loads determined to ensure that the maximum allowable stress intensity is not exceeded at any point.

The main sources of load to consider are:

1. Pressure.
2. Dead weight of vessel and contents.
3. Wind.
4. Earthquake (seismic).
5. External loads imposed by piping and attached equipment.

The primary stresses arising from these loads are considered in the following paragraphs, for cylindrical vessels; Figure 13.17.

Primary stresses

1. The longitudinal and circumferential stresses due to pressure (internal or external), given by:

$$\sigma_h = \frac{PD_i}{2t} \tag{13.63}$$

$$\sigma_L = \frac{PD_i}{4t} \tag{13.64}$$

2. The direct stress σ_w due to the weight of the vessel, its contents, and any attachments. The stress will be tensile (positive) for points below the plane of the vessel supports, and compressive (negative) for points above the supports, see Figure 13.18. The dead-weight stress will normally only be significant, compared to the magnitude of the other stresses, in tall vessels.

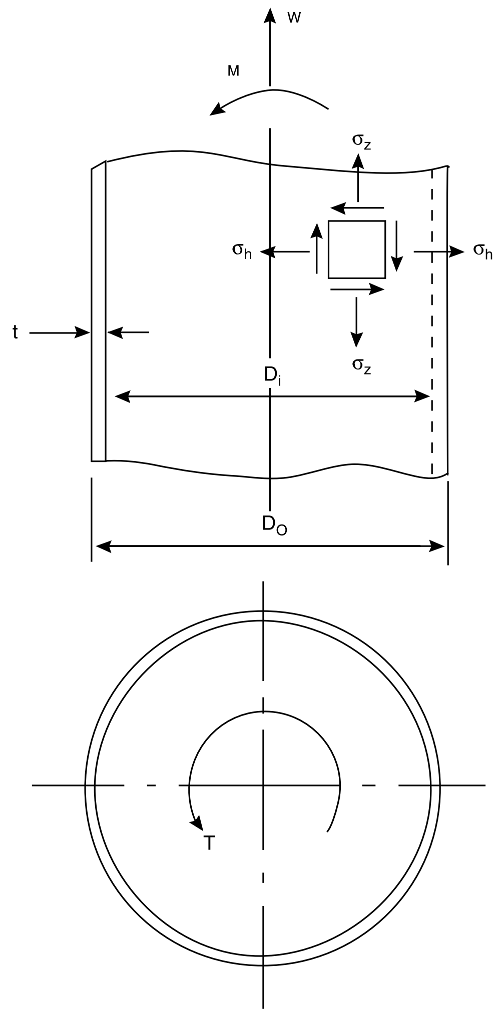


Figure 13.17. Stresses in a cylindrical shell under combined loading

$$\sigma_w = \frac{W}{\pi(D_i + t)t} \tag{13.65}$$

where W is the total weight which is supported by the vessel wall at the plane considered, see Section 13.8.1.

3. Bending stresses resulting from the bending moments to which the vessel is subjected. Bending moments will be caused by the following loading conditions:
   (a) The wind loads on tall self-supported vessels (Section 13.8.2).
   (b) Seismic (earthquake) loads on tall vessels (Section 13.8.3).
   (c) The dead weight and wind loads on piping and equipment which is attached to the vessel, but offset from the vessel centre line (Section 13.8.4).

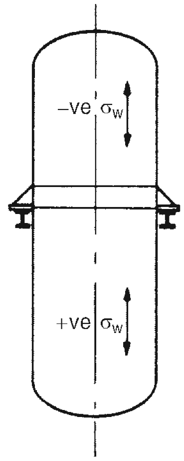


Figure 13.18. Stresses due to dead-weight loads

(d) For horizontal vessels with saddle supports, from the disposition of dead-weight load (see Section 13.9.1).

The bending stresses will be compressive or tensile, depending on location, and are given by:

$$\sigma_b = \pm\frac{M}{I_v}\left(\frac{D_i}{2} + t\right) \tag{13.66}$$

where M_v is the total bending moment at the plane being considered and I_v the second moment of area of the vessel about the plane of bending.

$$I_v = \frac{\pi}{64}(D_0^4 - D_i^4) \tag{13.67}$$

4. Torsional shear stresses τ resulting from torque caused by loads offset from the vessel axis. These loads will normally be small, and need not be considered in preliminary vessel designs.

   The torsional shear stress is given by:

$$\tau = \frac{T}{I_p}\left(\frac{D_i}{2} + t\right) \tag{13.68}$$

where T = the applied torque,
I_p = polar second moment of area = $(\pi/32)(D_0^4 - D_i^4)$

Principal stresses

The principal stresses will be given by:

$$\sigma_1 = \tfrac{1}{2}[\sigma_h + \sigma_z + \sqrt{(\sigma_h - \sigma_z)^2 + 4\tau^2}] \tag{13.69}$$

$$\sigma_2 = \tfrac{1}{2}[\sigma_h + \sigma_z - \sqrt{(\sigma_h - \sigma_z)^2 + 4\tau^2}] \tag{13.70}$$

where σ_z = total longitudinal stress
$= \sigma_L + \sigma_w \pm \sigma_b$

σ_w should be counted as positive if tension and negative if compressive.

τ is not usually significant.

The third principal stress, that in the radial direction σ_3, will usually be negligible for thin-walled vessels (see Section 13.1.1). As an approximation it can be taken as equal to one-half the pressure loading

$$\sigma_3 = 0.5P \tag{13.71}$$

σ_3 will be compressive (negative).

Allowable stress intensity

The maximum intensity of stress allowed will depend on the particular theory of failure adopted in the design method (see Section 13.3.2). The maximum shear-stress theory is normally used for pressure vessel design.

Using this criterion the maximum stress intensity at any point is taken for design purposes as the numerically greatest value of the following:

$$(\sigma_1 - \sigma_2)$$

$$(\sigma_1 - \sigma_3)$$

$$(\sigma_2 - \sigma_3)$$

The vessel wall thickness must be sufficient to ensure the maximum stress intensity does not exceed the design stress (nominal design strength) for the material of construction, at any point.

Compressive stresses and elastic stability

Under conditions where the resultant axial stress σ_z due to the combined loading is compressive, the vessel may fail by elastic instability (buckling) (see Section 13.3.3). Failure can occur in a thin-walled process column under an axial compressive load by buckling of the complete vessel, as with a strut (Euler buckling); or by local buckling, or wrinkling, of the shell plates. Local buckling will normally occur at a stress lower than that required to buckle the complete vessel. A column design must be checked to ensure that the maximum value of the resultant axial stress does not exceed the critical value at which buckling will occur.

For a curved plate subjected to an axial compressive load the critical buckling stress σ_c is given by (see Timoshenko, 1936):

$$\sigma_c = \frac{E}{\sqrt{3(1 - v^2)}}\left(\frac{t}{R_p}\right) \tag{13.72}$$

where R_p is the radius of curvature.

Taking Poisson's ratio as 0.3 gives:

$$\sigma_c = 0.60E\left(\frac{t}{R_p}\right) \tag{13.73}$$

By applying a suitable factor of safety, equation 13.72 can be used to predict the maximum allowable compressive stress to avoid failure by buckling. A large factor of safety is required, as experimental work has shown that cylindrical vessels will buckle at values well below that given by equation 13.72. For steels at ambient temperature $E = 200{,}000$ N/mm^2, and equation 13.72 with a factor of safety of 12 gives:

$$\sigma_c = 2 \times 10^4\left(\frac{t}{D_o}\right)\ \text{N/mm}^2 \tag{13.74}$$

The maximum compressive stress in a vessel wall should not exceed that given by equation 13.74; or the maximum allowable design stress for the material, whichever is the least.

Stiffening

As with vessels under external pressure, the resistance to failure buckling can be increased significantly by the use of stiffening rings, or longitudinal strips. Methods for estimating the critical buckling stress for stiffened vessels are given in the standards and codes.

Loading

The loads to which a vessel may be subjected will not all occur at the same time. For example, it is the usual practice to assume that the maximum wind load will not occur simultaneously with a major earthquake.

The vessel must be designed to withstand the worst combination of the loads likely to occur in the following situations:

1. During erection (or dismantling) of the vessel.
2. With the vessel erected but not operating.
3. During testing (the hydraulic pressure test).
4. During normal operation.

13.8.1. Weight loads

The major sources of dead weight loads are:

1. The vessel shell.
2. The vessel fittings: manways, nozzles.
3. Internal fittings: plates (plus the fluid on the plates); heating and cooling coils.
4. External fittings: ladders, platforms, piping.
5. Auxiliary equipment which is not self-supported; condensers, agitators.
6. Insulation.

7. The weight of liquid to fill the vessel. The vessel will be filled with water for the hydraulic pressure test; and may fill with process liquid due to misoperation.

Note: for vessels on a skirt support (see Section 13.9.2), the weight of the liquid to fill the vessel will be transferred directly to the skirt.

The weight of the vessel and fittings can be calculated from the preliminary design sketches. The weights of standard vessel components: heads, shell plates, manways, branches and nozzles, are given in various handbooks; Megyesy (2001) and Brownell and Young (1959).

For preliminary calculations the approximate weight of a cylindrical vessel with domed ends, and uniform wall thickness, can be estimated from the following equation:

$$W_v = C_v \pi \rho_m D_m g(H_v + 0.8D_m)t \times 10^{-3} \tag{13.75}$$

where W_v = total weight of the shell, excluding internal fittings, such as plates, N,
C_v = a factor to account for the weight of nozzles, manways, internal supports, etc; which can be taken as
= 1.08 for vessels with only a few internal fittings,
= 1.15 for distillation columns, or similar vessels, with several manways, and with plate support rings, or equivalent fittings,
H_v = height, or length, between tangent lines (the length of the cylindrical section), m,
g = gravitational acceleration, 9.81 m/s^2,
t = wall thickness, mm
ρ_m = density of vessel material, kg/m^3,
D_m = mean diameter of vessel = $(D_i + t \times 10^{-3})$, m.

For a steel vessel, equation 13.75 reduces to:

$$W_v = 240C_v D_m(H_v + 0.8D_m)t \tag{13.76}$$

The following values can be used as a rough guide to the weight of fittings; see Nelson (1963):

(a) caged ladders, steel, 360 N/m length,
(b) plain ladders, steel, 150 N/m length,
(c) platforms, steel, for vertical columns, 1.7 kN/m^2 area,
(d) contacting plates, steel, including typical liquid loading, 1.2 kN/m^2 plate area.

Typical values for the density of insulating materials are (all kg/m^3):

Foam glass	150
Mineral wool	130
Fibreglass	100
Calcium silicate	200

These densities should be doubled to allow for attachment fittings, sealing, and moisture absorption.

13.8.2. Wind loads (tall vessels)

Wind loading will only be important on tall columns installed in the open. Columns and chimney-stacks are usually free standing, mounted on skirt supports, and not attached to structural steel work. Under these conditions the vessel under wind loading acts as a cantilever beam, Figure 13.19. For a uniformly loaded cantilever the bending moment at any plane is given by:

$$M_x = \frac{wx^2}{2} \tag{13.77}$$

where x is the distance measured from the free end and w the load per unit length (Newtons per metre run).

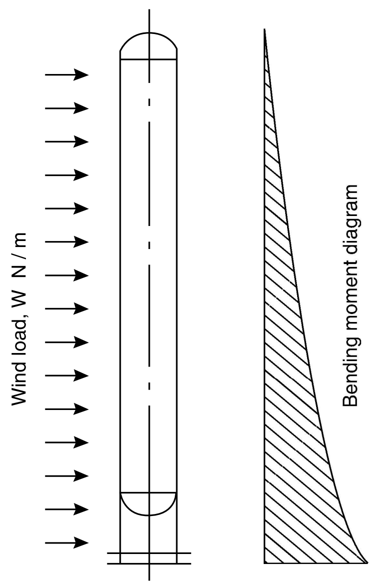


Figure 13.19. Wind loading on a tall column

So the bending moment, and hence the bending stress, will vary parabolically from zero at the top of the column to a maximum value at the base. For tall columns the bending stress due to wind loading will often be greater than direct stress due to pressure, and will determine the plate thickness required. The most economical design will be one in which the plate thickness is progressively increased from the top to the base of the column. The thickness at the top being sufficient for the pressure load, and that at the base sufficient for the pressure plus the maximum bending moment.

Any local increase in the column area presented to the wind will give rise to a local, concentrated, load, Figure 13.20. The bending moment at the column base caused by a concentrated load is given by:

$$M_p = F_p H_p \tag{13.78}$$

where F_p = local, concentrated, load,
H_p = the height of the concentrated load above the column base.

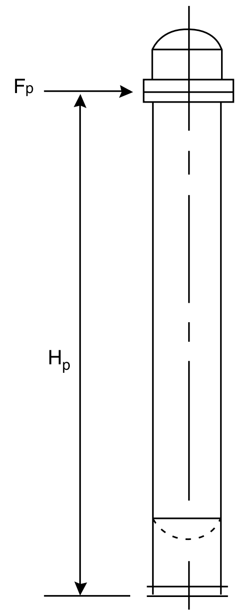


Figure 13.20. Local wind loading

Dynamic wind pressure

The load imposed on any structure by the action of the wind will depend on the shape of the structure and the wind velocity.

$$P_w = \frac{1}{2} C_d \rho_a u_w^2 \tag{13.79}$$

where P_w = wind pressure (load per unit area),
C_d = drag coefficient (shape factor),
ρ_a = density of air,
u_w = wind velocity.

The drag coefficient is a function of the shape of the structure and the wind velocity (Reynolds number).

For a smooth cylindrical column or stack the following semi-empirical equation can be used to estimate the wind pressure:

$$P_w = 0.05 u_w^2 \tag{13.79a}$$

where P_w = wind pressure, N/m^2,
u_w = wind speed, km/h.

If the column outline is broken up by attachments, such as ladders or pipe work, the factor of 0.05 in equation 13.79*a* should be increased to 0.07, to allow for the increased drag.

A column must be designed to withstand the highest wind speed that is likely to be encountered at the site during the life of the plant. The probability of a given wind speed occurring can be predicted by studying meteorological records for the site location.

Data and design methods for wind loading are given in the Engineering Sciences Data Unit (ESDU) Wind Engineering Series (www.ihsedsu.com).

Design loadings for locations in the United States are given by Moss (2003), Megyesy (2001) and Escoe (1994).

A wind speed of 160 km/h (100 mph) can be used for preliminary design studies; equivalent to a wind pressure of 1280 N/m^2 (25 lb/ft^2).

At any site, the wind velocity near the ground will be lower than that higher up (due to the boundary layer), and in some design methods a lower wind pressure is used at heights below about 20 m; typically taken as one-half of the pressure above this height.

The loading per unit length of the column can be obtained from the wind pressure by multiplying by the effective column diameter: the outside diameter plus an allowance for the thermal insulation and attachments, such as pipes and ladders.

$$F_w = P_w D_{\text{eff}} \tag{13.80}$$

An allowance of 0.4 m should be added for a caged ladder. The calculation of the wind load on a tall column, and the induced bending stresses, is illustrated in Example 13.3. Further examples of the design of tall columns are given by Brownell (1963), Henry (1973), Bednar (1990), Escoe (1994) and Jawad and Farr (1989).

Deflection of tall columns

Tall columns sway in the wind. The allowable deflection will normally be specified as less than 150 mm per 30 metres of height (6 in. per 100 ft).

For a column with a uniform cross-section, the deflection can be calculated using the formula for the deflection of a uniformly loaded cantilever. A method for calculating the deflection of a column where the wall thickness is not constant is given by Tang (1968).

Wind-induced vibrations

Vortex shedding from tall thin columns and stacks can induce vibrations which, if the frequency of shedding of eddies matches the natural frequency of the column, can be severe enough to cause premature failure of the vessel by fatigue. The effect of vortex shedding should be investigated for free standing columns with height to diameter ratios greater than 10. Methods for estimating the natural frequency of columns are given by Freese (1959) and DeGhetto and Long (1966).

Helical strakes (strips) are fitted to the tops of tall smooth chimneys to change the pattern of vortex shedding and so prevent resonant oscillation. The same effect will be achieved on a tall column by distributing any attachments (ladders, pipes and platforms) around the column.

13.8.3. Earthquake loading

The movement of the earth's surface during an earthquake produces horizontal shear forces on tall self-supported vessels, the magnitude of which increases from the base upward. The total shear force on the vessel will be given by:

$$F_s = a_e \left(\frac{W}{g}\right) \tag{13.81}$$

where a_e = the acceleration of the vessel due to the earthquake,
g = the acceleration due to gravity,
W = total weight of the vessel.

The term (a_e/g) is called the seismic constant C_e, and is a function of the natural period of vibration of the vessel and the severity of the earthquake. Values of the seismic constant have been determined empirically from studies of the damage caused by earthquakes, and are available for those geographical locations which are subject to earthquake activity. Values for sites in the United States, and procedures for determining the stresses induced in tall columns are given by Megyesy (2001), Escoe (1994) and Moss (2003).

A seismic stress analysis is not made as a routine procedure in the design of vessels for sites in the United Kingdom, except for nuclear installations, as the probability of an earthquake occurring of sufficient severity to cause significant damage is negligible. However, the possibility of earthquake damage may be considered if the site is a Major Hazards installation, see Chapter 9, Section 9.9.

13.8.4. Eccentric loads (tall vessels)

Ancillary equipment attached to a tall vessel will subject the vessel to a bending moment if the centre of gravity of the equipment does not coincide with the centre line of the vessel (Figure 13.21). The moment produced by small fittings, such as ladders, pipes and manways, will be small and can be neglected. That produced by heavy equipment, such as reflux condensers and side platforms, can be significant and should be considered. The moment is given by:

$$M_e = W_e L_o \tag{13.82}$$

where W_e = dead weight of the equipment,
L_o = distance between the centre of gravity of the equipment and the column centre line.

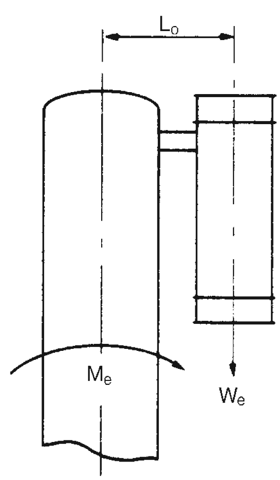


Figure 13.21. Bending moment due to offset equipment

13.8.5. Torque

Any horizontal force imposed on the vessel by ancillary equipment, the line of thrust of which does not pass through the centre line of the vessel, will produce a torque on the vessel. Such loads can arise through wind pressure on piping and other attachments. However, the torque will normally be small and usually can be disregarded. The pipe work and the connections for any ancillary equipment will be designed so as not to impose a significant load on the vessel.

Example 13.3

Make a preliminary estimate of the plate thickness required for the distillation column specified below:

Height, between tangent lines	50 m
Diameter	2 m
Skirt support, height	3 m
100 sieve plates, equally spaced	
Insulation, mineral wool	75 mm thick

Material of construction, stainless steel, design stress 135 N/mm^2 at design temperature 200°C

Operating pressure 10 bar (absolute)

Vessel to be fully radiographed (joint factor 1).

Solution

Design pressure; take as 10 per cent above operating pressure

$$= (10 - 1) \times 1.1 = 9.9 \text{ bar, say } 10 \text{ bar}$$

$$= 1.0 \text{ N/mm}^2$$

Minimum thickness required for pressure loading

$$= \frac{1 \times 2 \times 10^3}{2 \times 135 - 1} = 7.4 \text{ mm} \qquad (13.39)$$

A much thicker wall will be needed at the column base to withstand the wind and dead weight loads.

As a first trial, divide the column into five sections (courses), with the thickness increasing by 2 mm per section. Try 10, 12, 14, 16, 18 mm.

Dead weight of vessel

Though equation 13.76 only applies strictly to vessels with uniform thickness, it can be used to get a rough estimate of the weight of this vessel by using the average thickness in the equation, 14 mm.

Take $C_v = 1.15$, vessel with plates,

$D_m = 2 + 14 \times 10^{-3} = 2.014$ m,

$$H_v = 50 \text{ m},$$
$$t = 14 \text{ mm}$$
$$W_v = 240 \times 1.15 \times 2.014(50 + 0.8 \times 2.014)14 \quad (13.76)$$
$$= 401643 \text{ N}$$
$$= 402 \text{ kN}$$

Weight of plates:

plate area $= \pi/4 \times 2^2 = 3.14 \text{ m}^2$
weight of a plate (see page 761) $= 1.2 \times 3.14 = 3.8$ kN
100 plates $= 100 \times 3.8 = 380$ kN

Weight of insulation:

mineral wool density $= 130 \text{ kg/m}^3$
approximate volume of insulation $= \pi \times 2 \times 50 \times 75 \times 10^{-3}$
$= 23.6 \text{ m}^3$
weight $= 23.6 \times 130 \times 9.81 = 30{,}049$ N
double this to allow for fittings, etc. $= 60$ kN

Total weight:

shell	402
plates	380
insulation	60
	842 kN

Wind loading

Take dynamic wind pressure as 1280 N/m^2.
Mean diameter, including insulation $= 2 + 2(14 + 75) \times 10^{-3}$
$= 2.18$ m

$$\text{Loading (per linear metre) } F_w = 1280 \times 2.18 = 2790 \text{ N/m} \quad (13.80)$$

Bending moment at bottom tangent line:

$$M_x = \frac{2790}{2} \times 50^2 = 3{,}487{,}500 \text{ Nm} \quad (13.77)$$

Analysis of stresses

At bottom tangent line
Pressure stresses:

$$\sigma_L = \frac{1.0 \times 2 \times 10^3}{4 \times 18} = 27.8 \text{ N/mm}^2 \quad (13.64)$$

$$\sigma_h = \frac{1 \times 2 \times 10^3}{2 \times 18} = 55.6 \text{ N/mm}^2 \quad (13.63)$$

Dead weight stress:

$$\sigma_w = \frac{W_v}{\pi(D_i + t)t} = \frac{842 \times 10^3}{\pi(2000 + 18)18} \quad (13.65)$$

$$= 7.4 \text{ N/mm}^2 \text{ (compressive)}$$

Bending stresses:

$$D_o = 2000 + 2 \times 18 = 2036 \text{ mm}$$

$$I_v = \frac{\pi}{64}(2036^4 - 2000^4) = 5.81 \times 10^{10} \text{ mm}^4 \quad (13.67)$$

$$\sigma_b = \pm\frac{3{,}487{,}500 \times 10^3}{5.81 \times 10^{10}}\left(\frac{2000}{2} + 18\right) \quad (13.66)$$

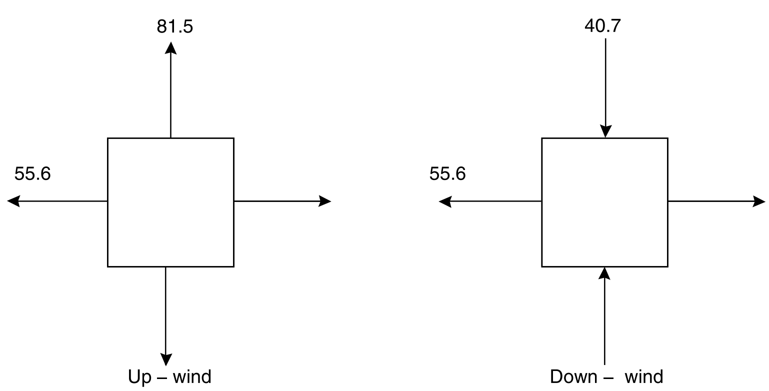
$$= \pm 61.1 \text{ N/mm}^2$$

The resultant longitudinal stress is:

$$\sigma_z = \sigma_L + \sigma_w \pm \sigma_b$$

σ_w is compressive and therefore negative.
σ_z (upwind) $= 27.8 - 7.4 + 61.1 = +81.5$ N/mm^2.
σ_z (downwind) $= 27.8 - 7.4 - 61.1 = -40.7$ N/mm^2.

As there is no torsional shear stress, the principal stresses will be σ_z and σ_h.
The radial stress is negligible, $\simeq (P_i/2) = 0.5$ N/mm^2.



The greatest difference between the principal stresses will be on the down-wind side

$$(55.6 - (-40.7)) = \underline{\underline{96.5 \text{ N/mm}^2}},$$

well below the maximum allowable design stress

Check elastic stability (buckling)

Critical buckling stress:

$$\sigma_c = 2 \times 10^4 \left(\frac{18}{2036}\right) = \underline{\underline{176.8 \text{ N/mm}^2}} \quad (13.74)$$

The maximum compressive stress will occur when the vessel is not under pressure = $7.4 + 61.1 = 68.5$, well below the critical buckling stress.

So design is satisfactory. Could reduce the plate thickness and recalculate.

13.9. VESSEL SUPPORTS

The method used to support a vessel will depend on the size, shape, and weight of the vessel; the design temperature and pressure; the vessel location and arrangement; and the internal and external fittings and attachments. Horizontal vessels are usually mounted on two saddle supports; Figure 13.22. Skirt supports are used for tall, vertical columns; Figure 13.23. Brackets, or lugs, are used for all types of vessel; Figure 13.24. The supports must be designed to carry the weight of the vessel and contents, and any superimposed loads, such as wind loads. Supports will impose localised loads on the vessel wall, and the design must be checked to ensure that the resulting stress concentrations are below the maximum allowable design stress. Supports should be designed to allow easy access to the vessel and fittings for inspection and maintenance.

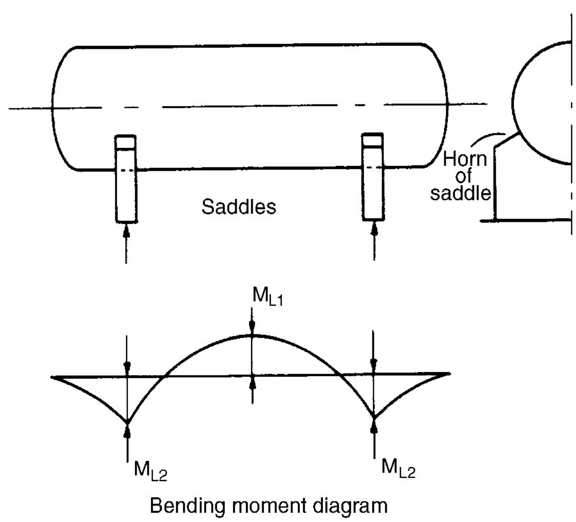


Figure 13.22. Horizontal cylindrical vessel on saddle supports

13.9.1. Saddle supports

Though saddles are the most commonly used support for horizontal cylindrical vessels, legs can be used for small vessels. A horizontal vessel will normally be supported at two cross-sections; if more than two saddles are used the distribution of the loading is uncertain.

A vessel supported on two saddles can be considered as a simply supported beam, with an essentially uniform load, and the distribution of longitudinal axial bending moment will be as shown in Figure 13.22. Maxima occur at the supports and at mid-span. The

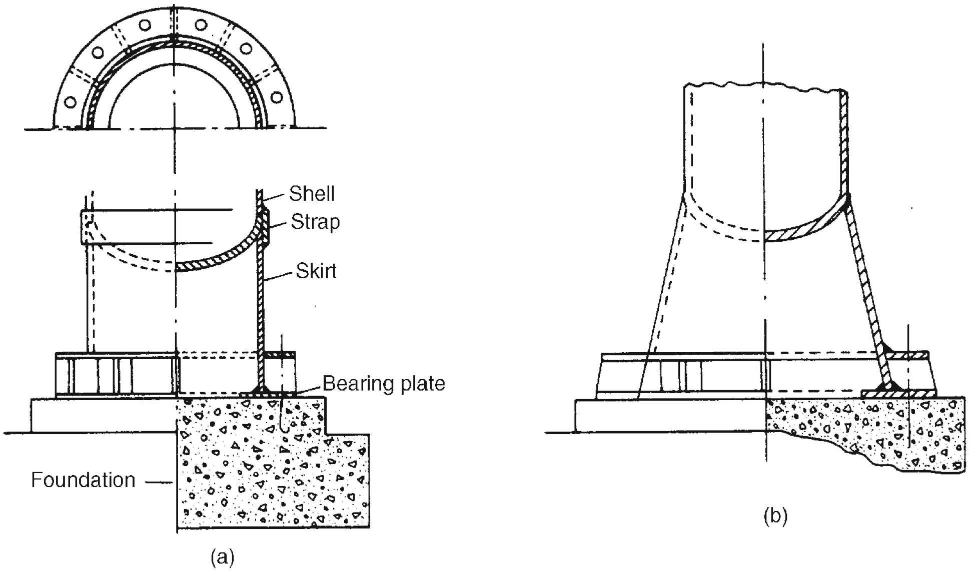


Figure 13.23. Typical skirt-support designs (*a*) Straight skirt (*b*) Conical skirt

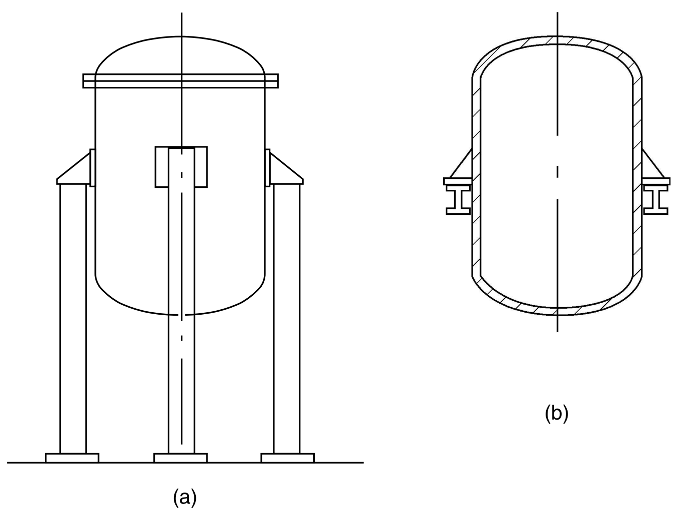


Figure 13.24. Bracket supports (*a*) Supported on legs (*b*) Supported from steel-work

theoretical optimum position of the supports to give the least maximum bending moment will be the position at which the maxima at the supports and at mid-span are equal in magnitude. For a uniformly loaded beam the position will be at 21 per cent of the span, in from each end. The saddle supports for a vessel will usually be located nearer the ends than this value, to make use of the stiffening effect of the ends.

Stress in the vessel wall

The longitudinal bending stress at the mid-span of the vessel is given by:

$$\sigma_{b1} = \frac{M_{L1}}{I_h} \times \frac{D}{2} \simeq \frac{4M_{L1}}{\pi D^2 t} \tag{13.83}$$

where M_{L1} = longitudinal bending stress at the mid-span,
I_h = second moment of area of the shell,
D = shell diameter,
t = shell thickness.

The resultant axial stress due to bending and pressure will be given by:

$$\sigma_z = \frac{PD}{4t} \pm \frac{4M_{L1}}{\pi D^2 t} \tag{13.84}$$

The magnitude of the longitudinal bending stress at the supports will depend on the local stiffness of the shell; if the shell does not remain circular under load a portion of the upper part of the cross-section is ineffective against longitudinal bending; see Figure 13.25. The stress is given by:

$$\sigma_{b2} = \frac{4M_{L2}}{C_h \pi D^2 t} \tag{13.85}$$

where M_{L2} = longitudinal bending moment at the supports,
C_h = an empirical constant; varying from 1.0 for a completely stiff shell to about 0.1 for a thin, unstiffened, shell.

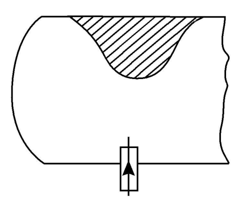


Figure 13.25. Saddle supports: shaded area is ineffective against longitudinal bending in an unstiffened shell

The ends of the vessels will stiffen the shell if the position of the saddles is less than $D/4$ from the ends. Ring stiffeners, located at the supports, are used to stiffen the shells of long thin vessels. The rings may be fitted inside or outside the vessel.

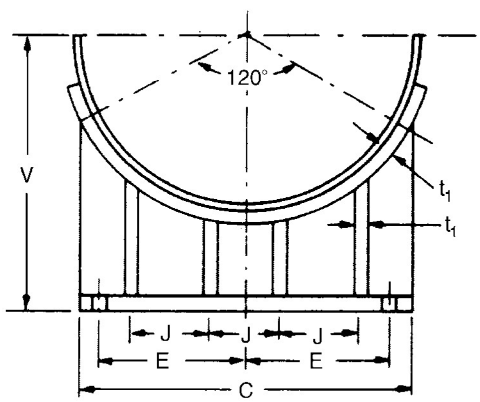
In addition to the longitudinal bending stress, a vessel supported on saddles will be subjected to tangential shear stresses, which transfer the load from the unsupported sections of the vessel to the supports; and to circumferential bending stresses. All these stresses need to be considered in the design of large, thin-walled, vessels, to ensure that the resultant stress does not exceed the maximum allowable design stress or the critical buckling stress for the material. A detailed stress analysis is beyond the scope of this book. A complete analysis of the stress induced in the shell by the supports is given by Zick (1951). Zick's method forms the basis of the design methods given in the national codes and standards. The method is also given by Brownell and Young (1959), Escoe (1994) and Megyesy (2001).

Design of saddles

The saddles must be designed to withstand the load imposed by the weight of the vessel and contents. They are constructed of bricks or concrete, or are fabricated from steel plate. The contact angle should not be less than 120°, and will not normally be greater than 150°. Wear plates are often welded to the shell wall to reinforce the wall over the area of contact with the saddle.

The dimensions of typical "standard" saddle designs are given in Figure 13.26. To take up any thermal expansion of the vessel, such as that in heat exchangers, the anchor bolt holes in one saddle can be slotted.

Procedures for the design of saddle supports are given by Brownell and Young (1959), Megyesy (2001), Escoe (1994) and Moss (2003).

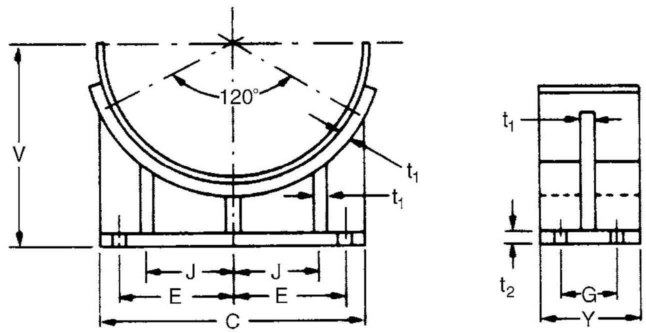


Vessel diam. (m)	Maximum weight (kN)	Dimensions (m)						mm			
		V	Y	C	E	J	G	t_2	t_1	Bolt diam.	Bolt holes
0.6	35	0.48	0.15	0.55	0.24	0.190	0.095	6	5	20	25
0.8	50	0.58	0.15	0.70	0.29	0.225	0.095	8	5	20	25
0.9	65	0.63	0.15	0.81	0.34	0.275	0.095	10	6	20	25
1.0	90	0.68	0.15	0.91	0.39	0.310	0.095	11	8	20	25
1.2	180	0.78	0.20	1.09	0.45	0.360	0.140	12	10	24	30

All contacting edges fillet welded

(a)

Figure 13.26. Standard steel saddles (adapted from Bhattacharyya, 1976). (*a*) for vessels up to 1.2 m



Vessel diam. (m)	Maximum weight (kN)	Dimensions (m)						mm			
		V	Y	C	E	J	G	t_2	t_1	Bolt diam.	Bolt holes
1.4	230	0.88	0.20	1.24	0.53	0.305	0.140	12	10	24	30
1.6	330	0.98	0.20	1.41	0.62	0.350	0.140	12	10	24	30
1.8	380	1.08	0.20	1.59	0.71	0.405	0.140	12	10	24	30
2.0	460	1.18	0.20	1.77	0.80	0.450	0.140	12	10	24	30
2.2	750	1.28	0.225	1.95	0.89	0.520	0.150	16	12	24	30
2.4	900	1.38	0.225	2.13	0.98	0.565	0.150	16	12	27	33
2.6	1000	1.48	0.225	2.30	1.03	0.590	0.150	16	12	27	33
2.8	1350	1.58	0.25	2.50	1.10	0.625	0.150	16	12	27	33
3.0	1750	1.68	0.25	2.64	1.18	0.665	0.150	16	12	27	33
3.2	2000	1.78	0.25	2.82	1.26	0.730	0.150	16	12	27	33
3.6	2500	1.98	0.25	3.20	1.40	0.815	0.150	16	12	27	33

All contacting edges fillet welded

(b)

Figure 13.26. (*b*) for vessels greater than 1.2 m

13.9.2. Skirt supports

A skirt support consists of a cylindrical or conical shell welded to the base of the vessel. A flange at the bottom of the skirt transmits the load to the foundations. Typical designs are shown in Figure 13.23. Openings must be provided in the skirt for access and for any connecting pipes; the openings are normally reinforced. The skirt may be welded to the bottom head of the vessel. Figure 13.27*a*; or welded flush with the shell, Figure 13.27*b*; or welded to the outside of the vessel shell, Figure 13.27*c*. The arrangement shown in Figure 13.27*b* is usually preferred.

Skirt supports are recommended for vertical vessels as they do not impose concentrated loads on the vessel shell; they are particularly suitable for use with tall columns subject to wind loading.

Skirt thickness

The skirt thickness must be sufficient to withstand the dead-weight loads and bending moments imposed on it by the vessel; it will not be under the vessel pressure.

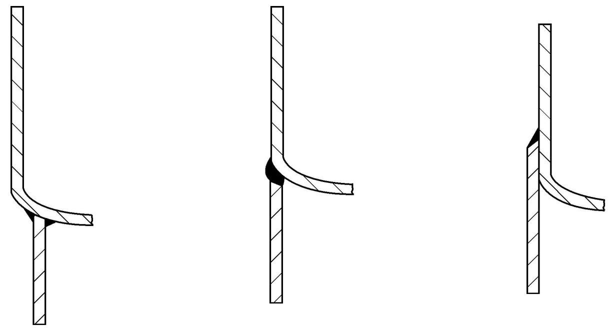


Figure 13.27. Skirt-support welds

The resultant stresses in the skirt will be:

$$\sigma_s \text{ (tensile)} = \sigma_{bs} - \sigma_{ws} \tag{13.86}$$

and

$$\sigma_s \text{ (compressive)} = \sigma_{bs} + \sigma_{ws} \tag{13.87}$$

where σ_{bs} = bending stress in the skirt

$$= \frac{4M_s}{\pi(D_s + t_s)t_s D_s}, \tag{13.88}$$

σ_{ws} = the dead weight stress in the skirt,

$$= \frac{W}{\pi(D_s + t_s)t_s} \tag{13.89}$$

where M_s = maximum bending moment, evaluated at the base of the skirt (due to wind, seismic and eccentric loads, see Section 13.8),
W = total weight of the vessel and contents (see Section 13.8),
D_s = inside diameter of the skirt, at the base,
t_s = skirt thickness.

The skirt thickness should be such that under the worst combination of wind and dead-weight loading the following design criteria are not exceeded:

$$\sigma_s \text{ (tensile)} \not> f_s J \sin\theta_s \tag{13.90}$$

$$\sigma_s \text{ (compressive)} \not> 0.125E\left(\frac{t_s}{D_s}\right)\sin\theta_s \tag{13.91}$$

where f_s = maximum allowable design stress for the skirt material, normally taken at ambient temperature, 20°C,

J = weld joint factor, if applicable,

θ_s = base angle of a conical skirt, normally 80° to 90°.

The minimum thickness should be not less than 6 mm.

Where the vessel wall will be at a significantly higher temperature than the skirt, discontinuity stresses will be set up due to differences in thermal expansion. Methods for calculating the thermal stresses in skirt supports are given by Weil and Murphy (1960) and Bergman (1963).

Base ring and anchor bolt design

The loads carried by the skirt are transmitted to the foundation slab by the skirt base ring (bearing plate). The moment produced by wind and other lateral loads will tend to overturn the vessel; this will be opposed by the couple set up by the weight of the vessel and the tensile load in the anchor bolts. A variety of base ring designs is used with skirt supports. The simplest types, suitable for small vessels, are the rolled angle and plain flange rings shown in Figure 13.28*a* and *b*. For larger columns a double ring stiffened by gussets, Figure 13.18*c*, or chair supports, Figure 13.30, are used. Design methods for base rings, and methods for sizing the anchor bolts, are given by Brownell and Young (1959). For preliminary design, the short-cut method and nomographs given by Scheiman (1963) can be used. Scheiman's method is based on a more detailed procedure for the design of base rings and foundations for columns and stacks given by Marshall (1958). Scheiman's method is outlined below and illustrated in Example 13.4.

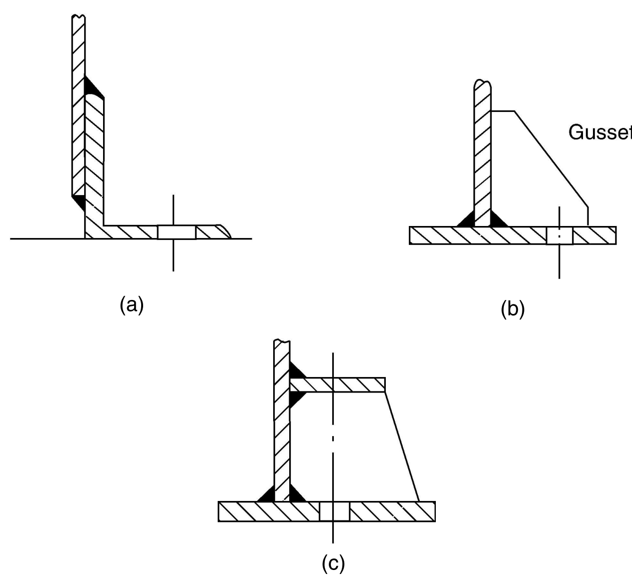


Figure 13.28. Flange ring designs (*a*) Rolled-angle (*b*) Single plate with gusset (*c*) Double plate with gusset

The anchor bolts are assumed to share the overturning load equally, and the bolt area required is given by:

$$A_b = \frac{1}{N_b f_b}\left[\frac{4M_s}{D_b} - W\right] \tag{13.92}$$

where A_b = area of one bolt at the root of the thread, mm^2,
N_b = number of bolts,
f_b = maximum allowable bolt stress, N/mm^2;
typical design value 125 N/mm^2 (18,000 psi),
M_s = bending (overturning) moment at the base, Nm,
W = weight of the vessel, N,
D_b = bolt circle diameter, m.

Scheiman gives the following guide rules which can be used for the selection of the anchor bolts:

1. Bolts smaller than 25 mm (1 in.) diameter should not be used.
2. Minimum number of bolts 8.
3. Use multiples of 4 bolts.
4. Bolt pitch should not be less than 600 mm (2 ft).

If the minimum bolt pitch cannot be accommodated with a cylindrical skirt, a conical skirt should be used.

The base ring must be sufficiently wide to distribute the load to the foundation. The total compressive load on the base ring is given by:

$$F_b = \left[\frac{4M_s}{\pi D_s^2} + \frac{W}{\pi D_s}\right] \tag{13.93}$$

where F_b = the compressive load on the base ring, Newtons per linear metre,
D_s = skirt diameter, m.

The minimum width of the base ring is given by:

$$L_b = \frac{F_b}{f_c} \times \frac{1}{10^3} \tag{13.94}$$

where L_b = base ring width, mm (Figure 13.29),
f_c = the maximum allowable bearing pressure on the concrete foundation pad, which will depend on the mix used, and will typically range from 3.5 to 7 N/mm^2 (500 to 1000 psi).

The required thickness for the base ring is found by treating the ring as a cantilever beam.

The minimum thickness is given by:

$$t_b = L_r\sqrt{\frac{3f'_c}{f_r}} \tag{13.95}$$

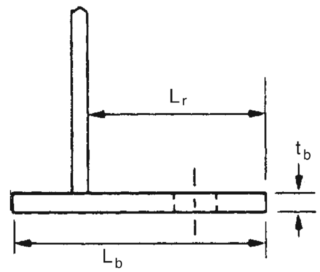
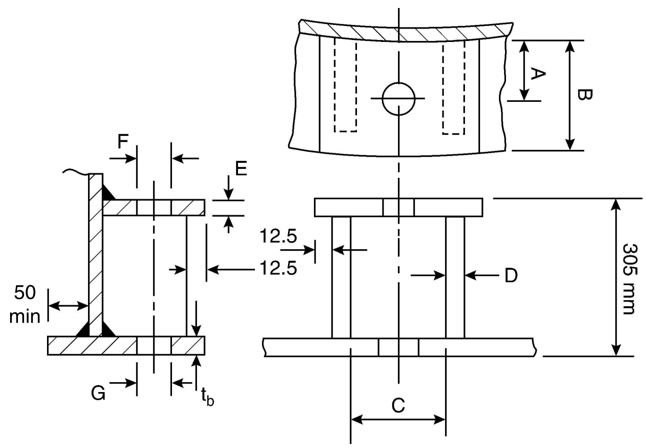


Figure 13.29. Flange ring dimensions



All contacting edges fillet welded

Dimensions mm								
Bolt size	Root area	*A*	*B*	*C*	*D*	*E*	*F*	*G*
M24	353	45	76	64	13	19	30	36
M30	561	50	76	64	13	25	36	42
M36	817	57	102	76	16	32	42	48
M42	1120	60	102	76	16	32	48	54
M48	1470	67	127	89	19	38	54	60
M56	2030	75	150	102	25	45	60	66
M64	2680	83	152	102	25	50	70	76
70	—	89	178	127	32	64	76	83
76	—	95	178	127	32	64	83	89

Bolt size = Nominal dia. (BS 4190: 1967)

Figure 13.30. Anchor bolt chair design

where L_r = the distance from the edge of the skirt to the outer edge of the ring, mm; Figure 13.29,

t_b = base ring thickness, mm,

f'_c = actual bearing pressure on base, N/mm^2,

f_r = allowable design stress in the ring material, typically 140 N/mm^2.

Standard designs will normally be used for the bolting chairs. The design shown in Figure 13.30 has been adapted from that given by Scheiman.

Example 13.4

Design a skirt support for the column specified in Example 13.3.

Solution

Try a straight cylindrical skirt ($\theta_s = 90^\circ$) of plain carbon steel, design stress 135 N/mm^2 and Young's modulus 200,000 N/mm^2 at ambient temperature.

The maximum dead weight load on the skirt will occur when the vessel is full of water.

$$\text{Approximate weight} = \left(\frac{\pi}{4} \times 2^2 \times 50\right) 1000 \times 9.81$$

$$= 1{,}540{,}951 \text{ N}$$

$$= 1541 \text{ kN}$$

Weight of vessel, from Example 13.3 = 842 kN

Total weight = 1541 + 842 = 2383 kN

Wind loading, from Example 13.4 = 2.79 kN/m

$$\text{Bending moment at base of skirt} = 2.79 \times \frac{53^2}{2} \tag{13.77}$$

$$= 3919 \text{ kNm}$$

As a first trial, take the skirt thickness as the same as that of the bottom section of the vessel, 18 mm.

$$\sigma_{bs} = \frac{4 \times 3919 \times 10^3 \times 10^3}{\pi(2000 + 18)2000 \times 18} \tag{13.88}$$

$$= 68.7 \text{ N/mm}^2$$

$$\sigma_{ws} \text{ (test)} = \frac{1543 \times 10^3}{\pi(2000 + 18)18} = 13.5 \text{ N/mm}^2 \tag{13.89}$$

$$\sigma_{ws} \text{ (operating)} = \frac{842 \times 10^3}{\pi(2000 + 18)18} = 7.4 \text{ N/mm}^2 \tag{13.89}$$

Note: the "test" condition is with the vessel full of water for the hydraulic test. In estimating total weight, the weight of liquid on the plates has been counted twice. The weight has not been adjusted to allow for this as the error is small, and on the "safe side".

$$\text{Maximum } \hat{\sigma}_s \text{ (compressive)} = 68.7 + 13.5 = 82.2 \text{ N/mm}^2 \tag{13.87}$$

$$\text{Maximum } \hat{\sigma}_s \text{ (tensile)} = 68.7 - 7.4 = 61.3 \text{ N/mm}^2 \tag{13.86}$$

Take the joint factor J as 0.85.

Criteria for design:

$$\hat{\sigma}_s \text{ (tensile)} \not> f_s J \sin\theta \qquad (13.90)$$

$$61.3 \not> 0.85 \times 135 \sin 90$$

$$61.3 \not> 115$$

$$\hat{\sigma}_s \text{ (compressive)} \not> 0.125E\left(\frac{t_s}{D_s}\right)\sin\theta \qquad (13.91)$$

$$82.2 \not> 0.125 \times 200{,}000\left(\frac{18}{2000}\right)\sin 90$$

$$82.2 \not> 225$$

Both criteria are satisfied, add 2 mm for corrosion, gives a design thickness of 20 mm

Base ring and anchor bolts

Approximate pitch circle dia., say, 2.2 m
Circumference of bolt circle $= 2200\pi$
Number of bolts required, at minimum recommended bolt spacing

$$= \frac{2200\pi}{600} = 11.5$$

Closest multiple of $4 = 12$ bolts
Take bolt design stress $= 125$ N/mm^2
$M_s = 3919$ kN m

Take W = operating value = 842 kN.

$$A_b = \frac{1}{12 \times 125}\left[\frac{4 \times 3919 \times 10^3}{2.2} - 842 \times 10^3\right] \qquad (13.92)$$

$$= 4190 \text{ mm}^2$$

$$\text{Bolt root dia.} = \sqrt{\frac{4190 \times 4}{\pi}} = 73 \text{ mm, looks too large.}$$

Total compressive load on the base ring per unit length

$$F_b = \left[\frac{4 \times 3919 \times 10^3}{\pi \times 2.0^2} + \frac{842 \times 10^3}{\pi \times 2.0}\right] \qquad (13.93)$$

$$= 1381 \times 10^3 \text{ N/m}$$

Taking the bearing pressure as 5 N/mm^2

$$L_b = \frac{1381 \times 10^3}{5 \times 10^3} = 276 \text{ mm} \qquad (13.94)$$

Rather large — consider a flared skirt.

Take the skirt bottom dia. as 3 m

$$\text{Skirt base angle } \theta_s = \tan^{-1}\frac{3}{\frac{1}{2}(3-2)} = 80.5^\circ$$

Keep the skirt thickness the same as that calculated for the cylindrical skirt. Highest stresses will occur at the top of the skirt; where the values will be close to those calculated for the cylindrical skirt. Sin $80.5^\circ = 0.99$, so this term has little effect on the design criteria.

Assume bolt circle dia. = 3.2 m.

Take number of bolts as 16.

$$\text{Bolt spacing} = \frac{\pi \times 3.2 \times 10^3}{16} = 628 \text{ mm satisfactory.}$$

$$A_b = \frac{1}{16 \times 125}\left[\frac{4 \times 3919 \times 10^3}{3.2} - 842 \times 10^3\right]$$

$$= \underline{\underline{2029 \text{ mm}^2}}$$

Use M56 bolts (BS 4190:1967) root area = 2030 mm^2,

$$F_b = \left[\frac{4 \times 3919 \times 10^3}{\pi \times 3.0^2} + \frac{842 \times 10^3}{\pi \times 3.0}\right]$$

$$= 644 \text{ kN/m.}$$

$$L_b = \frac{644 \times 10^3}{5 \times 10^3} = 129 \text{ mm}$$

This is the minimum width required; actual width will depend on the chair design.

Actual width required (Figure 13.30):

$$= L_r + t_s + 50 \text{ mm}$$

$$= 150 + 20 + 50 = \underline{\underline{220 \text{ mm}}}$$

Actual bearing pressure on concrete foundation:

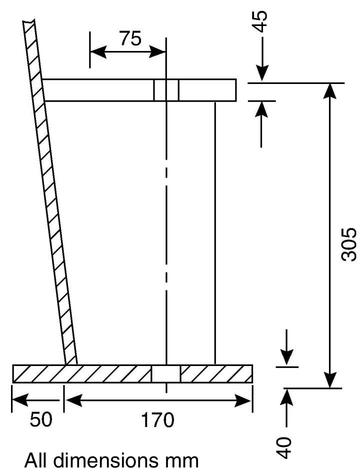
$$f'_c = \frac{644 \times 10^3}{220 \times 10^3} = 2.93 \text{ N/mm}^2$$

$$t_b = 150\sqrt{\frac{3 \times 2.93}{140}} = 37.6 \text{ mm} \qquad (13.95)$$

$$\text{round off to } \underline{\underline{40 \text{ mm}}}$$

Chair dimensions from Figure 13.30 for bolt size M56.

Skirt to be welded flush with outer diameter of column shell.



13.9.3. Bracket supports

Brackets, or lugs, can be used to support vertical vessels. The bracket may rest on the building structural steel work, or the vessel may be supported on legs; Figure 13.24.

The main load carried by the brackets will be the weight of the vessel and contents; in addition the bracket must be designed to resist the load due to any bending moment due to wind, or other loads. If the bending moment is likely to be significant skirt supports should be considered in preference to bracket supports.

As the reaction on the bracket is eccentric, Figure 13.31, the bracket will impose a bending moment on the vessel wall. The point of support, at which the reaction acts, should be made as close to the vessel wall as possible; allowing for the thickness of any insulation. Methods for estimating the magnitude of the stresses induced in the vessel

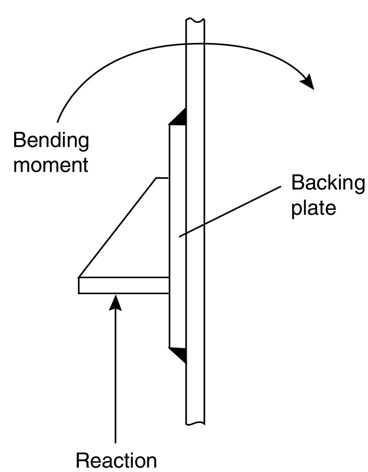


Figure 13.31. Loads on a bracket support

wall by bracket supports are given by Brownell and Young (1959) and by Wolosewick (1951). Backing plates are often used to carry the bending loads.

The brackets, and supporting steel work, can be designed using the usual methods for structural steelwork. Suitable methods are given by Bednar (1986) and Moss (2003).

A quick method for sizing vessel reinforcing rings (backing plates) for bracket supports is given by Mahajan (1977).

Typical bracket designs are shown in Figures 13.32*a* and *b*. The loads which steel brackets with these proportions will support are given by the following formula:

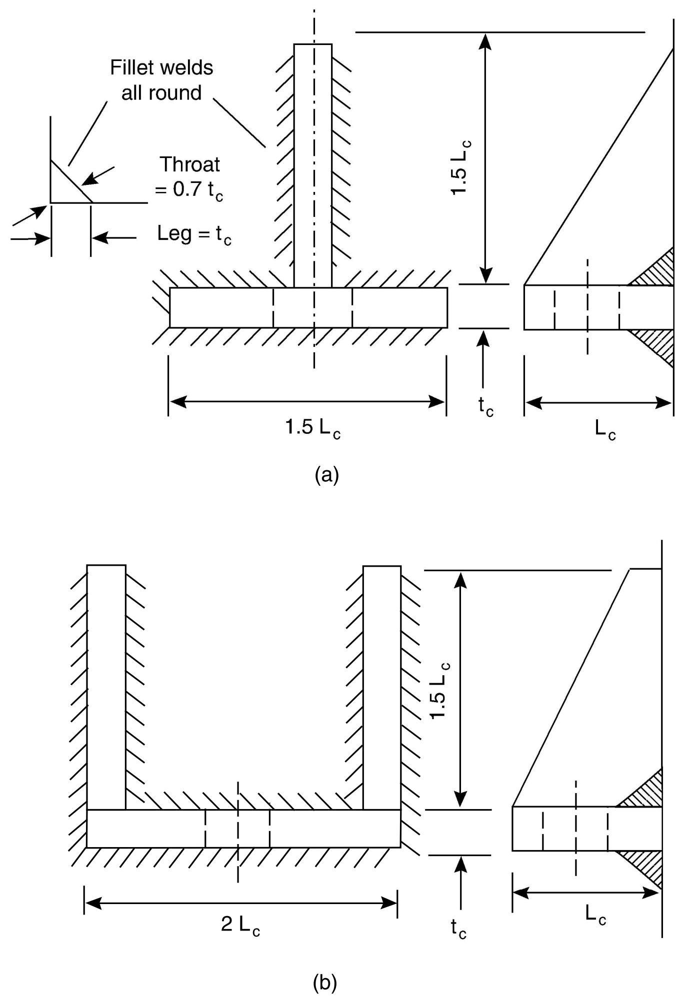


Figure 13.32. Bracket designs (*a*) Single gusset plate (*b*) Double gusset plate

Single-gusset plate design, Figure 13.32*a*:

$$F_{bs} = 60L_c t_c \tag{13.96}$$

Double-gusset plate design, Figure 13.32*b*:

$$F_{bs} = 120 L_c t_c \tag{13.97}$$

where F_{bs} = maximum design load per bracket, N,
L_c = the characteristic dimension of bracket (depth), mm,
t_c = thickness of plate, mm.

13.10. BOLTED FLANGED JOINTS

Flanged joints are used for connecting pipes and instruments to vessels, for manhole covers, and for removable vessel heads when ease of access is required. Flanges may also be used on the vessel body, when it is necessary to divide the vessel into sections for transport or maintenance. Flanged joints are also used to connect pipes to other equipment, such as pumps and valves. Screwed joints are often used for small-diameter pipe connections, below 40 mm. Flanged joints are also used for connecting pipe sections where ease of assembly and dismantling is required for maintenance, but pipework will normally be welded to reduce costs.

Flanges range in size from a few millimetres diameter for small pipes, to several metres diameter for those used as body or head flanges on vessels.

13.10.1. Types of flange, and selection

Several different types of flange are used for various applications. The principal types used in the process industries are:

1. Welding-neck flanges.
2. Slip-on flanges, hub and plate types.
3. Lap-joint flanges.
4. Screwed flanges.
5. Blank, or blind, flanges.

Welding-neck flanges, Figure 13.33*a*: have a long tapered hub between the flange ring and the welded joint. This gradual transition of the section reduces the discontinuity stresses between the flange and branch, and increases the strength of the flange assembly. Welding-neck flanges are suitable for extreme service conditions; where the flange is likely to be subjected to temperature, shear and vibration loads. They will normally be specified for the connections and nozzles on process vessels and process equipment.

Slip-on flanges, Figure 13.33*b*: slip over the pipe or nozzle and are welded externally, and usually also internally. The end of the pipe is set back from 0 to 2.0 mm. The strength of a slip-on flange is from one-third to two-thirds that of the corresponding standard welding-neck flange. Slip-on flanges are cheaper than welding-neck flanges and are easier to align, but have poor resistance to shock and vibration loads. Slip-on flanges are generally used for pipe work. Figure 13.33*b* shows a forged flange with a hub; for light duties slip-on flanges can be cut from plate.

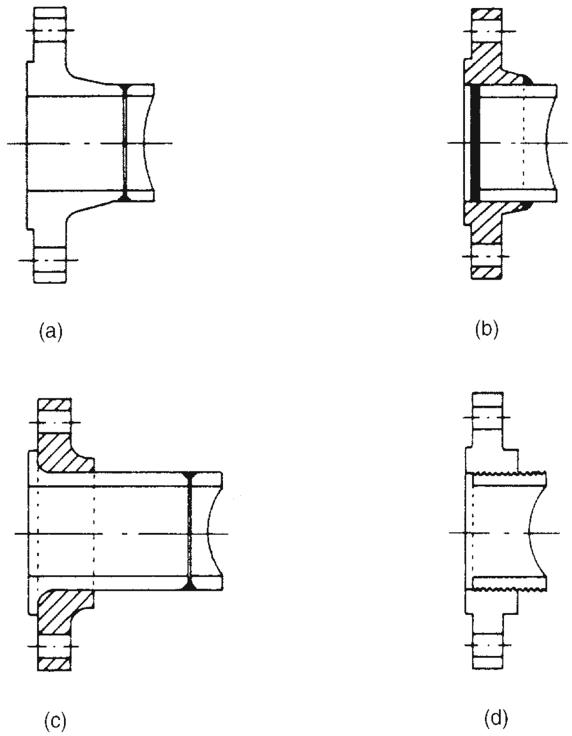


Figure 13.33. Flange types (*a*) Welding-neck (*b*) Slip-on (*c*) Lap-joint (*d*) Screwed

Lap-joint flanges, Figure 13.33*c*: are used for piped work. They are economical when used with expensive alloy pipe, such as stainless steel, as the flange can be made from inexpensive carbon steel. Usually a short lapped nozzle is welded to the pipe, but with some schedules of pipe the lap can be formed on the pipe itself, and this will give a cheap method of pipe assembly.

Lap-joint flanges are sometimes known as "Van-stone flanges".

Screwed flanges, Figure 13.33*d*: are used to connect screwed fittings to flanges. They are also sometimes used for alloy pipe which is difficult to weld satisfactorily.

Blind flanges (blank flanges): are flat plates, used to blank off flange connections, and as covers for manholes and inspection ports.

13.10.2. Gaskets

Gaskets are used to make a leak-tight joint between two surfaces. It is impractical to machine flanges to the degree of surface finish that would be required to make a satisfactory seal under pressure without a gasket. Gaskets are made from "semi-plastic" materials; which will deform and flow under load to fill the surface irregularities between the flange faces, yet retain sufficient elasticity to take up the changes in the flange alignment that occur under load.

Table 13.4. Gasket materials
(Based on a similar table in BS 5500: 1991; see BS PD 5500–2003)

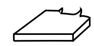
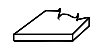
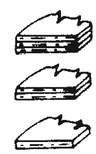
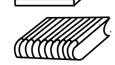
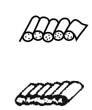
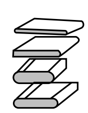
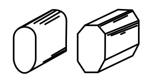
Gasket material		Gasket factor *m*	Min. design seating stress *y*(N/mm^2)	Sketches	Minimum gasket width (mm)
Rubber without fabric or a high percentage of asbestos fibre; hardness:					
below 75° IRH		0.50	0		10
75° IRH or higher		1.00	1.4		
Asbestos with a suitable binder for the operating conditions	3.2 mm thick	2.00	11.0		
	1.6 mm thick	2.75	25.5		10
	0.8 mm thick	3.50	44.8		
Rubber with cotton fabric insertion		1.25	2.8		10
Rubber with asbestos fabric insertion, with or without wire reinforcement	3-ply	2.25	15.2		
	2-ply	2.50	20.0		10
	1-ply	2.75	25.5		
Vegetable fibre		1.75	7.6		10
Spiral-wound metal, asbestos filled	Carbon	2.50	20.0		
	Stainless or monel	3.00	31.0		10
Corrugated metal, asbestos inserted or Corrugated metal, jacketed asbestos filled	Soft aluminium	2.50	20.0		
	Soft copper or brass	2.75	25.5		
	Iron or soft steel	3.00	31.0		10
	Monel or 4 to 6 per cent chrome	3.25	37.9		
	Stainless steels	3.50	44.8		
Corrugated metal	Soft aluminium	2.75	25.5		
	Soft copper or brass	3.00	31.0		
	Iron or soft steel	3.25	37.9		10
	Monel or 4 to 6 per cent chrome	3.50	44.8		
	Stainless steels	3.75	52.4		
Flat metal jacketed asbestos filled	Soft aluminium	3.25	37.9		
	Soft copper or brass	3.50	44.8		
	Iron or soft steel	3.75	52.4		10
	Monel	3.50	55.1		
	4 to 6 per cent chrome	3.75	62.0		
	Stainless steels	3.75	62.0		
Grooved metal	Soft aluminium	3.25	37.9		
	Soft copper or brass	3.50	44.8		
	Iron or soft steel	3.75	52.4		10
	Monel or 4 to 6 per cent chrome	3.75	62.0		
	Stainless steels	4.25	69.5		
	Soft aluminium	4.00	60.6		
	Soft copper or brass	4.75	89.5		

Table 13.4. (*continued*)

Gasket material		Gasket factor m	Min. design seating stress y(N/mm^2)	Sketches	Minimum gasket width (mm)
Solid flat metal	Iron or soft steel	5.50	124		6
	Monel or 4 to 6 per cent chrome	6.00	150		
	Stainless steels	6.50	179		
Ring joint	Iron or soft steel	5.50	124		
	Monel or 4 to 6 per cent chrome	6.00	150		6
	Stainless steels	6.50	179		

A great variety of proprietary gasket materials is used, and reference should be made to the manufacturers' catalogues and technical manuals when selecting gaskets for a particular application. Design data for some of the more commonly used gasket materials are given in Table 13.4. Further data can be found in the pressure vessel codes and standards and in various handbooks; Perry *et al*. (1997). The minimum seating stress y is the force per unit area (pressure) on the gasket that is required to cause the material to flow and fill the surface irregularities in the gasket face.

The gasket factor m is the ratio of the gasket stress (pressure) under the operating conditions to the internal pressure in the vessel or pipe. The internal pressure will force the flanges' faces apart, so the pressure on the gasket under operating conditions will be lower than the initial tightening-up pressure. The gasket factor gives the minimum pressure that must be maintained on the gasket to ensure a satisfactory seal.

The following factors must be considered when selecting a gasket material:

1. The process conditions: pressure, temperature, corrosive nature of the process fluid.
2. Whether repeated assembly and disassembly of the joint is required.
3. The type of flange and flange face (see Section 13.10.3).

Up to pressures of 20 bar, the operating temperature and corrosiveness of the process fluid will be the controlling factor in gasket selection. Vegetable fibre and synthetic rubber gaskets can be used at temperatures of up to 100°C. Solid polyfluorocarbon (Teflon) and compressed asbestos gaskets can be used to a maximum temperature of about 260°C. Metal-reinforced gaskets can be used up to around 450°C. Plain soft metal gaskets are normally used for higher temperatures.

13.10.3. Flange faces

Flanges are also classified according to the type of flange face used. There are two basic types:

1. Full-faced flanges, Figure 13.34*a*: where the face contact area extends outside the circle of bolts; over the full face of the flange.

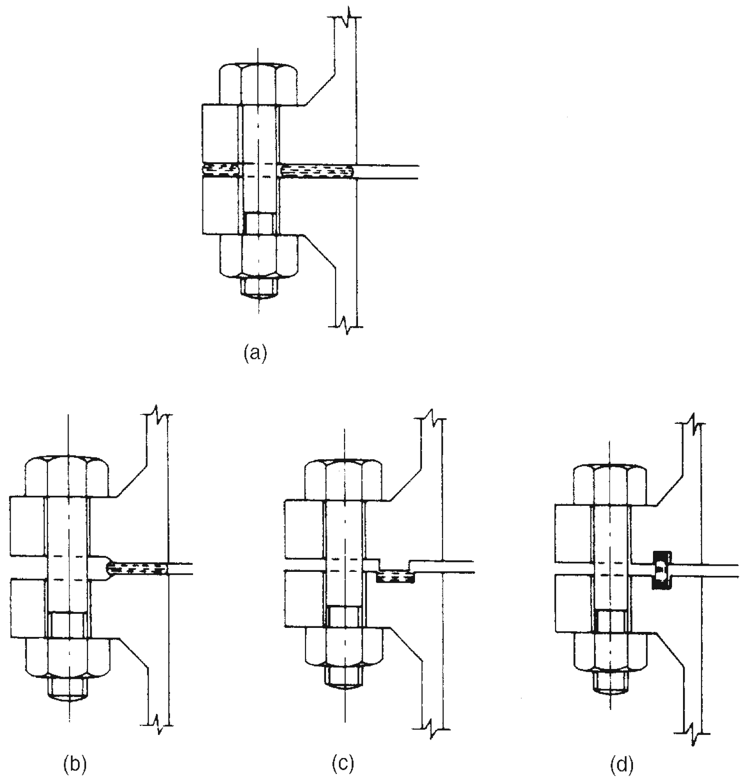


Figure 13.34. Flange types and faces (*a*) Full-face (*b*) Gasket within bolt circle (*c*) Spigot and socket (*d*) Ring type joint

2. Narrow-faced flanges, Figure 13.34*b*, *c*, *d*: where the face contact area is located within the circle of bolts.

Full face, wide-faced, flanges are simple and inexpensive, but are only suitable for low pressures. The gasket area is large, and an excessively high bolt tension would be needed to achieve sufficient gasket pressure to maintain a good seal at high operating pressures.

The raised face, narrow-faced, flange shown in Figure 13.34*b* is probably the most commonly used type of flange for process equipment.

Where the flange has a plain face, as in Figure 13.34*b*, the gasket is held in place by friction between the gasket and flange surface. In the spigot and socket, and tongue and grooved faces, Figure 13.34*c*, the gasket is confined in a groove, which prevents failure by "blow-out". Matched pairs of flanges are required, which increases the cost, but this type is suitable for high pressure and high vacuum service. Ring joint flanges, Figure 13.34*d*, are used for high temperatures and high pressure services.

13.10.4. Flange design

Standard flanges will be specified for most applications (see Section 13.10.5). Special designs would be used only if no suitable standard flange were available; or for large flanges, such as the body flanges of vessels, where it may be cheaper to size a flange specifically for the duty required rather than to accept the nearest standard flange, which of necessity would be over-sized.

Figure 13.35 shows the forces acting on a flanged joint. The bolts hold the faces together, resisting the forces due to the internal pressure and the gasket sealing pressure. As these forces are offset the flange is subjected to a bending moment. It can be considered as a cantilever beam with a concentrated load. A flange assembly must be sized so as to have sufficient strength and rigidity to resist this bending moment. A flange that lacks sufficient rigidity will rotate slightly, and the joint will leak; Figure 13.36. The principles of flange design are discussed by Singh and Soler (1992), and Azbel and Cheremisinoff (1982). Singh and Soler give a computer programme for flange design.

Design procedures and work sheets for non-standard flanges are given in the national codes and standards.

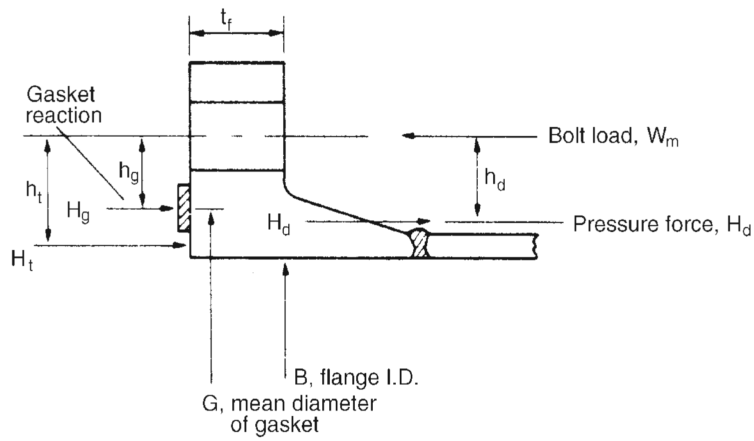


Figure 13.35. Forces acting on an integral flange

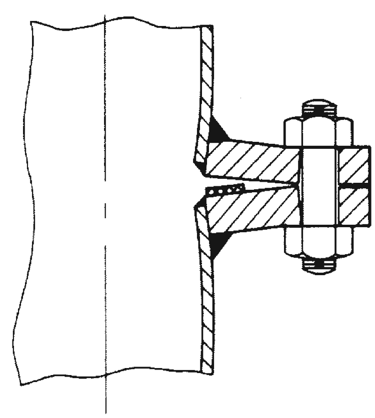


Figure 13.36. Deflection of a weak flange (exaggerated)

For design purposes, flanges are classified as integral or loose flanges.

Integral flanges are those in which the construction is such that the flange obtains support from its hub and the connecting nozzle (or pipe). The flange assembly and nozzle neck form an "integral" structure. A welding-neck flange would be classified as an integral flange.

Loose flanges are attached to the nozzle (or pipe) in such a way that they obtain no significant support from the nozzle neck and cannot be classified as an integral attachment. Screwed and lap-joint flanges are typical examples of loose flanges.

The design procedures given in the codes and standards can be illustrated by considering the forces and moments which act on an integral flange, Figure 13.35.

The total moment M_{op} acting on the flange is given by:

$$M_{op} = H_d h_d + H_t h_t + H_g h_g \tag{13.98}$$

Where H_g = gasket reaction (pressure force), = $\pi G(2b)mP_i$
H_t = pressure force on the flange face = $H - H_d$,
H = total pressure force = $(\pi/4)G^2 P_i$,
H_d = pressure force on the area inside the flange = $(\pi/4)B^2 P_i$,
G = mean diameter of the gasket,
B = inside diameter of the flange,
$2b$ = effective gasket pressure width,
b = effective gasket sealing width,
h_d, h_g and h_t are defined in Figure 13.35.

The minimum required bolt load under the operating conditions is given by:

$$W_{m1} = H + H_g \tag{13.99}$$

The forces and moments on the flange must also be checked under the bolting-up conditions.

The moment M_{atm} is given by:

$$M_{atm} = W_{m2} h_g \tag{13.100}$$

where W_{m2} is the bolt load required to seat the gasket, given by:

$$W_{m2} = y\pi G b \tag{13.101}$$

where y is the gasket seating pressure (stress).

The flange stresses are given by:

longitudinal hub stress, $$\sigma_{hb} = F_1 M \tag{13.102}$$

radial flange stress, $$\sigma_{rd} = F_2 M \tag{13.103}$$

tangential flange stress, $$\sigma_{tg} = F_3 M - F_4 \sigma_{rd} \tag{13.104}$$

where M is taken as M_{op} or M_{atm}, whichever is the greater; and the factors F_1 to F_4 are functions of the flange type and dimensions, and are obtained from equations and graphs given in the codes and standards (BS 5500, clause 3.8).

The flange must be sized so that the stresses given by equations 13.102 to 13.104 satisfy the following criteria:

$$\sigma_{hb} \ngtr 1.5 f_{f0} \tag{13.105}$$

$$\sigma_{rd} \ngtr f_{f0} \tag{13.106}$$

$$\tfrac{1}{2}(\sigma_{hb} + \sigma_{rd}) \ngtr f_{f0} \tag{13.107}$$

$$\tfrac{1}{2}(\sigma_{hb} + \sigma_{tg}) \ngtr f_{f0} \tag{13.108}$$

where f_{f0} is the maximum allowable design stress for the flange material at the operating conditions.

The minimum bolt area required A_{bf} will be given by:

$$A_{bf} = \frac{W_m}{f_b} \tag{13.109}$$

where W_m is the greater value of W_{m1} or W_{m2}, and f_b the maximum allowable bolt stress. Standard size bolts should be chosen, sufficient to give the required area. The bolt size will not normally be less than 12 mm, as smaller sizes can be sheared off by over-tightening.

The bolt spacing must be selected to give a uniform compression of the gasket. It will not normally be less than 2.5 times the bolt diameter, to give sufficient clearance for tightening with a wrench or spanner. The following formula can be used to determine the maximum bolt spacing:

$$p_b = 2d_b + \frac{6t_f}{(m + 0.5)} \tag{13.110}$$

where p_b = bolt pitch (spacing), mm,
d_b = bolt diameter, mm,
t_f = flange thickness, mm,
m = gasket factor.

13.10.5. Standard flanges

Standard flanges are available in a range of types, sizes and materials; and are used extensively for pipes, nozzles and other attachments to pressure vessels.

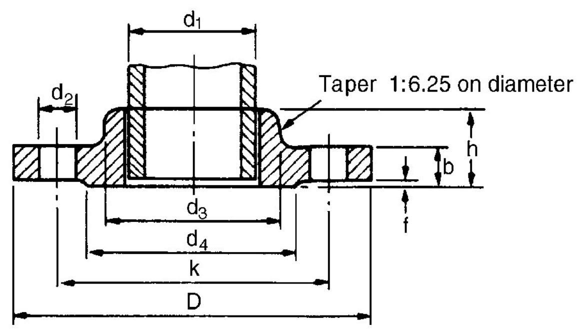
The proportions of standard flanges are set out in various codes and standards. A typical example of a standard flange design is shown in Figure 13.37. This was taken from BS 4504, which has now been superseded by the European standard BS EN 1092. The design of standard flanges is also specified in BS 1560.

In the United States, flanges are covered by the standards issued by the American National Standards Institute (ANSI). An abstract of the American standards is given by Perry *et al.* (1997).

Standard flanges are designated by class numbers, or rating numbers, which correspond to the primary service (pressure) rating of the flange at room temperature.

STEEL SLIP-ON BOSS FLANGE FOR WELDING

Nominal pressure 6 bar



Nom. size	Pipe o.d. d_1 ≈	Flange			Raised face		Bolting	Drilling			Boss
		D	b	h	d_4	f		No.	d_2	k	d_3
10	17.2	75	12	20	35	2	M10	4	11	50	25
15	21.3	80	12	20	40	2	M10	4	11	55	30
20	26.9	90	14	24	50	2	M10	4	11	65	40
25	33.7	100	14	24	60	2	M10	4	11	75	50
32	42.4	120	14	26	70	2	M12	4	14	90	60
40	48.3	130	14	26	80	3	M12	4	14	100	70
50	60.3	140	14	28	90	3	M12	4	14	110	80
65	76.1	160	14	32	110	3	M12	4	14	130	100
80	88.9	190	16	34	128	3	M16	4	18	150	110
100	114.3	210	16	40	148	3	M16	4	18	170	130
125	139.7	240	18	44	178	3	M16	8	18	200	160
150	168.3	265	18	44	202	3	M16	8	18	225	185
200	219.1	320	20	44	258	3	M16	8	18	280	240
250	273	375	22	44	312	3	M16	12	18	335	295
300	323.9	440	22	44	365	4	M20	12	22	395	355

Figure 13.37. Typical standard flange design (All dimensions mm)

The flange class number required for a particular application will depend on the design pressure and temperature, and the material of construction. The reduction in strength at elevated temperatures is allowed for by selecting a flange with a higher rating than the design pressure. For example, for a design pressure of 10 bar (150 psi) a BS 1560 carbon steel flange class 150 flange would be selected for a service temperature below 300°C; whereas for a service temperature of, say, 300°C a 300 pound flange would be specified. A typical pressure–temperature relationship for carbon steel flanges is shown in Table 13.5. Pressure–temperature ratings for a full range of materials can be obtained from the standards.

Typical designs, dimensioned, for welding-neck flanges over a range of pressure ratings are given in Appendix E. These can be used for preliminary designs. The current standards and suppliers' catalogues should be consulted before firming up the design.

Table 13.5. Typical pressure-temperature ratings for carbon steel flanges, BS 4504.

Nominal pressure (bar)	Design pressure at temperature, °C (bar)						
	up to 120	150	200	250	300	350	400
2.5	2.5	2.3	2.0	1.8	1.5	1.3	0.9
6	6.0	5.4	4.8	4.2	3.6	3.0	2.1
10	10	9.0	8.0	7.0	6.0	5.0	3.5
16	16	14.4	12.8	11.2	9.6	8.0	5.6
25	25	2.5	20.0	17.5	15.0	12.5	8.8
40	40	36.0	32.0	28.0	24.0	20.0	14.0

13.11. HEAT-EXCHANGER TUBE-PLATES

The tube-plates (tube-sheets) in shell and tube heat exchangers support the tubes, and separate the shell and tube side fluids (see Chapter 12). One side is subject to the shell-side pressure and the other the tube-side pressure. The plates must be designed to support the maximum differential pressure that is likely to occur. Radial and tangential bending stresses will be induced in the plate by the pressure load and, for fixed-head exchangers, by the load due to the differential expansion of the shell and tubes.

A tube-plate is essentially a perforated plate with an unperforated rim, supported at its periphery. The tube holes weaken the plate and reduce its flexual rigidity. The equations developed for the stress analysis of unperforated plates (Section 13.3.5) can be used for perforated plates by substituting "virtual" (effective) values for the elastic constants E and v, in place of the normal values for the plate material. The virtual elastic constants E' and v' are functions of the plate ligament efficiency, Figure 13.38; see O'Donnell and Langer (1962). The ligament efficiency of a perforated plate is defined as:

$$\lambda = \frac{p_h - d_h}{p_h} \tag{13.111}$$

where p_h = hole pitch,
d_h = hole diameter.

The "ligament" is the material between the holes (that which holds the holes together). In a tube-plate the presence of the tubes strengthens the plate, and this is taken into account when calculating the ligament efficiency by using the inside diameter of the tubes in place of the hole diameter in equation 13.111.

Design procedures for tube-plates are given in BS PD 5500, and in the TEMA heat exchanger standards (see Chapter 12). The tube-plate must be thick enough to resist the bending and shear stresses caused by the pressure load and any differential expansion of the shell and tubes. The minimum plate thickness to resist bending can be estimated using an equation of similar form to that for plate end closures (Section 13.5.3).

$$t_p = C_{ph} D_p \sqrt{\frac{\Delta P'}{\lambda f_p}} \tag{13.112}$$

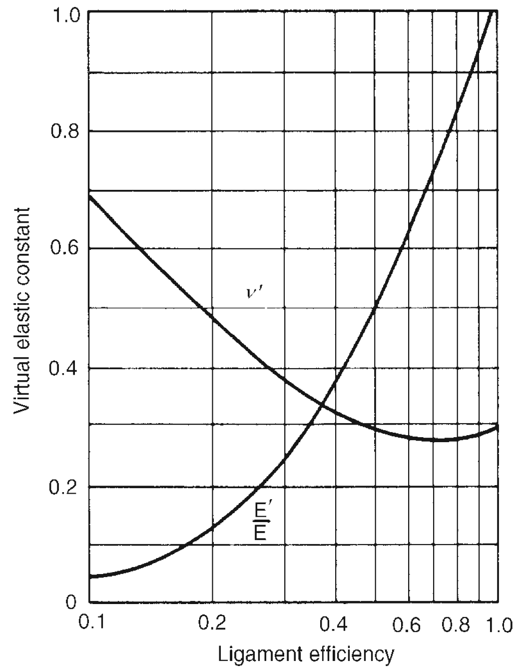


Figure 13.38. Virtual elastic constants

where t_p = the minimum plate thickness,
$\Delta P'$ = the effective tube plate design pressure,
λ = ligament efficiency,
f_p = maximum allowable design stress for the plate,
C_{ph} = a design factor,
D_p = plate diameter.

The value of the design factor C_{ph} will depend on the type of head, the edge support (clamped or simply supported), the plate dimensions, and the elastic constants for the plate and tube material.

The tube-sheet design pressure $\Delta P'$ depends on the type of exchanger. For an exchanger with confined heads or U-tubes it is taken as the maximum difference between the shell-side and tube-side operating pressures; with due consideration being given to the possible loss of pressure on either side. For exchangers with unconfined heads (plates fixed to the shell) the load on the tube-sheets due to differential expansion of the shell and tubes must be added to that due to the differential pressure.

The shear stress in the tube-plate can be calculated by equating the pressure force on the plate to the shear force in the material at the plate periphery. The minimum plate thickness to resist shear is given by:

$$t_p = \frac{0.155 D_p \Delta P'}{\lambda \tau_p} \tag{13.113}$$

where $\tau_p =$ the maximum allowable shear stress, taken as half the maximum allowable design stress for the material (see Section 13.3.2).

The design plate thickness is taken as the greater of the values obtained from equations 13.112 and 13.113 and must be greater than the minimum thickness given below:

Tube o.d. (mm)	Minimum plate thickness (mm)
25	0.75 × tube o.d.
25–30	22
30–40	25
40–50	30

For exchangers with fixed tube-plates the longitudinal stresses in the tubes and shell must be checked to ensure that the maximum allowable design stresses for the materials are not exceeded. Methods for calculating these stresses are given in the standards.

A detailed account of the methods used for the stresses analysis of tube sheets is given by Jawad and Farr (1989), and Singh and Soler (1992). Singh and Soler give computer programs for the design of the principal types of tube-plate.

13.12. WELDED JOINT DESIGN

Process vessels are built up from preformed parts: cylinders, heads, and fittings, joined by fusion welding. Riveted construction was used extensively in the past (prior to the 1940s) but is now rarely seen.

Cylindrical sections are usually made up from plate sections rolled to the required curvature. The sections (strakes) are made as large as is practicable to reduce the number of welds required. The longitudinal welded seams are offset to avoid a conjunction of welds at the corners of the plates.

Many different forms of welded joint are needed in the construction of a pressure vessel. Some typical forms are shown in Figures 13.39 to 13.41.

The design of a welded joint should satisfy the following basic requirements:

1. Give good accessibility for welding and inspection.
2. Require the minimum amount of weld metal.
3. Give good penetration of the weld metal; from both sides of the joint, if practicable.
4. Incorporate sufficient flexibility to avoid cracking due to differential thermal expansion.

The preferred types of joint, and recommended designs and profiles, are given in the codes and standards.

The correct form to use for a given joint will depend on the material, the method of welding (machine or hand), the plate thickness, and the service conditions. Double-sided V- or U-sections are used for thick plates, and single V- or U-profiles for thin plates. A backing strip is used where it is not possible to weld from both sides. Lap joints are seldom used for pressure vessels construction, but are used for atmospheric pressure storage tanks.

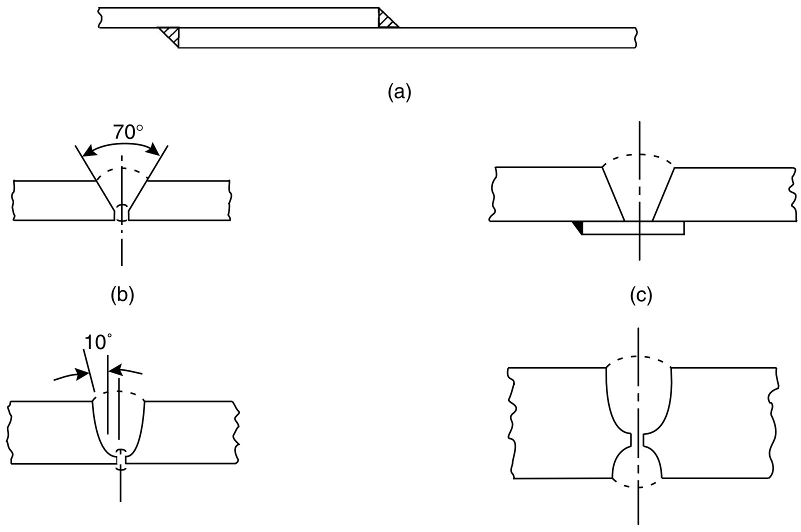


Figure 13.39. Weld profiles; (*b* to *e*) butt welds (*a*) Lap joint (*b*) Single 'V' (*c*) Backing strip (*d*) Single 'U' (*e*) Double 'U'

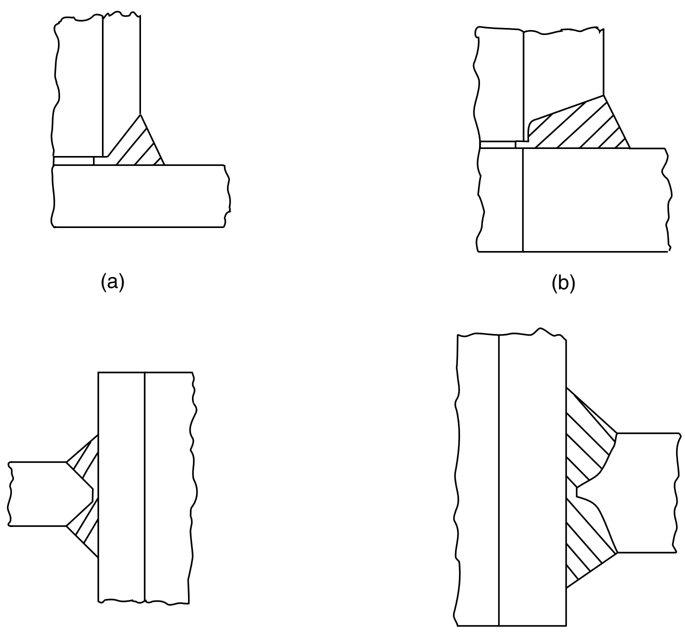


Figure 13.40. Typical weld profiles — Branches (*a*), (*b*) Set-on branches (*c*), (*d*) Set-in branches

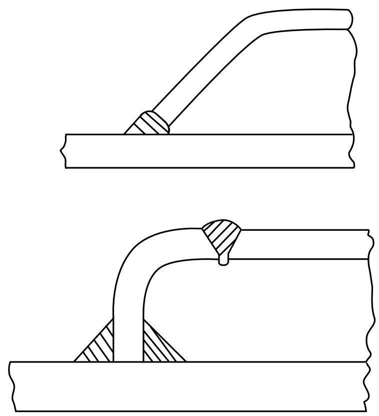


Figure 13.41. Typical construction methods for welded jackets

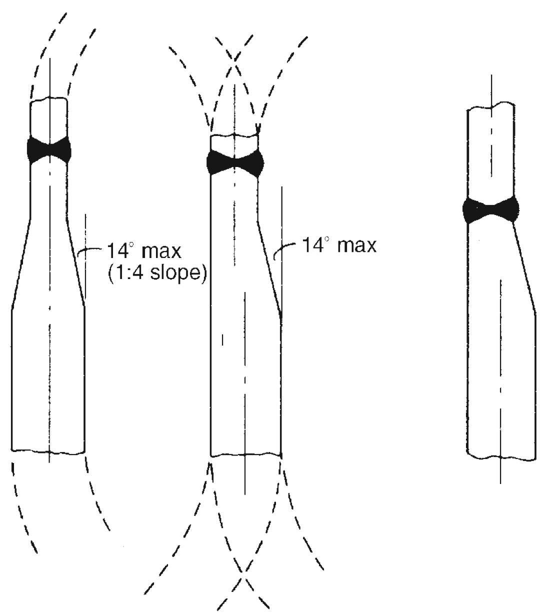


Figure 13.42. Transition between plates of unequal thickness

Where butt joints are made between plates of different thickness, the thicker plate is reduced in thickness with a slope of not greater than 1 in 4 (14°) (Figure 13.42).

The local heating, and consequent expansion, that occurs during welding can leave the joint in a state of stress. These stresses are relieved by post-welding heat treatment. Not all vessels will be stress relieved. Guidance on the need for post-welding heat treatment is given in the codes and standards, and will depend on the service and conditions, materials of construction, and plate thickness.

To ensure that a satisfactory quality of welding is maintained, welding-machine operators and welders working on the pressure parts of vessels are required to pass welder approval tests; which are designed to test their competence to make sound welds.

13.13. FATIGUE ASSESSMENT OF VESSELS

During operation the shell, or components of the vessel, may be subjected to cyclic stresses. Stress cycling can arise from the following causes:

1. Periodic fluctuations in operating pressure.
2. Temperature cycling.
3. Vibration.
4. "Water hammer".
5. Periodic fluctuation of external loads.

A detailed fatigue analysis is required if any of these conditions is likely to occur to any significant extent. Fatigue failure will occur during the service life of the vessel if the endurance limit (number of cycles for failure) at the particular value of the cyclic stress is exceeded. The codes and standards should be consulted to determine when a detailed fatigue analysis must be undertaken.

13.14. PRESSURE TESTS

The national pressure vessel codes and standards require that all pressure vessels be subjected to a pressure test to prove the integrity of the finished vessel. A hydraulic test is normally carried out, but a pneumatic test can be substituted under circumstances where the use of a liquid for testing is not practical. Hydraulic tests are safer because only a small amount of energy is stored in the compressed liquid. A standard pressure test is used when the required thickness of the vessel parts can be calculated in accordance with the particular code or standard. The vessel is tested at a pressure above the design pressure, typically 25 to 30 per cent. The test pressure is adjusted to allow for the difference in strength of the vessel material at the test temperature compared with the design temperature, and for any corrosion allowance.

Formulae for determining the appropriate test pressure are given in the codes and standards; such as that given below:

$$\text{Test pressure} = 1.25\left[P_d \frac{f_a}{f_n} \times \frac{t}{(t-c)}\right] \tag{13.114}$$

where P_d = design pressure, N/mm^2,
f_a = nominal design strength (design stress) at the test temperature, N/mm^2,
f_n = nominal design strength at the design temperature, N/mm^2,
c = corrosion allowance, mm,
t = actual plate thickness, mm.

When the required thickness of the vessel component parts cannot be determined by calculation in accordance with the methods given, the codes and standards require that a hydraulic proof test be carried out. In a proof test the stresses induced in the vessel during the test are monitored using strain gauges, or similar techniques. The requirements for the proof testing of vessels are set out in the codes and standards.

13.15. HIGH-PRESSURE VESSELS

High pressures are required for many commercial chemical processes. For example, the synthesis of ammonia is carried out at reactor pressures of up to 1000 bar, and high-density polyethylene processes operate up to 1500 bar.

Only a brief discussion of the design of vessels for operation at high pressures will be given in this section; sufficient to show the fundamental limitations of single-wall (monobloc) vessels, and the construction techniques that are used to overcome this limitation. A full discussion of the design and construction of high-pressure vessels and ancillary equipment (pumps, compressors, valves and fittings) is given in the books by Fryer and Harvey (1997) and Jawad and Farr (1989); see also the relevant ASME code, ASME (2004).

13.15.1. Fundamental equations

Thick walls are required to contain high pressures, and the assumptions made in the earlier sections of this chapter to develop the design equations for "thin-walled" vessels will not be valid. The radial stress will not be negligible and the tangential (hoop) stress will vary across the wall.

Consider the forces acting on the elemental section of the wall of the cylinder shown in Figure 13.43. The cylinder is under an internal pressure P_i and an external pressure P_e. The conditions for static equilibrium, with the forces resolved radially, give:

$$\sigma_r r\delta\phi - 2\sigma_t\delta r \sin\frac{\delta\phi}{2} - (\sigma_r + \delta\sigma_r)(r + \delta r)\delta\phi = 0$$

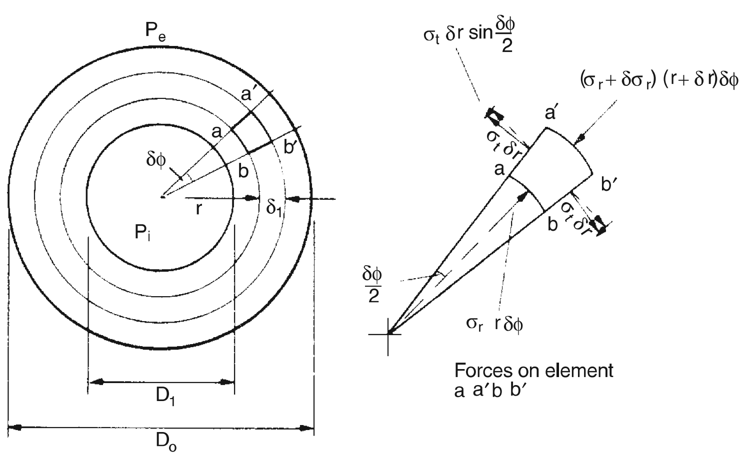


Figure 13.43. Thick cylinder

multiplying out taking the limit gives:

$$\sigma_t + r\frac{d\sigma_r}{dr} + \sigma_r = 0 \tag{13.115}$$

A second equation relating the radial and tangential stresses can be written if the longitudinal strain ε_L and stress σ_L are taken to be constant across the wall; that is, that there is no distortion of plane sections, which will be true for sections away from the ends. The longitudinal strain is given by:

$$\varepsilon_L = \frac{1}{E}[\sigma_L - (\sigma_t - \sigma_r)v] \tag{13.116}$$

If ε_L and σ_L are constant, then the term $(\sigma_t - \sigma_r)$ must also be constant, and can be written as:

$$(\sigma_t - \sigma_r) = 2A \tag{13.117}$$

where A is an arbitrary constant.

Substituting for σ_t in equation 13.115 gives:

$$2\sigma_r + r\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} = -2A$$

and integrating

$$\sigma_r = -A + \frac{B'}{r^2} \tag{13.118}$$

where B' is the constant of integration.

In terms of the cylinder diameter, the equations can be written as:

$$\sigma_r = -A + \frac{B}{d^2} \tag{13.119}$$

$$\sigma_t = A + \frac{B}{d^2} \tag{13.120}$$

These are the fundamental equations for the design of thick cylinders and are often referred to as Lamé's equations, as they were first derived by Lamé and Clapeyron (1833). The constants A and B are determined from the boundary conditions for the particular loading condition.

Most high-pressure process vessels will be under internal pressure only, the atmospheric pressure outside a vessel will be negligible compared with the internal pressure. The boundary conditions for this loading condition will be:

$$\sigma_r = P_i \text{ at } d = D_i$$

$$\sigma_r = 0 \text{ at } d = D_o$$

Substituting these values in equation 13.119 gives

$$P_i = -A + \frac{B}{D_i^2}$$

and
$$0 = -A + \frac{B}{D_o^2}$$

subtracting gives
$$P_i = B\left[\frac{1}{D_i^2} - \frac{1}{D_o^2}\right]$$

hence
$$B = P_i\frac{(D_i^2 D_o^2)}{(D_o^2 - D_i^2)}$$

and
$$A = P_i\frac{D_i^2}{(D_o^2 - D_i^2)}$$

Substituting in equations 13.119 and 13.120 gives:

$$\sigma_r = P_i\left[\frac{D_i^2(D_o^2 - d^2)}{d^2(D_o^2 - D_i^2)}\right] \qquad (13.121)$$

$$\sigma_t = P_i\left[\frac{D_i^2(D_o^2 + d^2)}{d^2(D_o^2 - D_i^2)}\right] \qquad (13.122)$$

The stress distribution across the vessel wall is shown plotted in Figure 13.44. The maximum values will occur at the inside surface, at $d = D_i$.

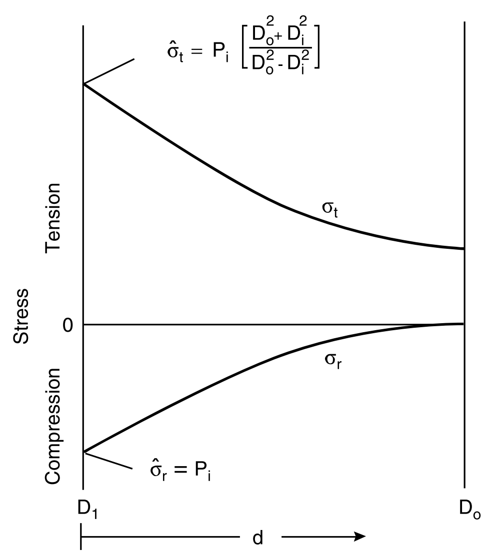


Figure 13.44. Stress distribution in wall of a monobloc cylinder

Putting $K = D_o/D_i$, the maximum values are given by:

$$\hat{\sigma}_r = P_i \text{ (compressive)} \qquad (13.123)$$