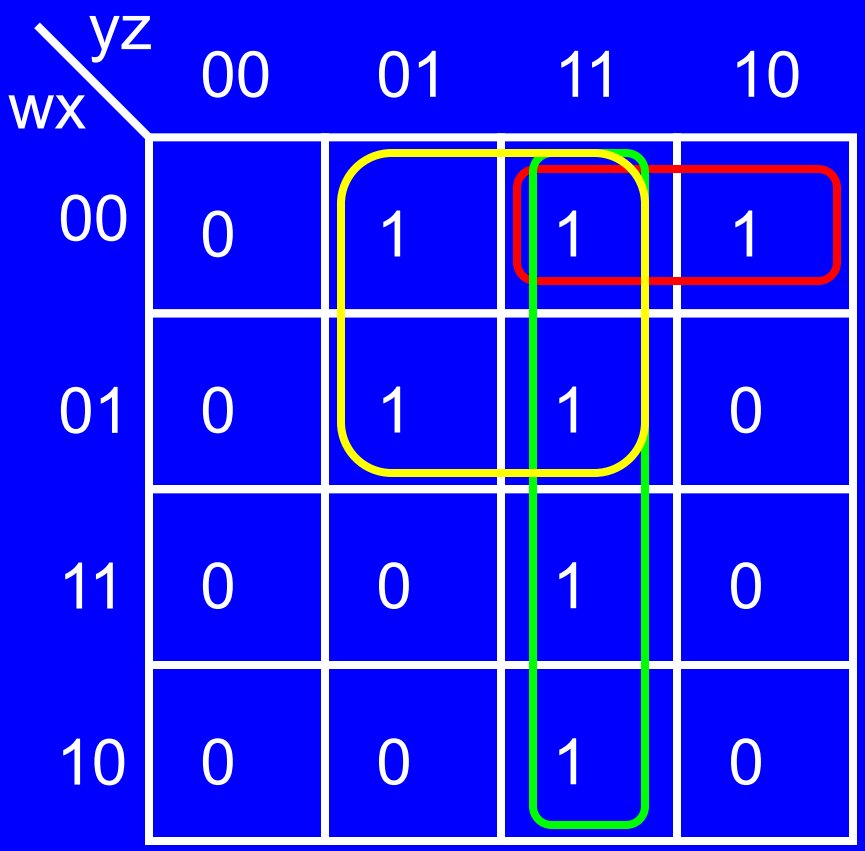
Some definitions

A function f covers a function g, if f assumes the value 1 when the same occurs with g.

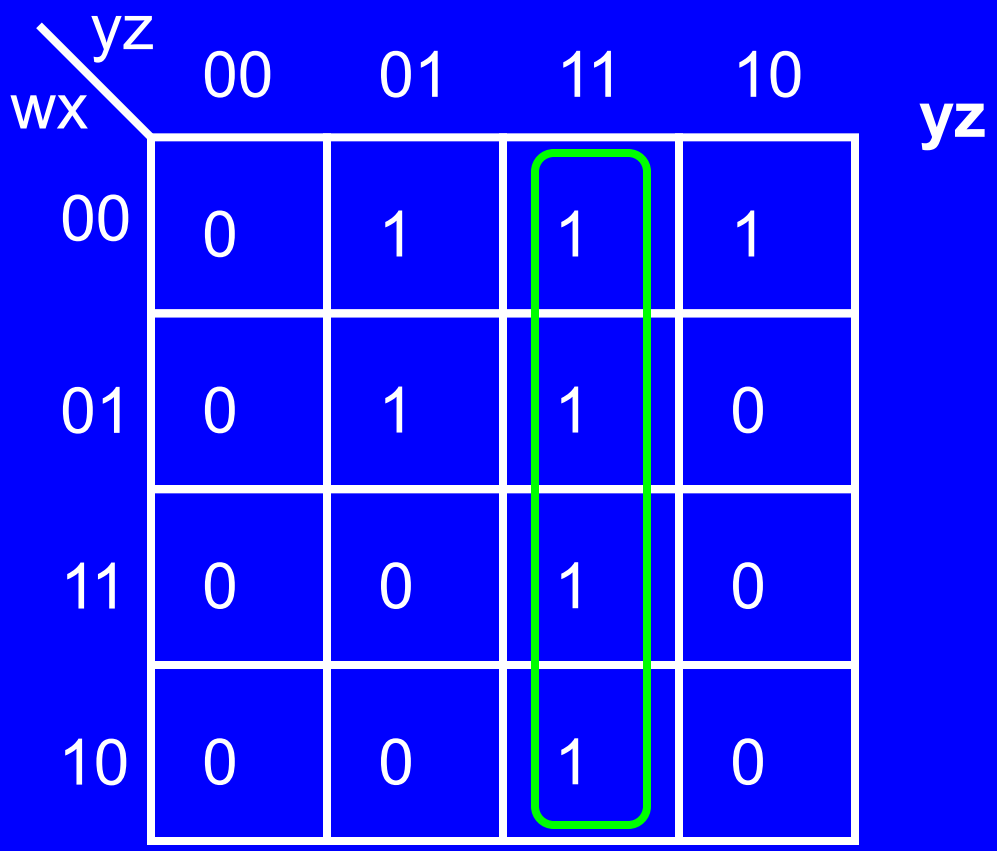
Prime implicant p of a function f is a product term which is covered by f and the deletion of any variable from p results in a function which is not covered by f.

Example

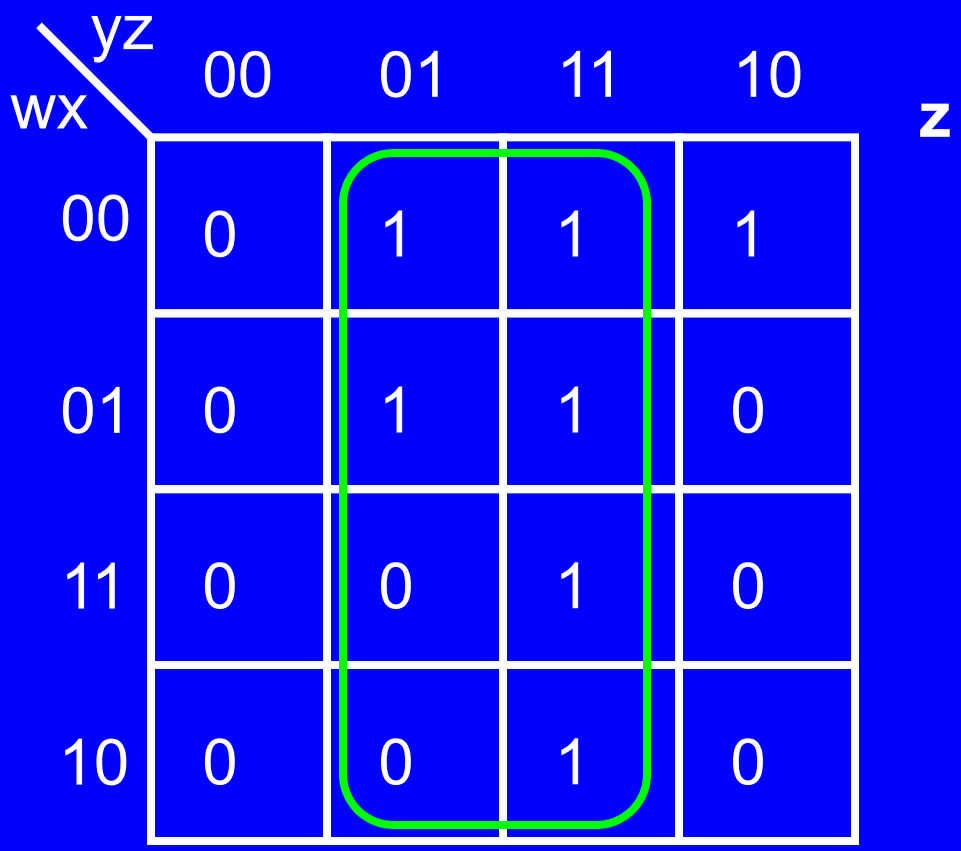


F=w'z+yz+w'x'y

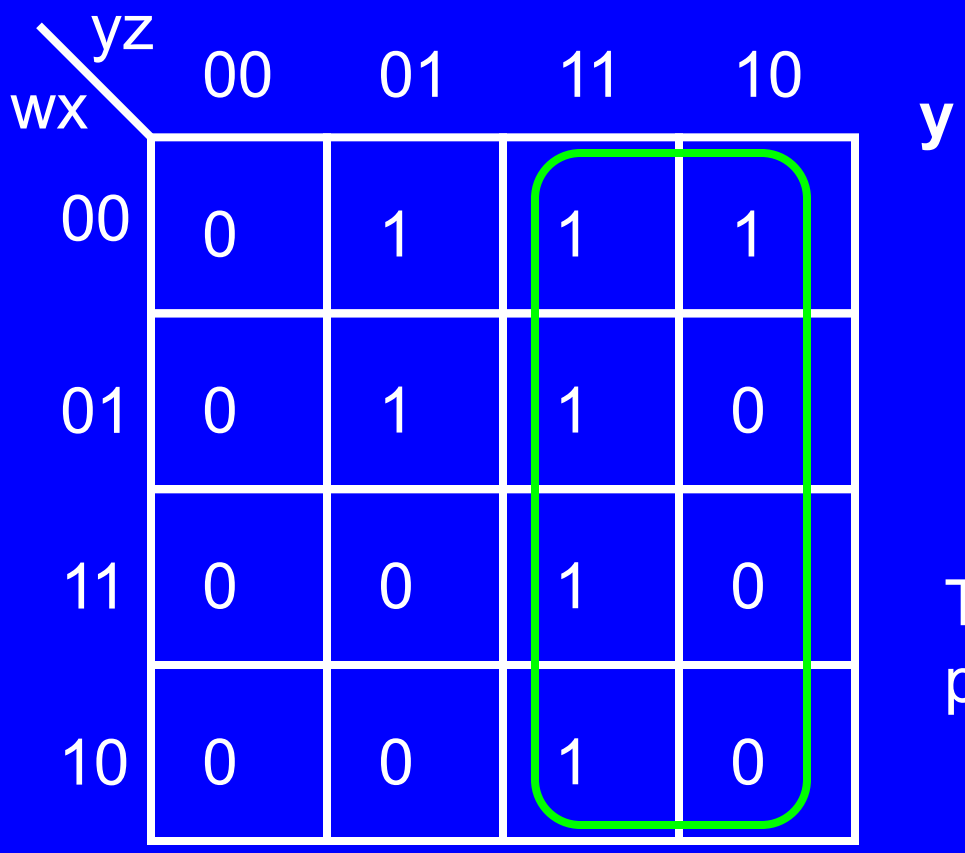
Four-variable Karnaugh map



Four-variable Karnaugh map

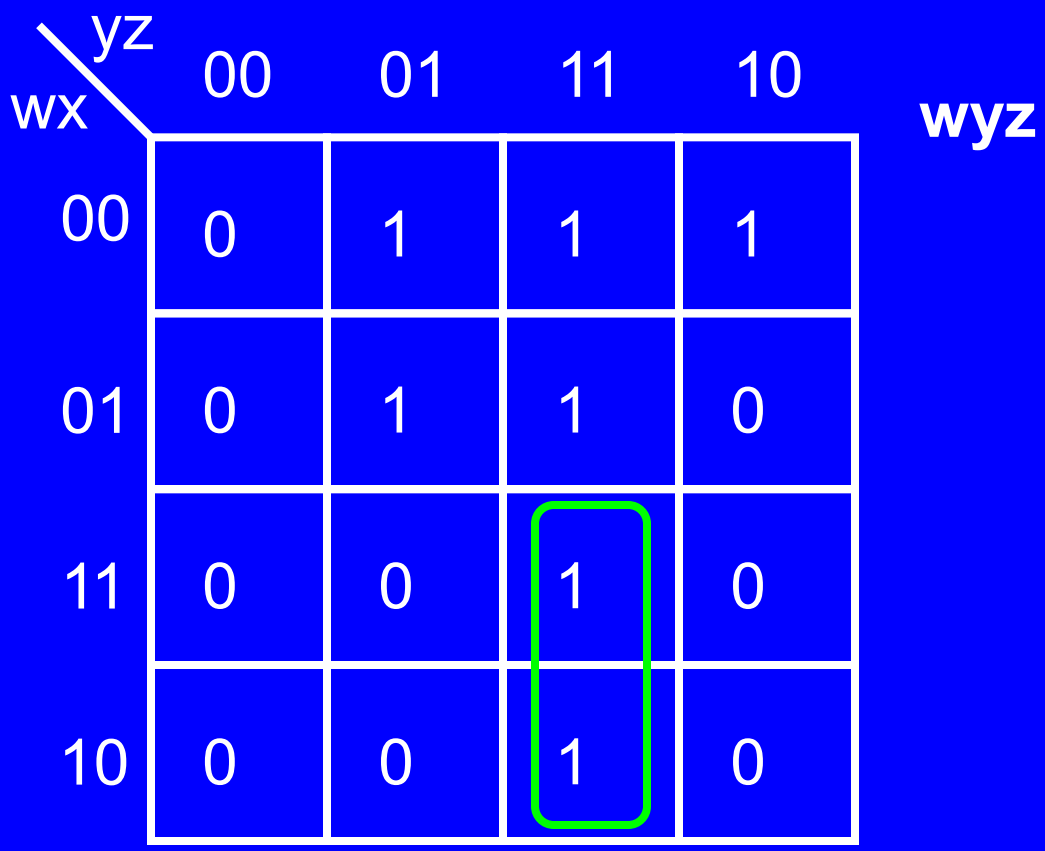


Four-variable Karnaugh map

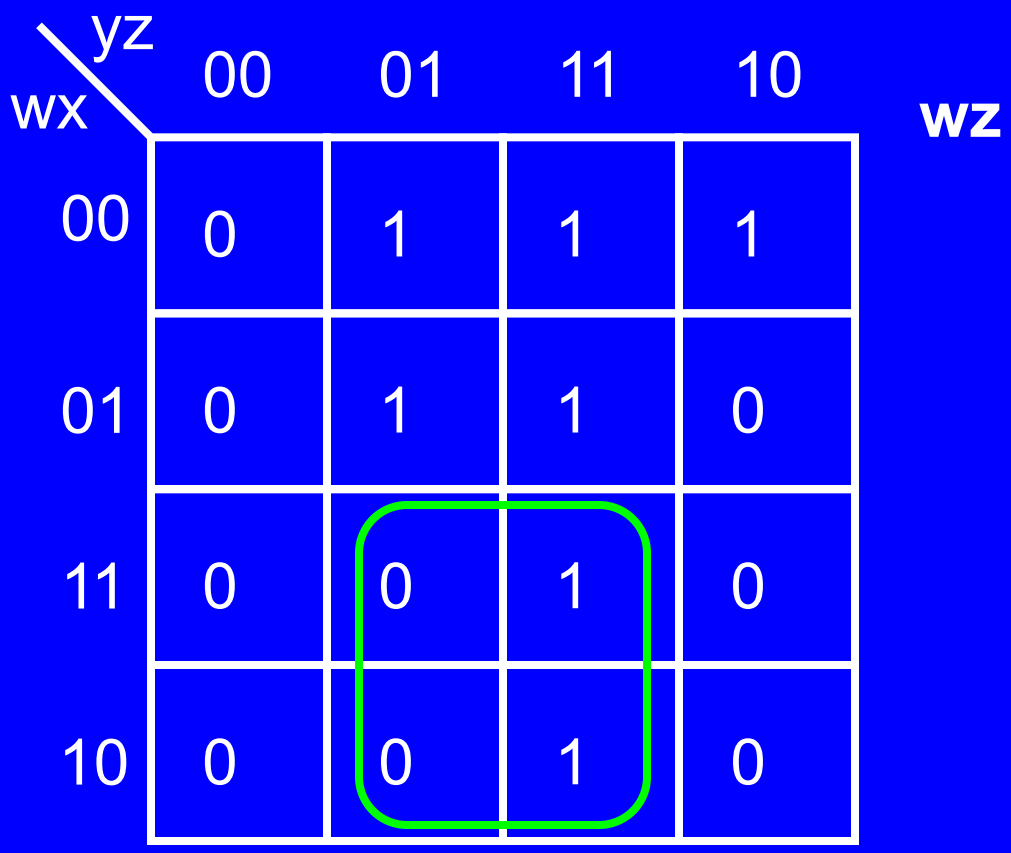


Therefore, yz is a prime implicant

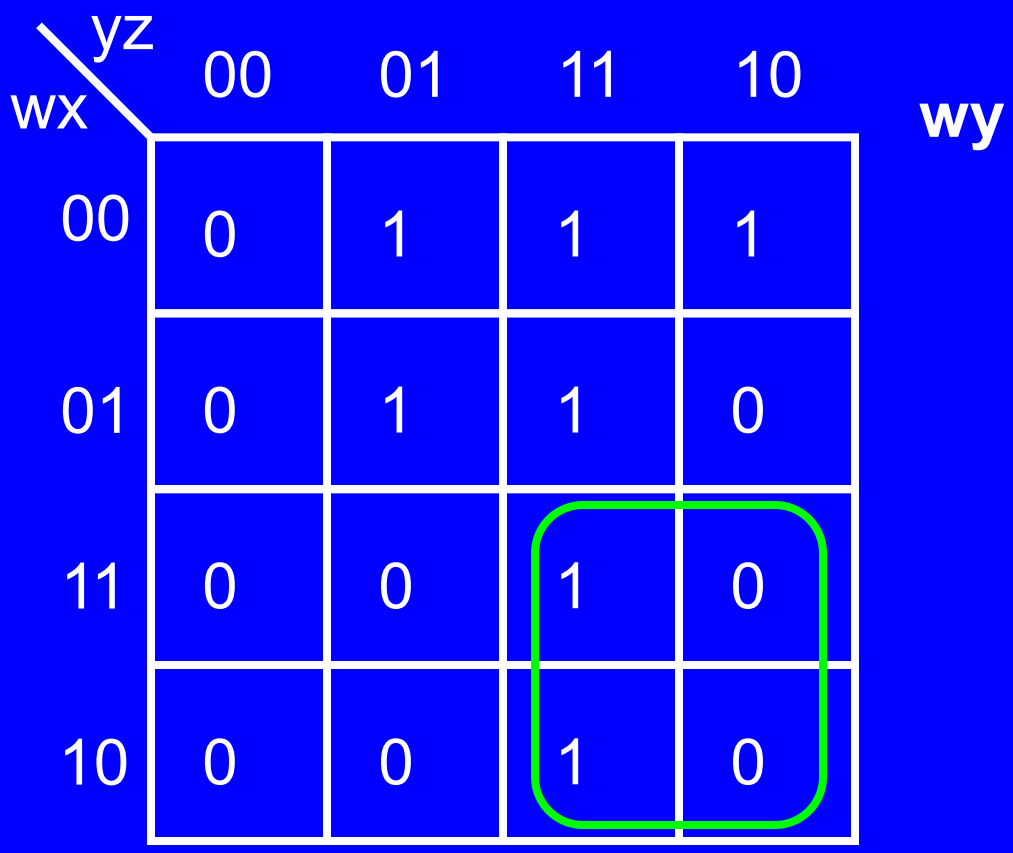
Four-variable Karnaugh map



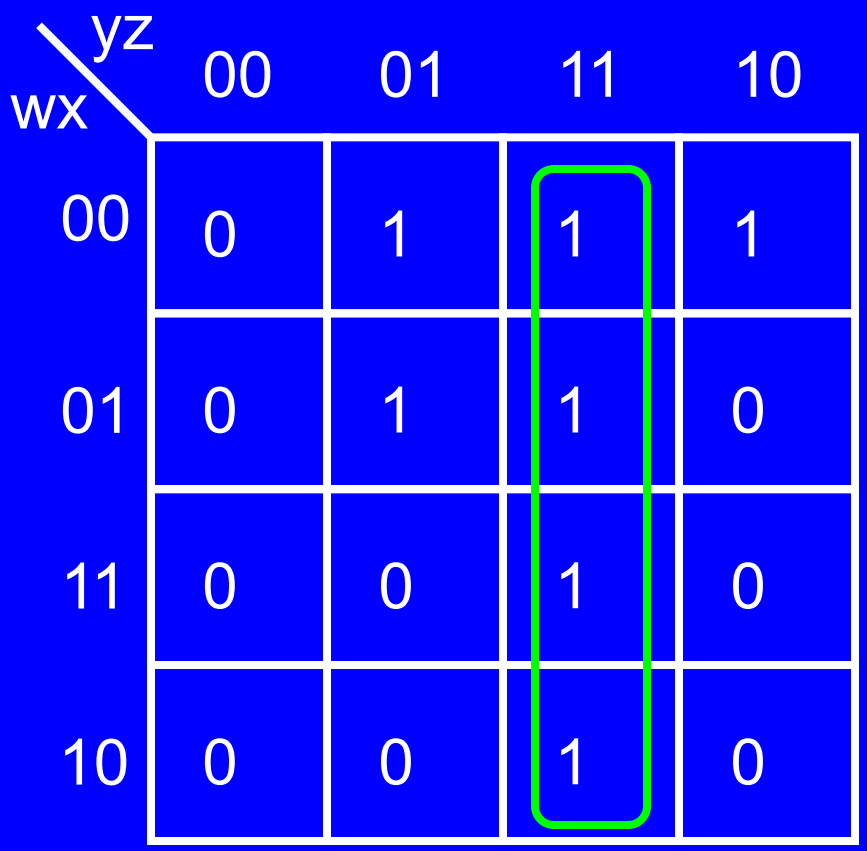
Four-variable Karnaugh map



Four-variable Karnaugh map



Four-variable Karnaugh map



Therefore, wyz is not a prime implicant

$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$

	w	x	y	z
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
8	1	0	0	0
5	0	1	0	1
9	1	0	0	1
10	1	0	1	0
7	0	1	1	1
13	1	1	0	1
15	1	1	1	1

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

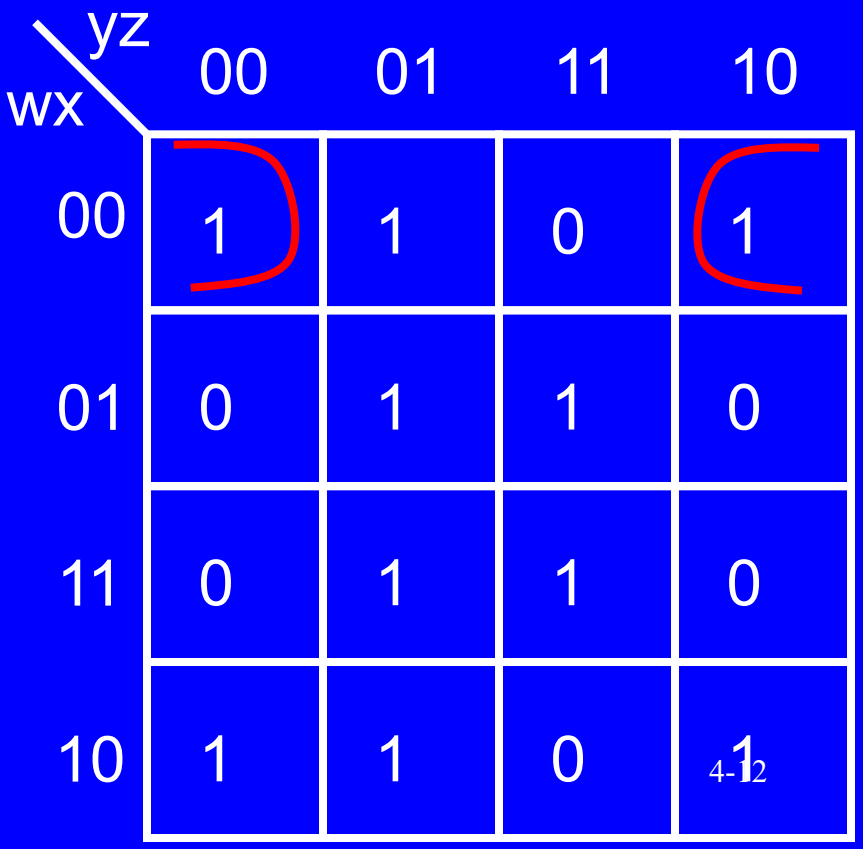
	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	
8	1 0 0 0	
5	0 1 0 1	
9	1 0 0 1	
10	1 0 1 0	
7	0 1 1 1	
13	1 1 0 1	
15	1 1 1 1	

	w x y z
0,1	0 0 0 -

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z			w x y z
0	0 0 0 0 ✓		0,1	0 0 0 -
1	0 0 0 1 ✓		0,2	0 0 - 0
2	0 0 1 0 ✓			
8	1 0 0 0			
5	0 1 0 1			
9	1 0 0 1			
10	1 0 1 0			
7	0 1 1 1			
13	1 1 0 1			
15	1 1 1 1			



$f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	
9	1 0 0 1	
10	1 0 1 0	
7	0 1 1 1	
13	1 1 0 1	
15	1 1 1 1	

	w x y z
0,1	0 0 0 -
0,2	0 0 - 0
0,8	- 0 0 0

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	
10	1 0 1 0	
7	0 1 1 1	
13	1 1 0 1	
15	1 1 1 1	

	w x y z
0,1	0 0 0 -
0,2	0 0 - 0
0,8	- 0 0 0
1,5	0 - 0 1

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	
7	0 1 1 1	
13	1 1 0 1	
15	1 1 1 1	

	w x y z
0,1	0 0 0 -
0,2	0 0 - 0
0,8	- 0 0 0
1,5	0 - 0 1
1,9	- 0 0 1

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	
13	1 1 0 1	
15	1 1 1 1	

	w x y z
0,1	0 0 0 -
0,2	0 0 - 0
0,8	- 0 0 0
1,5	0 - 0 1
1,9	- 0 0 1
2,10	- 0 1 0

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	
13	1 1 0 1	
15	1 1 1 1	

	w x y z
0,1	0 0 0 -
0,2	0 0 - 0
0,8	- 0 0 0
1,5	0 - 0 1
1,9	- 0 0 1
2,10	- 0 1 0
8,9	1 0 0 -

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	
13	1 1 0 1	
15	1 1 1 1	

	w x y z
0,1	0 0 0 -
0,2	0 0 - 0
0,8	- 0 0 0
1,5	0 - 0 1
1,9	- 0 0 1
2,10	- 0 1 0
8,9	1 0 0 -
8,10	1 0 - 0

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	
15	1 1 1 1	

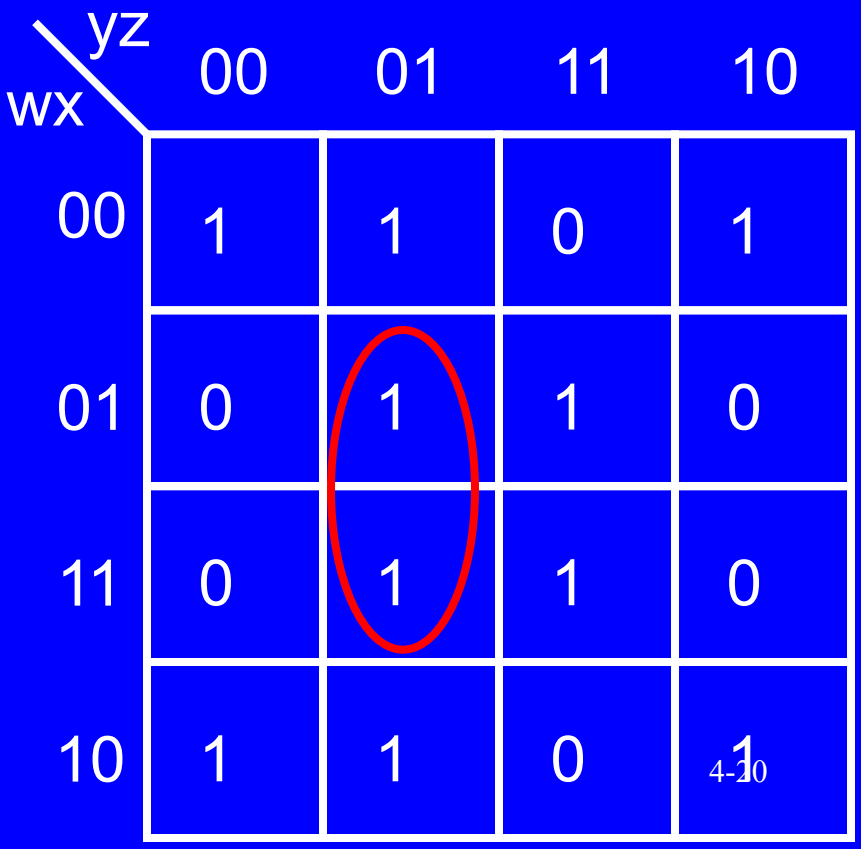
	w x y z
0,1	0 0 0 -
0,2	0 0 - 0
0,8	- 0 0 0
1,5	0 - 0 1
1,9	- 0 0 1
2,10	- 0 1 0
8,9	1 0 0 -
8,10	1 0 - 0
5,7	0 1 - 1

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	

	w x y z
0,1	0 0 0 -
0,2	0 0 - 0
0,8	- 0 0 0
1,5	0 - 0 1
1,9	- 0 0 1
2,10	- 0 1 0
8,9	1 0 0 -
8,10	1 0 - 0
5,7	0 1 - 1
5,13	- 1 0 1



f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	

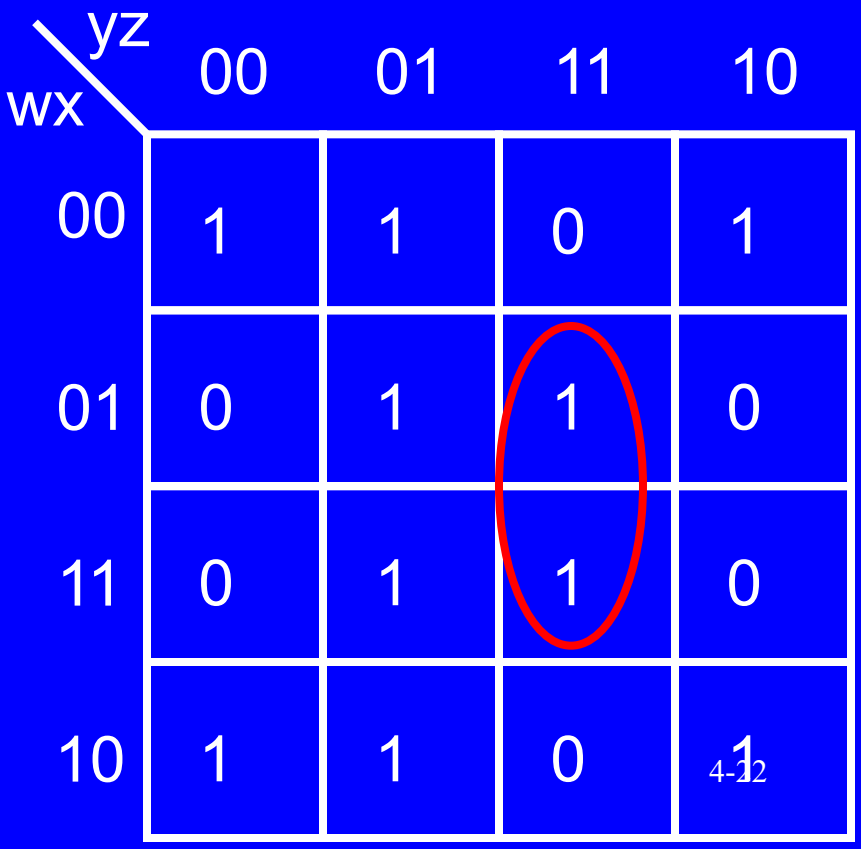
	w x y z
0,1	0 0 0 -
0,2	0 0 - 0
0,8	- 0 0 0
1,5	0 - 0 1
1,9	- 0 0 1
2,10	- 0 1 0
8,9	1 0 0 -
8,10	1 0 - 0
5,7	0 1 - 1
5,13	- 1 0 1
9,13	1 - 0 1

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

	w x y z
0,1	0 0 0 -
0,2	0 0 - 0
0,8	- 0 0 0
1,5	0 - 0 1
1,9	- 0 0 1
2,10	- 0 1 0
8,9	1 0 0 -
8,10	1 0 - 0
5,7	0 1 - 1
5,13	- 1 0 1
9,13	1 - 0 1
7,15	- 1 1 1



f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

	w x y z
0,1	0 0 0 -
0,2	0 0 - 0
0,8	- 0 0 0
1,5	0 - 0 1
1,9	- 0 0 1
2,10	- 0 1 0
8,9	1 0 0 -
8,10	1 0 - 0
5,7	0 1 - 1
5,13	- 1 0 1
9,13	1 - 0 1
7,15	- 1 1 1
13,15	1 1 - 1

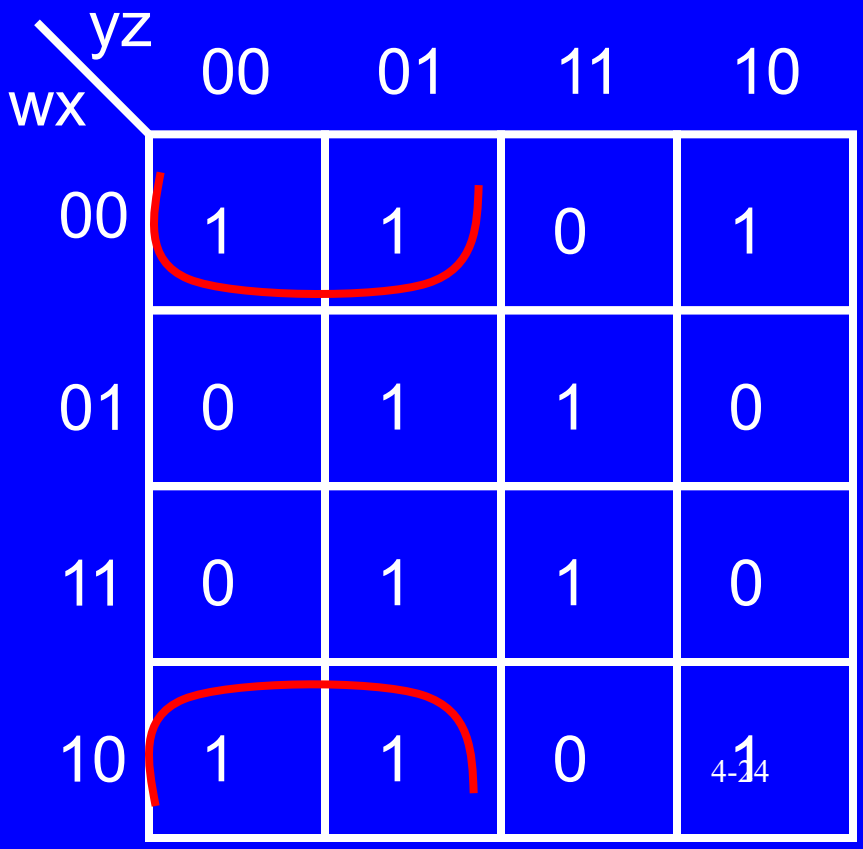
wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

	w x y z	
0,1	0 0 0 -	✓
0,2	0 0 - 0	
0,8	- 0 0 0	
1,5	0 - 0 1	
1,9	- 0 0 1	
2,10	- 0 1 0	
8,9	1 0 0 -	✓
8,10	1 0 - 0	
5,7	0 1 - 1	
5,13	- 1 0 1	
9,13	1 - 0 1	
7,15	- 1 1 1	
13,15	1 1 - 1	

	w x y z
0,1,8,9	- 0 0 -



f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

	w x y z	
0,1	0 0 0 -	✓
0,2	0 0 - 0	✓
0,8	- 0 0 0	
1,5	0 - 0 1	
1,9	- 0 0 1	
2,10	- 0 1 0	
8,9	1 0 0 -	✓
8,10	1 0 - 0	✓
5,7	0 1 - 1	
5,13	- 1 0 1	
9,13	1 - 0 1	
7,15	- 1 1 1	
13,15	1 1 - 1	

	w x y z
0,1,8,9	- 0 0 -
0,2,8,10	- 0 – 0

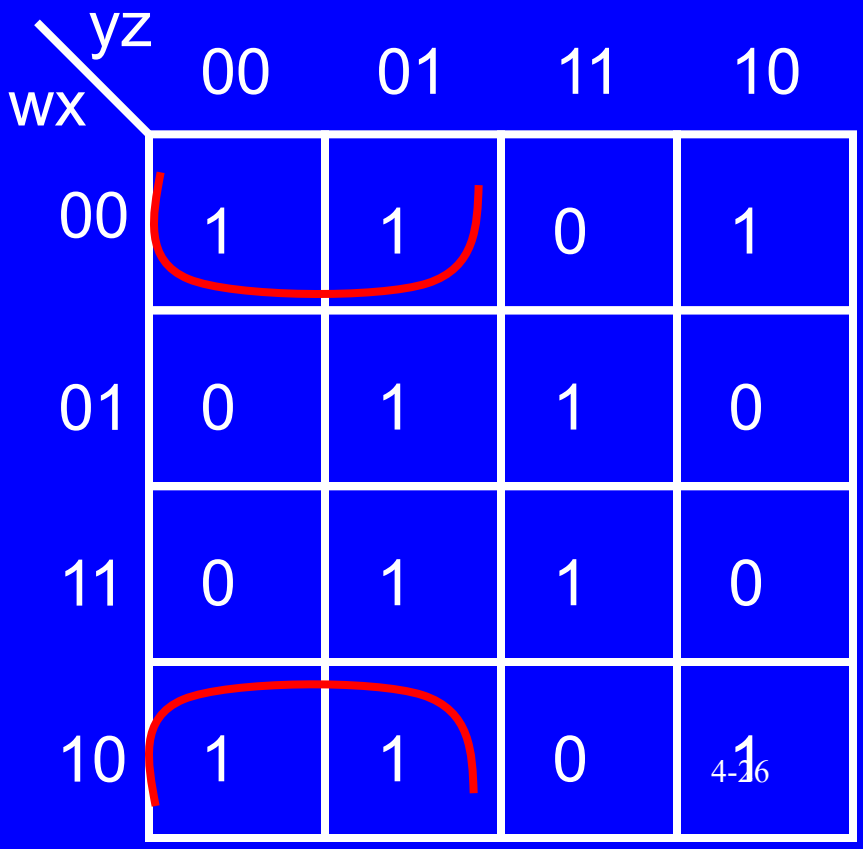
wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

	w x y z	
0,1	0 0 0 -	✓
0,2	0 0 - 0	✓
0,8	- 0 0 0	✓
1,5	0 - 0 1	
1,9	- 0 0 1	✓
2,10	- 0 1 0	
8,9	1 0 0 -	✓
8,10	1 0 - 0	✓
5,7	0 1 - 1	
5,13	- 1 0 1	
9,13	1 - 0 1	
7,15	- 1 1 1	
13,15	1 1 - 1	

	w x y z
0,1,8,9	- 0 0 -
0,2,8,10	- 0 – 0



f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

	w x y z	
0,1	0 0 0 -	✓
0,2	0 0 - 0	✓
0,8	- 0 0 0	✓
1,5	0 - 0 1	
1,9	- 0 0 1	✓
2,10	- 0 1 0	✓
8,9	1 0 0 -	✓
8,10	1 0 - 0	✓
5,7	0 1 - 1	
5,13	- 1 0 1	
9,13	1 - 0 1	
7,15	- 1 1 1	
13,15	1 1 - 1	

	w x y z
0,1,8,9	- 0 0 -
0,2,8,10	- 0 – 0

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

	w x y z	
0,1	0 0 0 -	✓
0,2	0 0 - 0	✓
0,8	- 0 0 0	✓
1,5	0 - 0 1	✓
1,9	- 0 0 1	✓
2,10	- 0 1 0	✓
8,9	1 0 0 -	✓
8,10	1 0 - 0	✓
5,7	0 1 - 1	
5,13	- 1 0 1	
9,13	1 - 0 1	✓
7,15	- 1 1 1	
13,15	1 1 - 1	

	w x y z
0,1,8,9	- 0 0 -
0,2,8,10	- 0 – 0
1,5,9,13	- - 0 1

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z
0	0 0 0 0 ✓
1	0 0 0 1 ✓
2	0 0 1 0 ✓
8	1 0 0 0 ✓
5	0 1 0 1 ✓
9	1 0 0 1 ✓
10	1 0 1 0 ✓
7	0 1 1 1 ✓
13	1 1 0 1 ✓
15	1 1 1 1 ✓

	w x y z
0,1	0 0 0 - ✓
0,2	0 0 - 0 ✓
0,8	- 0 0 0 ✓
1,5	0 - 0 1 ✓
1,9	- 0 0 1 ✓
2,10	- 0 1 0 ✓
8,9	1 0 0 - ✓
8,10	1 0 - 0 ✓
5,7	0 1 - 1
5,13	- 1 0 1 ✓
9,13	1 - 0 1 ✓
7,15	- 1 1 1
13,15	1 1 - 1

	w x y z
0,1,8,9	- 0 0 -
0,2,8,10	- 0 – 0
1,5,9,13	- - 0 1

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

	w x y z	
0,1	0 0 0 -	✓
0,2	0 0 - 0	✓
0,8	- 0 0 0	✓
1,5	0 - 0 1	✓
1,9	- 0 0 1	✓
2,10	- 0 1 0	✓
8,9	1 0 0 -	✓
8,10	1 0 - 0	✓
5,7	0 1 - 1	✓
5,13	- 1 0 1	✓
9,13	1 - 0 1	✓
7,15	- 1 1 1	
13,15	1 1 - 1	✓

	w x y z
0,1,8,9	- 0 0 -
0,2,8,10	- 0 – 0
1,5,9,13	- - 0 1
5,7,13,15	- 1 – 1

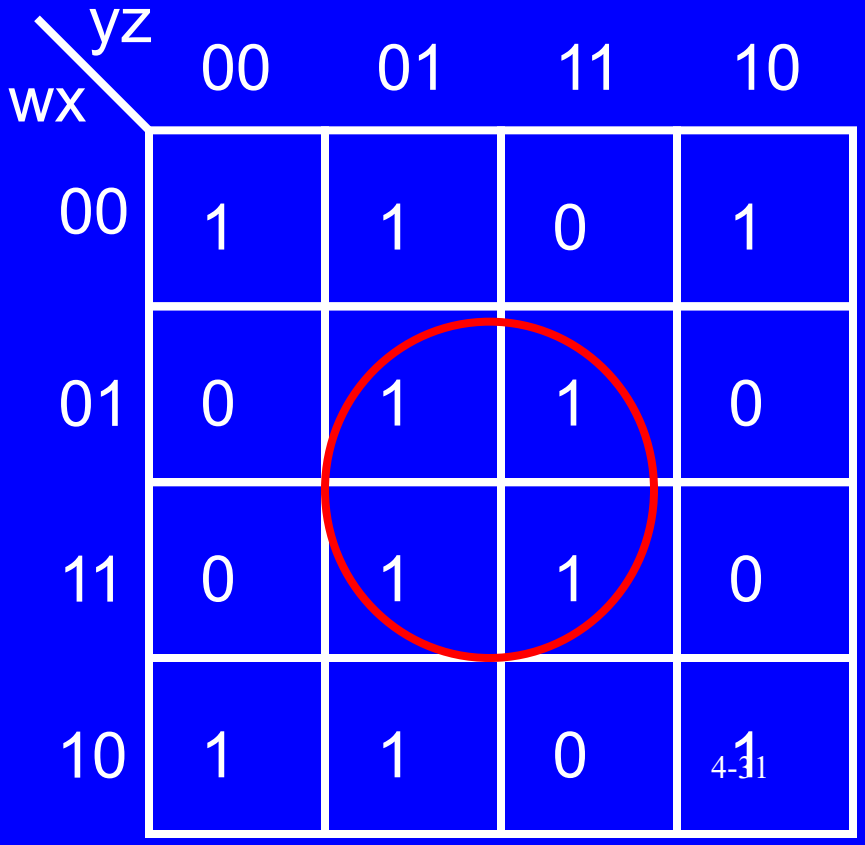
wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

	w x y z	
0,1	0 0 0 -	✓
0,2	0 0 - 0	✓
0,8	- 0 0 0	✓
1,5	0 - 0 1	✓
1,9	- 0 0 1	✓
2,10	- 0 1 0	✓
8,9	1 0 0 -	✓
8,10	1 0 - 0	✓
5,7	0 1 - 1	✓
5,13	- 1 0 1	✓
9,13	1 - 0 1	✓
7,15	- 1 1 1	✓
13,15	1 1 - 1	✓

	w x y z
0,1,8,9	- 0 0 -
0,2,8,10	- 0 – 0
1,5,9,13	- - 0 1
5,7,13,15	- 1 – 1



f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

	w x y z	
0,1	0 0 0 -	✓
0,2	0 0 - 0	✓
0,8	- 0 0 0	✓
1,5	0 - 0 1	✓
1,9	- 0 0 1	✓
2,10	- 0 1 0	✓
8,9	1 0 0 -	✓
8,10	1 0 - 0	✓
5,7	0 1 - 1	✓
5,13	- 1 0 1	✓
9,13	1 - 0 1	✓
7,15	- 1 1 1	✓
13,15	1 1 - 1	✓

	w x y z
0,1,8,9	- 0 0 -
0,2,8,10	- 0 – 0
1,5,9,13	- - 0 1
5,7,13,15	- 1 – 1

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

	w x y z	
0,1	0 0 0 -	✓
0,2	0 0 - 0	✓
0,8	- 0 0 0	✓
1,5	0 - 0 1	✓
1,9	- 0 0 1	✓
2,10	- 0 1 0	✓
8,9	1 0 0 -	✓
8,10	1 0 - 0	✓
5,7	0 1 - 1	✓
5,13	- 1 0 1	✓
9,13	1 - 0 1	✓
7,15	- 1 1 1	✓
13,15	1 1 - 1	✓

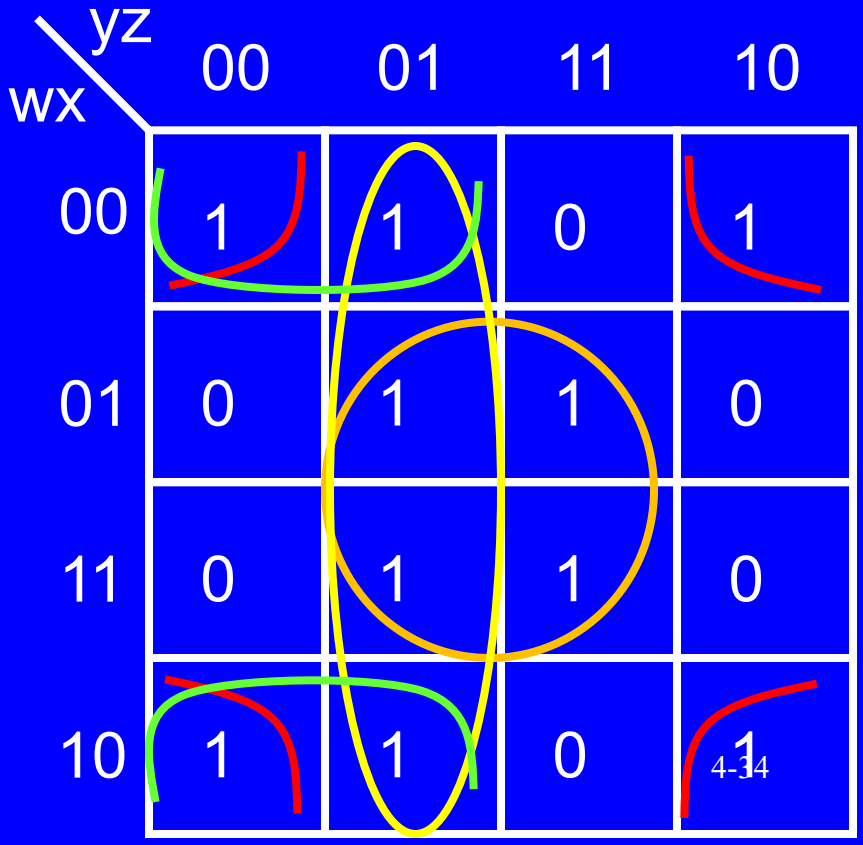
	w x y z	
0,1,8,9	- 0 0 -	**A = x'y'**
0,2,8,10	- 0 - 0	**B = x'z'**
1,5,9,13	- - 0 1	**C = y'z**
5,7,13,15	- 1 - 1	**D = xz**

f(w,x,y,z) = Σ(0,1,2,5,7,8,9,10,13,15)

	w x y z	
0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
9	1 0 0 1	✓
10	1 0 1 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

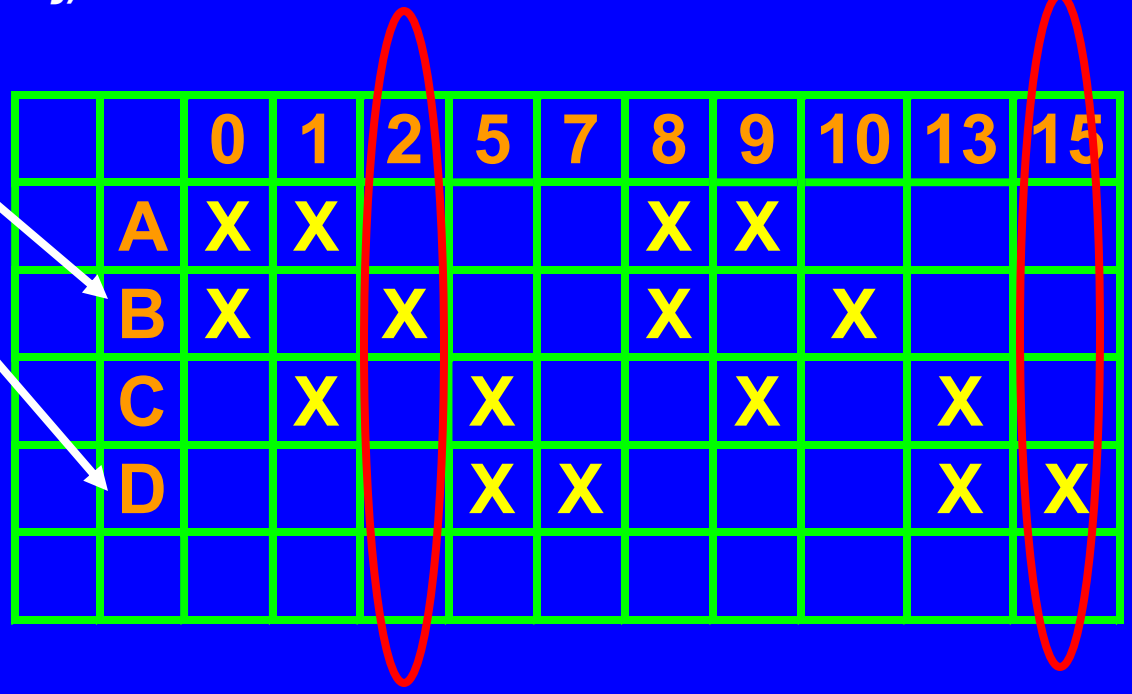
	w x y z	
0,1	0 0 0 -	✓
0,2	0 0 - 0	✓
0,8	- 0 0 0	✓
1,5	0 - 0 1	✓
1,9	- 0 0 1	✓
2,10	- 0 1 0	✓
8,9	1 0 0 -	✓
8,10	1 0 - 0	✓
5,7	0 1 - 1	✓
5,13	- 1 0 1	✓
9,13	1 - 0 1	✓
7,15	- 1 1 1	✓
13,15	1 1 - 1	✓

	w x y z	
0,1,8,9	- 0 0 -	A = x'y'
0,2,8,10	- 0 - 0	B = x'z'
1,5,9,13	- - 0 1	C = y'z
5,7,13,15	- 1 - 1	D = xz



Prime implicant selection

Essential (ουσιώδεις)



		0	1	2	5	7	8	9	10	13	15
	A	X	X				X	X			
	B	X		X			X		X		
	C		X		X			X		X	
	D				X	X				X	X

Prime implicant selection

		0	1	2	5	7	8	9	10	13	15
	A	X	X				X	X			
*	B	X		X			X		X		
	C		X		X			X		X	
*	D				X	X				X	X
		X		X			X		X		

Prime implicant selection

		0	1	2	5	7	8	9	10	13	15
	A	X	X				X	X			
*	B	X		X			X		X		
	C		X		X			X		X	
*	D				X	X				X	X
		X		X	X	X	X		X	X	X

Prime implicant selection

		0	1	2	5	7	8	9	10	13	15
√	A	X	X				X	X			
*	B	X		X			X		X		
	C		X		X			X		X	
*	D				X	X				X	X
		X	X	X	X	X	X	X	X	X	X

Prime implicant selection

		0	1	2	5	7	8	9	10	13	15
√	A	X	X				X	X			
*	B	X		X			X		X		
	C		X		X			X		X	
*	D				X	X				X	X
		X	X	X	X	X	X	X	X	X	X

F = xz + x'z' + x'y'

Prime implicant selection

		0	1	2	5	7	8	9	10	13	15
	A	X	X				X	X			
*	B	X		X			X		X		
√	C		X		X			X		X	
*	D				X	X				X	X
		X	X	X	X	X	X	X	X	X	X

F = xz + x'z' + y'z

Prime implicant selection

F = xz + x'z' + x'y'

F = xz + x'z' + y'z

Prime implicant selection

F = xz + x'z' + x'y'

F = xz + x'z' + y'z

F = Σ(0,1,2,5,7,8,9,10,13,15)

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1
2

12
17
18
20
24

13
19
21
25

15
23
27
29

31

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2		
12		
17	✓	
18		
20		
24		
13		
19		
21		
25		
15		
23		
27		
29		
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12		
17	✓	
18	✓	
20		
24		
13		
19		
21		
25		
15		
23		
27		
29		
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	
18	✓	
20		
24		
13	✓	
19		
21		
25		
15		
23		
27		
29		
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	
20		
24		
13	✓	
19	✓	
21		
25		
15		
23		
27		
29		
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20		
24		
13	✓	
19	✓	
21	✓	
25		
15		
23		
27		
29		
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20		17,25 (8)
24		
13	✓	
19	✓	
21	✓	
25	✓	
15		
23		
27		
29		
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20		17,25 (8)
24		18,19 (1)
13	✓	
19	✓	
21	✓	
25	✓	
15		
23		
27		
29		
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
24		18,19 (1)
13	✓	20,21 (1)
19	✓	
21	✓	
25	✓	
15		
23		
27		
29		
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
24	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	24,25 (1)
21	✓	
25	✓	
15		
23		
27		
29		
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
24	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	24,25 (1)
21	✓	13,15 (2)
25	✓	
15	✓	
23		
27		
29		
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
24	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	24,25 (1)
21	✓	13,15 (2)
25	✓	13,29 (16)
15	✓	
23		
27		
29	✓	
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
24	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	24,25 (1)
21	✓	13,15 (2)
25	✓	13,29 (16)
15	✓	19,23 (4)
23	✓	
27		
29	✓	
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
24	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	24,25 (1)
21	✓	13,15 (2)
25	✓	13,29 (16)
15	✓	19,23 (4)
23	✓	19,27 (8)
27	✓	
29	✓	
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
24	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	24,25 (1)
21	✓	13,15 (2)
25	✓	13,29 (16)
15	✓	19,23 (4)
23	✓	19,27 (8)
27	✓	21,23 (2)
29	✓	
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
24	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	24,25 (1)
21	✓	13,15 (2)
25	✓	13,29 (16)
15	✓	19,23 (4)
23	✓	19,27 (8)
27	✓	21,23 (2)
29	✓	21,29 (8)
31		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)
2	✓	2,18 (16)
12	✓	12,13 (1)
17	✓	17,19 (2)
18	✓	17,21 (4)
20	✓	17,25 (8)
24	✓	18,19 (1)
13	✓	20,21 (1)
19	✓	24,25 (1)
21	✓	13,15 (2)
25	✓	13,29 (16)
15	✓	19,23 (4)
23	✓	19,27 (8)
27	✓	21,23 (2)
29	✓	21,29 (8)
31		25,27 (2)

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	25,29 (4)
2	✓	2,18 (16)	
12	✓	12,13 (1)	
17	✓	17,19 (2)	
18	✓	17,21 (4)	
20	✓	17,25 (8)	
24	✓	18,19 (1)	
13	✓	20,21 (1)	
19	✓	24,25 (1)	
21	✓	13,15 (2)	
25	✓	13,29 (16)	
15	✓	19,23 (4)	
23	✓	19,27 (8)	
27	✓	21,23 (2)	
29	✓	21,29 (8)	
31		25,27 (2)	

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	25,29 (4)
2	✓	2,18 (16)	15,31 (16)
12	✓	12,13 (1)	
17	✓	17,19 (2)	
18	✓	17,21 (4)	
20	✓	17,25 (8)	
24	✓	18,19 (1)	
13	✓	20,21 (1)	
19	✓	24,25 (1)	
21	✓	13,15 (2)	
25	✓	13,29 (16)	
15	✓	19,23 (4)	
23	✓	19,27 (8)	
27	✓	21,23 (2)	
29	✓	21,29 (8)	
31	✓	25,27 (2)	

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	25,29 (4)
2	✓	2,18 (16)	15,31 (16)
12	✓	12,13 (1)	23,31 (8)
17	✓	17,19 (2)	
18	✓	17,21 (4)	
20	✓	17,25 (8)	
24	✓	18,19 (1)	
13	✓	20,21 (1)	
19	✓	24,25 (1)	
21	✓	13,15 (2)	
25	✓	13,29 (16)	
15	✓	19,23 (4)	
23	✓	19,27 (8)	
27	✓	21,23 (2)	
29	✓	21,29 (8)	
31	✓	25,27 (2)	

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	25,29 (4)
2	✓	2,18 (16)	15,31 (16)
12	✓	12,13 (1)	23,31 (8)
17	✓	17,19 (2)	27,31 (4)
18	✓	17,21 (4)	
20	✓	17,25 (8)	
24	✓	18,19 (1)	
13	✓	20,21 (1)	
19	✓	24,25 (1)	
21	✓	13,15 (2)	
25	✓	13,29 (16)	
15	✓	19,23 (4)	
23	✓	19,27 (8)	
27	✓	21,23 (2)	
29	✓	21,29 (8)	
31	✓	25,27 (2)	

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	25,29 (4)
2	✓	2,18 (16)	15,31 (16)
12	✓	12,13 (1)	23,31 (8)
17	✓	17,19 (2)	27,31 (4)
18	✓	17,21 (4)	29,31 (2)
20	✓	17,25 (8)	
24	✓	18,19 (1)	
13	✓	20,21 (1)	
19	✓	24,25 (1)	
21	✓	13,15 (2)	
25	✓	13,29 (16)	
15	✓	19,23 (4)	
23	✓	19,27 (8)	
27	✓	21,23 (2)	
29	✓	21,29 (8)	
31	✓	25,27 (2)	

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4)
2	✓	2,18 (16)		15,31 (16)
12	✓	12,13 (1)		23,31 (8)
17	✓	17,19 (2)	✓	27,31 (4)
18	✓	17,21 (4)		29,31 (2)
20	✓	17,25 (8)		
24	✓	18,19 (1)		17,19,21,23 (2,4)
13	✓	20,21 (1)		
19	✓	24,25 (1)		
21	✓	13,15 (2)		
25	✓	13,29 (16)		
15	✓	19,23 (4)		
23	✓	19,27 (8)		
27	✓	21,23 (2)	✓	
29	✓	21,29 (8)		
31	✓	25,27 (2)		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4)
2	✓	2,18 (16)		15,31 (16)
12	✓	12,13 (1)		23,31 (8)
17	✓	17,19 (2)	✓	27,31 (4)
18	✓	17,21 (4)		29,31 (2)
20	✓	17,25 (8)		
24	✓	18,19 (1)		17,19,21,23 (2,4)
13	✓	20,21 (1)		17,19,25,27 (2,8)
19	✓	24,25 (1)		
21	✓	13,15 (2)		
25	✓	13,29 (16)		
15	✓	19,23 (4)		
23	✓	19,27 (8)		
27	✓	21,23 (2)	✓	
29	✓	21,29 (8)		
31	✓	25,27 (2)	✓	

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4)
2	✓	2,18 (16)		15,31 (16)
12	✓	12,13 (1)		23,31 (8)
17	✓	17,19 (2)	✓	27,31 (4)
18	✓	17,21 (4)	✓	29,31 (2)
20	✓	17,25 (8)		
24	✓	18,19 (1)		17,19,21,23 (2,4)
13	✓	20,21 (1)		17,19,25,27 (2,8)
19	✓	24,25 (1)		
21	✓	13,15 (2)		
25	✓	13,29 (16)		
15	✓	19,23 (4)	✓	
23	✓	19,27 (8)		
27	✓	21,23 (2)	✓	
29	✓	21,29 (8)		
31	✓	25,27 (2)	✓	

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4) ✓
2	✓	2,18 (16)		15,31 (16)
12	✓	12,13 (1)		23,31 (8)
17	✓	17,19 (2)	✓	27,31 (4)
18	✓	17,21 (4)	✓	29,31 (2)
20	✓	17,25 (8)		
24	✓	18,19 (1)		17,19,21,23 (2,4)
13	✓	20,21 (1)		17,19,25,27 (2,8)
19	✓	24,25 (1)		17,21,25,29 (4,8)
21	✓	13,15 (2)		
25	✓	13,29 (16)		
15	✓	19,23 (4)	✓	
23	✓	19,27 (8)		
27	✓	21,23 (2)	✓	
29	✓	21,29 (8)		
31	✓	25,27 (2)	✓	

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4) ✓
2	✓	2,18 (16)		15,31 (16)
12	✓	12,13 (1)		23,31 (8)
17	✓	17,19 (2)	✓	27,31 (4)
18	✓	17,21 (4)	✓	29,31 (2)
20	✓	17,25 (8)	✓	
24	✓	18,19 (1)		17,19,21,23 (2,4)
13	✓	20,21 (1)		17,19,25,27 (2,8)
19	✓	24,25 (1)		17,21,25,29 (4,8)
21	✓	13,15 (2)		
25	✓	13,29 (16)		
15	✓	19,23 (4)	✓	
23	✓	19,27 (8)	✓	
27	✓	21,23 (2)	✓	
29	✓	21,29 (8)		
31	✓	25,27 (2)	✓	

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4) ✓
2	✓	2,18 (16)		15,31 (16)
12	✓	12,13 (1)		23,31 (8)
17	✓	17,19 (2)	✓	27,31 (4)
18	✓	17,21 (4)	✓	29,31 (2)
20	✓	17,25 (8)	✓	
24	✓	18,19 (1)		17,19,21,23 (2,4)
13	✓	20,21 (1)		17,19,25,27 (2,8)
19	✓	24,25 (1)		17,21,25,29 (4,8)
21	✓	13,15 (2)		
25	✓	13,29 (16)		
15	✓	19,23 (4)	✓	
23	✓	19,27 (8)	✓	
27	✓	21,23 (2)	✓	
29	✓	21,29 (8)	✓	
31	✓	25,27 (2)	✓	

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4)	✓
2	✓	2,18 (16)		15,31 (16)	
12	✓	12,13 (1)		23,31 (8)	
17	✓	17,19 (2)	✓	27,31 (4)	
18	✓	17,21 (4)	✓	29,31 (2)	✓
20	✓	17,25 (8)	✓		
24	✓	18,19 (1)		17,19,21,23 (2,4)	
13	✓	20,21 (1)		17,19,25,27 (2,8)	
19	✓	24,25 (1)		17,21,25,29 (4,8)	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	
25	✓	13,29 (16)			
15	✓	19,23 (4)	✓		
23	✓	19,27 (8)	✓		
27	✓	21,23 (2)	✓		
29	✓	21,29 (8)	✓		
31	✓	25,27 (2)	✓		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4)	✓
2	✓	2,18 (16)		15,31 (16)	✓
12	✓	12,13 (1)		23,31 (8)	
17	✓	17,19 (2)	✓	27,31 (4)	
18	✓	17,21 (4)	✓	29,31 (2)	✓
20	✓	17,25 (8)	✓		
24	✓	18,19 (1)		17,19,21,23 (2,4)	
13	✓	20,21 (1)		17,19,25,27 (2,8)	
19	✓	24,25 (1)		17,21,25,29 (4,8)	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	
25	✓	13,29 (16)	✓		
15	✓	19,23 (4)	✓		
23	✓	19,27 (8)	✓		
27	✓	21,23 (2)	✓		
29	✓	21,29 (8)	✓		
31	✓	25,27 (2)	✓		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4)	✓
2	✓	2,18 (16)		15,31 (16)	✓
12	✓	12,13 (1)		23,31 (8)	
17	✓	17,19 (2)	✓	27,31 (4)	✓
18	✓	17,21 (4)	✓	29,31 (2)	✓
20	✓	17,25 (8)	✓		
24	✓	18,19 (1)		17,19,21,23 (2,4)	
13	✓	20,21 (1)		17,19,25,27 (2,8)	
19	✓	24,25 (1)		17,21,25,29 (4,8)	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	
15	✓	19,23 (4)	✓		
23	✓	19,27 (8)	✓		
27	✓	21,23 (2)	✓		
29	✓	21,29 (8)	✓		
31	✓	25,27 (2)	✓		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4)	✓
2	✓	2,18 (16)		15,31 (16)	✓
12	✓	12,13 (1)		23,31 (8)	✓
17	✓	17,19 (2)	✓	27,31 (4)	✓
18	✓	17,21 (4)	✓	29,31 (2)	✓
20	✓	17,25 (8)	✓		
24	✓	18,19 (1)		17,19,21,23 (2,4)	
13	✓	20,21 (1)		17,19,25,27 (2,8)	
19	✓	24,25 (1)		17,21,25,29 (4,8)	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	
15	✓	19,23 (4)	✓		
23	✓	19,27 (8)	✓		
27	✓	21,23 (2)	✓		
29	✓	21,29 (8)	✓		
31	✓	25,27 (2)	✓		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4)	✓
2	✓	2,18 (16)		15,31 (16)	✓
12	✓	12,13 (1)		23,31 (8)	✓
17	✓	17,19 (2)	✓	27,31 (4)	✓
18	✓	17,21 (4)	✓	29,31 (2)	✓
20	✓	17,25 (8)	✓		
24	✓	18,19 (1)		17,19,21,23 (2,4)	
13	✓	20,21 (1)		17,19,25,27 (2,8)	
19	✓	24,25 (1)		17,21,25,29 (4,8)	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	
23	✓	19,27 (8)	✓		
27	✓	21,23 (2)	✓		
29	✓	21,29 (8)	✓		
31	✓	25,27 (2)	✓		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4)	✓
2	✓	2,18 (16)		15,31 (16)	✓
12	✓	12,13 (1)		23,31 (8)	✓
17	✓	17,19 (2)	✓	27,31 (4)	✓
18	✓	17,21 (4)	✓	29,31 (2)	✓
20	✓	17,25 (8)	✓		
24	✓	18,19 (1)		17,19,21,23 (2,4)	
13	✓	20,21 (1)		17,19,25,27 (2,8)	
19	✓	24,25 (1)		17,21,25,29 (4,8)	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	
27	✓	21,23 (2)	✓		
29	✓	21,29 (8)	✓		
31	✓	25,27 (2)	✓		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4)	✓
2	✓	2,18 (16)		15,31 (16)	✓
12	✓	12,13 (1)		23,31 (8)	✓
17	✓	17,19 (2)	✓	27,31 (4)	✓
18	✓	17,21 (4)	✓	29,31 (2)	✓
20	✓	17,25 (8)	✓		
24	✓	18,19 (1)		17,19,21,23 (2,4)	✓
13	✓	20,21 (1)		17,19,25,27 (2,8)	
19	✓	24,25 (1)		17,21,25,29 (4,8)	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓
27	✓	21,23 (2)	✓		
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	
31	✓	25,27 (2)	✓		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4)	✓
2	✓	2,18 (16)		15,31 (16)	✓
12	✓	12,13 (1)		23,31 (8)	✓
17	✓	17,19 (2)	✓	27,31 (4)	✓
18	✓	17,21 (4)	✓	29,31 (2)	✓
20	✓	17,25 (8)	✓		
24	✓	18,19 (1)		17,19,21,23 (2,4)	✓
13	✓	20,21 (1)		17,19,25,27 (2,8)	✓
19	✓	24,25 (1)		17,21,25,29 (4,8)	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓
27	✓	21,23 (2)	✓		
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	
31	✓	25,27 (2)	✓		

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)		25,29 (4)	✓	
2	✓	2,18 (16)		15,31 (16)	✓	
12	✓	12,13 (1)		23,31 (8)	✓	
17	✓	17,19 (2)	✓	27,31 (4)	✓	
18	✓	17,21 (4)	✓	29,31 (2)	✓	
20	✓	17,25 (8)	✓			
24	✓	18,19 (1)		17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)		17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)		17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)		
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)		
31	✓	25,27 (2)	✓			

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1 ✓	1,17 (16)	H	25,29 (4) ✓			
2 ✓	2,18 (16)	G	15,31 (16) ✓			
12 ✓	12,13 (1)	F	23,31 (8) ✓			
17 ✓	17,19 (2)	✓	27,31 (4) ✓			
18 ✓	17,21 (4)	✓	29,31 (2) ✓			
20 ✓	17,25 (8)	✓				
24 ✓	18,19 (1)	E	17,19,21,23 (2,4)	✓		
13 ✓	20,21 (1)	D	17,19,25,27 (2,8)	✓		
19 ✓	24,25 (1)	C	17,21,25,29 (4,8)	✓		
21 ✓	13,15 (2)	✓	13,15,29,31 (2,16)	B		
25 ✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓		
15 ✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓		
23 ✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓		
27 ✓	21,23 (2)	✓				
29 ✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A		
31 ✓	25,27 (2)	✓				

uwxyz
10001 (17)
10011 (19)
10101 (21)
10111 (23)
11001 (25)
11011 (27)
11101 (29)
11111 (31)

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	H	25,29 (4)	✓	
2	✓	2,18 (16)	G	15,31 (16)	✓	
12	✓	12,13 (1)	F	23,31 (8)	✓	
17	✓	17,19 (2)	✓	27,31 (4)	✓	
18	✓	17,21 (4)	✓	29,31 (2)	✓	
20	✓	17,25 (8)	✓			
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

uwxyz	
10001	(17)
10011	(19)
10101	(21)
10111	(23)
11001	(25)
11011	(27)
11101	(29)
11111	(31)

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	H	25,29 (4)	✓	
2	✓	2,18 (16)	G	15,31 (16)	✓	
12	✓	12,13 (1)	F	23,31 (8)	✓	
17	✓	17,19 (2)	✓	27,31 (4)	✓	
18	✓	17,21 (4)	✓	29,31 (2)	✓	
20	✓	17,25 (8)	✓			
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

A=uz

uwxyz	
10001	(17)
10011	(19)
10101	(21)
10111	(23)
11001	(25)
11011	(27)
11101	(29)
11111	(31)

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz
2	✓	2,18 (16)	G	15,31 (16)	✓	B=wxz
12	✓	12,13 (1)	F	23,31 (8)	✓	
17	✓	17,19 (2)	✓	27,31 (4)	✓	
18	✓	17,21 (4)	✓	29,31 (2)	✓	
20	✓	17,25 (8)	✓			
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

uwxyz
01101 (13)
01111 (15)
11101 (29)
11111 (31)

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz
2	✓	2,18 (16)	G	15,31 (16)	✓	B=wxz
12	✓	12,13 (1)	F	23,31 (8)	✓	C=uwx'y'
17	✓	17,19 (2)	✓	27,31 (4)	✓	
18	✓	17,21 (4)	✓	29,31 (2)	✓	
20	✓	17,25 (8)	✓			
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	uwxyz
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	11000 (24)
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	11001 (25)
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz
2	✓	2,18 (16)	G	15,31 (16)	✓	B=wxz
12	✓	12,13 (1)	F	23,31 (8)	✓	C=uwx'y'
17	✓	17,19 (2)	✓	27,31 (4)	✓	D=uw'xy'
18	✓	17,21 (4)	✓	29,31 (2)	✓	
20	✓	17,25 (8)	✓			
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	uwxyz
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	10100 (20)
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	10101 (21)
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz
2	✓	2,18 (16)	G	15,31 (16)	✓	B=wxz
12	✓	12,13 (1)	F	23,31 (8)	✓	C=uwx'y'
17	✓	17,19 (2)	✓	27,31 (4)	✓	D=uw'xy'
18	✓	17,21 (4)	✓	29,31 (2)	✓	E=uw'x'y
20	✓	17,25 (8)	✓			
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	uwxyz
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	10010 (18)
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	10011 (19)
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	H	25,29 (4)	✓		
2	✓	2,18 (16)	G	15,31 (16)	✓		
12	✓	12,13 (1)	F	23,31 (8)	✓		
17	✓	17,19 (2)	✓	27,31 (4)	✓		
18	✓	17,21 (4)	✓	29,31 (2)	✓		
20	✓	17,25 (8)	✓				
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓		
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓		
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓		
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B		
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓		
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓		
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓		
27	✓	21,23 (2)	✓				
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A		
31	✓	25,27 (2)	✓				

A=uz
B=wxz
C=uwx'y'
D=uw'xy'
E=uw'x'y
F=u'wxy'

uwxyz
01100 (12)
01101 (13)

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	H	25,29 (4)	✓		
2	✓	2,18 (16)	G	15,31 (16)	✓		
12	✓	12,13 (1)	F	23,31 (8)	✓		
17	✓	17,19 (2)	✓	27,31 (4)	✓		
18	✓	17,21 (4)	✓	29,31 (2)	✓		
20	✓	17,25 (8)	✓				
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓		
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓		
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓		
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B		
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓		
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓		
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓		
27	✓	21,23 (2)	✓				
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A		
31	✓	25,27 (2)	✓				

A=uz
B=wxz
C=uwx'y'
D=uw'xy'
E=uw'x'y
F=u'wxy'
G=w'x'yz'

uwxyz
00010 (2)
10010 (18)

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	H	25,29 (4)	✓		
2	✓	2,18 (16)	G	15,31 (16)	✓		
12	✓	12,13 (1)	F	23,31 (8)	✓		
17	✓	17,19 (2)	✓	27,31 (4)	✓		
18	✓	17,21 (4)	✓	29,31 (2)	✓		
20	✓	17,25 (8)	✓				
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓		
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓		
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓		
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B		
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓		
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓		
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓		
27	✓	21,23 (2)	✓				
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A		
31	✓	25,27 (2)	✓				

A=uz
B=wxz
C=uwx'y'
D=uw'xy'
E=uw'x'y
F=u'wxy'
G=w'x'yz'
H=w'x'y'z

uwxyz
00001 (1)
10001 (17)

$f(u,w,x,y,z) = \Sigma(13,15,17,18,19,20,21,23,25,27,29,31) + \Sigma_{\varphi}(1,2,12,24)$

1	✓	1,17 (16)	H	25,29 (4)	✓	A=uz
2	✓	2,18 (16)	G	15,31 (16)	✓	B=wxz
12	✓	12,13 (1)	F	23,31 (8)	✓	C=uwx'y'
17	✓	17,19 (2)	✓	27,31 (4)	✓	D=uw'xy'
18	✓	17,21 (4)	✓	29,31 (2)	✓	E=uw'x'y
20	✓	17,25 (8)	✓			F=u'wxy'
24	✓	18,19 (1)	E	17,19,21,23 (2,4)	✓	G=w'x'yz'
13	✓	20,21 (1)	D	17,19,25,27 (2,8)	✓	H=w'x'y'z
19	✓	24,25 (1)	C	17,21,25,29 (4,8)	✓	
21	✓	13,15 (2)	✓	13,15,29,31 (2,16)	B	
25	✓	13,29 (16)	✓	19,23,27,31 (4,8)	✓	
15	✓	19,23 (4)	✓	21,23,29,31 (2,8)	✓	
23	✓	19,27 (8)	✓	25,27,29,31 (2,4)	✓	
27	✓	21,23 (2)	✓			
29	✓	21,29 (8)	✓	17,19,21,23,25,27,29,31 (2,4,8)	A	
31	✓	25,27 (2)	✓			

Prime implicant selection

essential

		13	15	17	18	19	20	21	23	25	27	29	31
*	A			X		X		X	X	X	X	X	X
*	B	X	X									X	X
	C									X			
*	D						X	X					
√	E				X	X							
	F	X											
	G				X								
	H			X									
		X	X	X	X	X	X	X	X	X	X	X	X

Prime implicant selection

essential

		13	15	17	18	19	20	21	23	25	27	29	31
*	A			X		X		X	X	X	X	X	X
*	B	X	X									X	X
	C									X			
*	D						X	X					
√	E				X	X							
	F	X											
	G				X								
	H			X									
		X	X	X	X	X	X	X	X	X	X	X	X

F = uz + wxz + uw'xy' + uw'x'y

Binary Decision Diagrams

A binary decision diagram is defined as a rooted, directed acyclic graph, which has one or two terminal nodes corresponding to logic 0 and logic 1, and a set of non-terminal nodes out of which originate two edges.

Binary Decision Diagrams

Each of the two edges correspond the functions Fx και Fx' from Shannon's theorem

$$F(x_1,x_2,\ldots,x_i,\ldots,x_n) =$$
$$x_i\, F(x_1,x_2,\ldots,1,\ldots,x_n) + x_i'\, F(x_1,x_2,\ldots,0,\ldots,x_n)$$

Exactly one variable corresponds to each node.

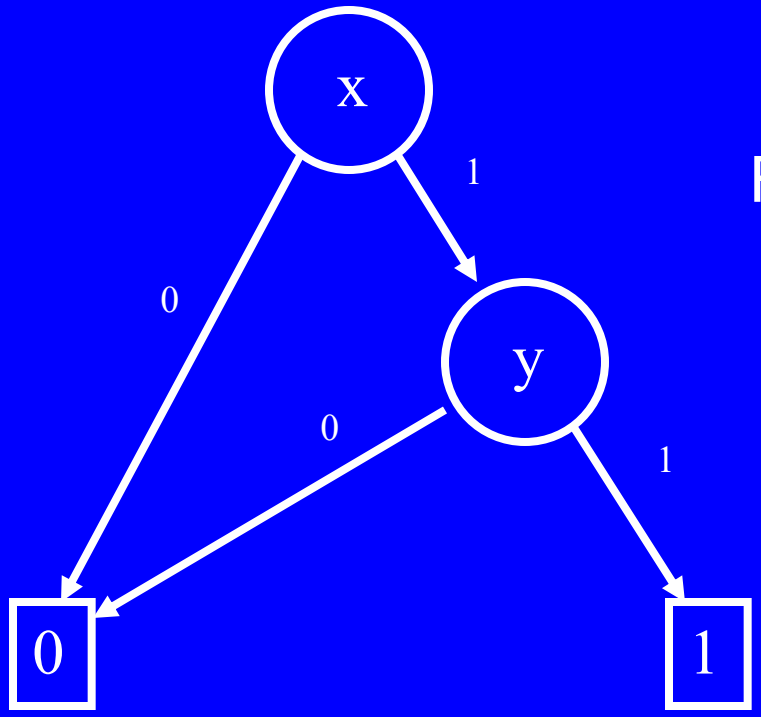
Binary Decision Diagrams

A BDD is defined as ordered if in all paths inside the graph, variable always appear in the same order ($x_1 < x_2 < ... < x_i < ... < x_n$)

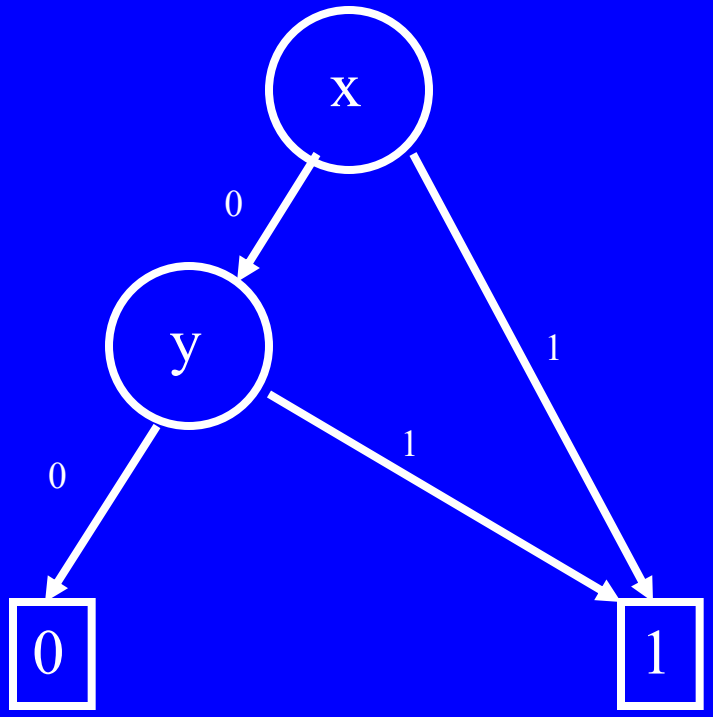
A BDD is defined as reduced if:

- there are no two nodes corresponding to the same variable that have the same subgraphs, and
- no node has identical subgraphs

Binary Decision Diagrams



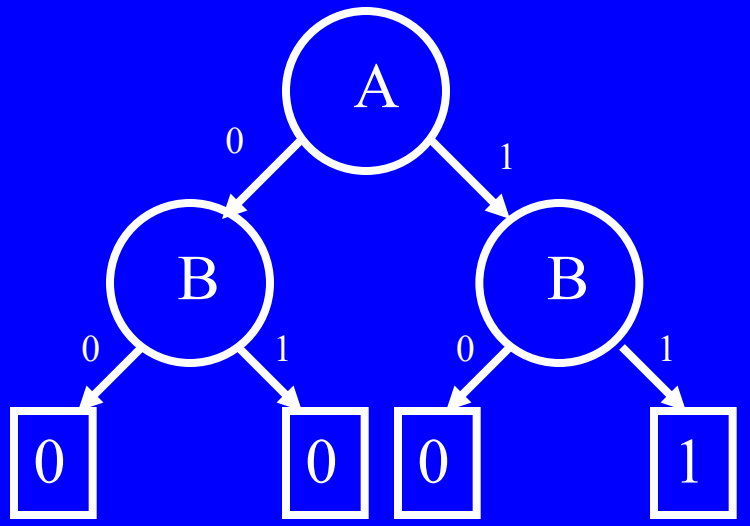
Binary Decision Diagrams



F = x+y

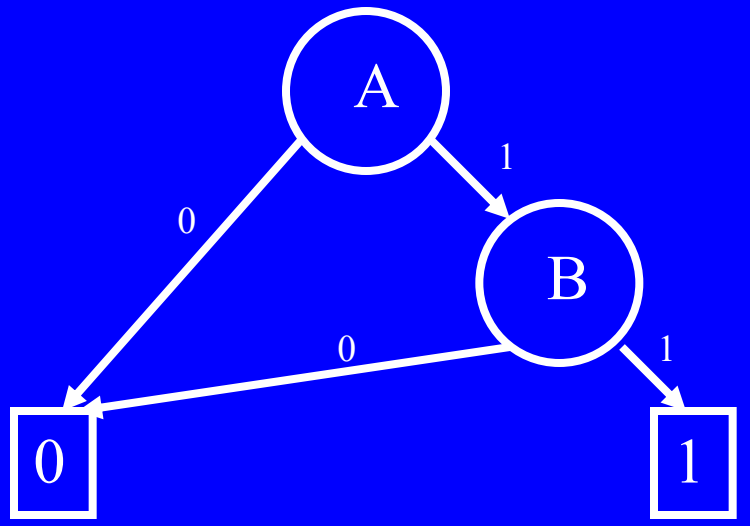
Binary Decision Diagrams

A	0	0	1	1
B	0	1	0	1
F	0	0	0	1



Binary Decision Diagrams

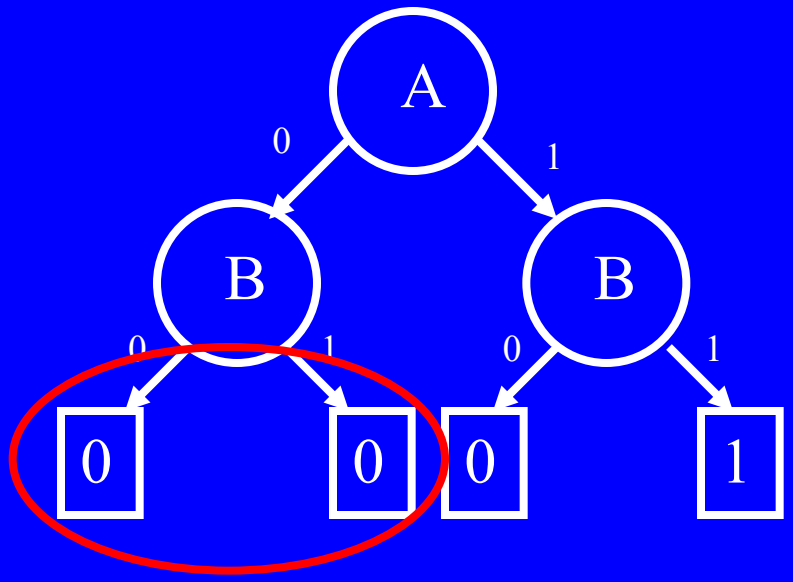
A	0	0	1	1
B	0	1	0	1
F	0	0	0	1



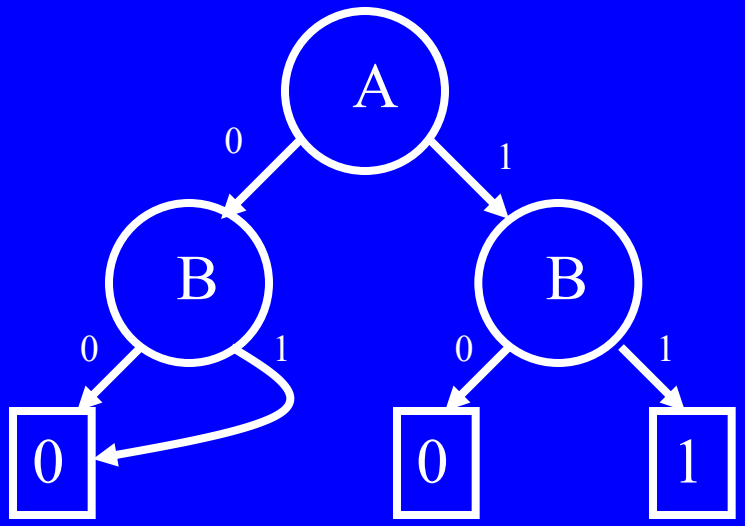
Reduction process

- Join identical nodes
- If both edges orginating from node X point to the same node Y, then we remove node X and we directly connect the higher-order node to node Y.
- We repeat the process until we cannot join or remove any more nodes.

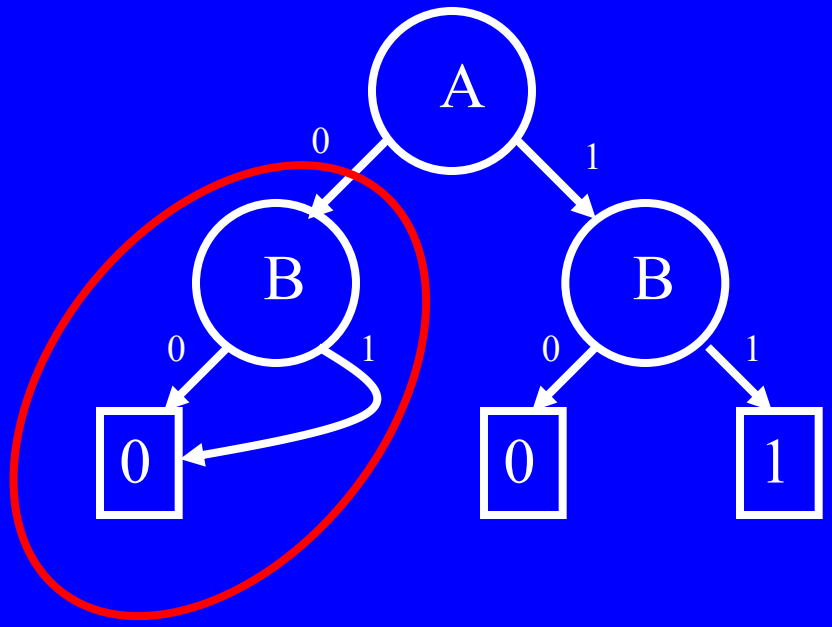
Reduction process



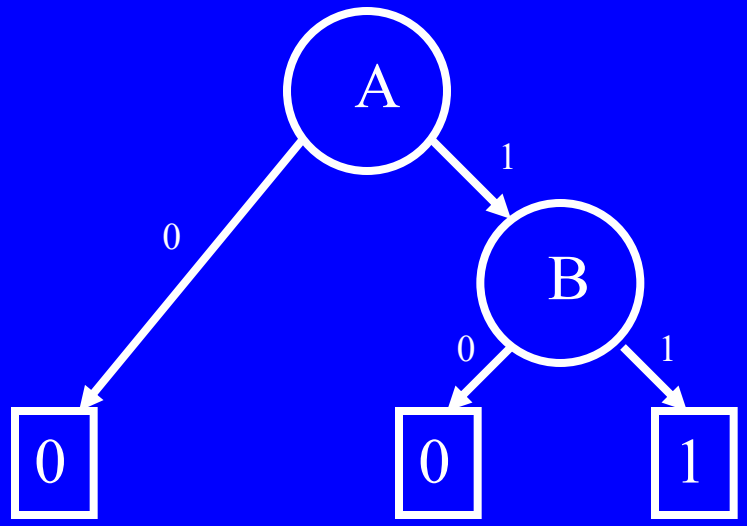
Reduction process



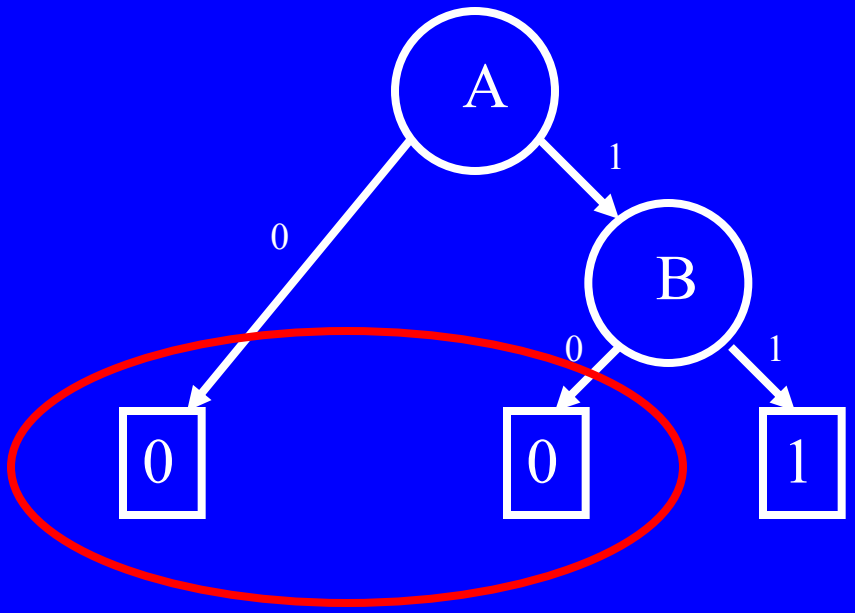
Reduction process



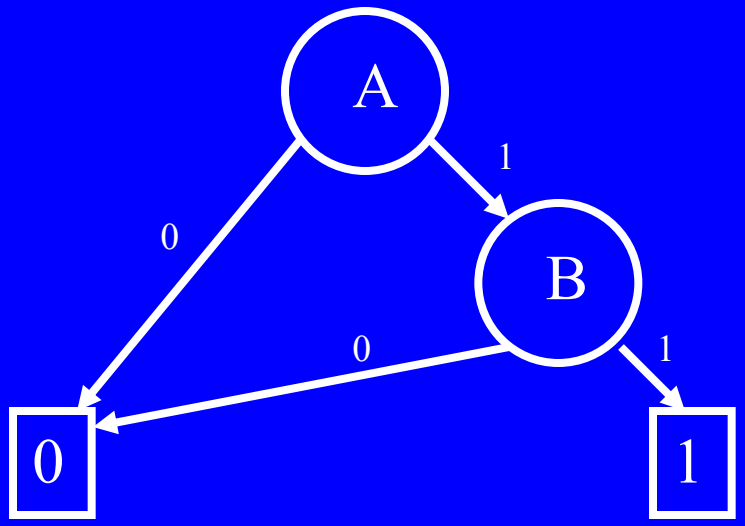
Reduction process



Reduction process



Reduction process

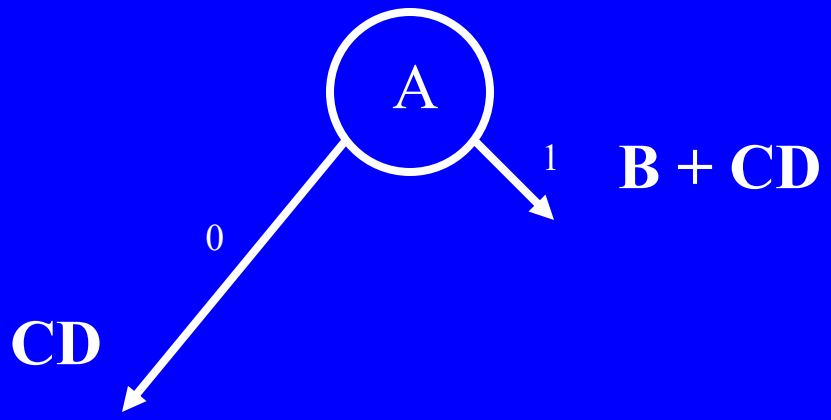


Example



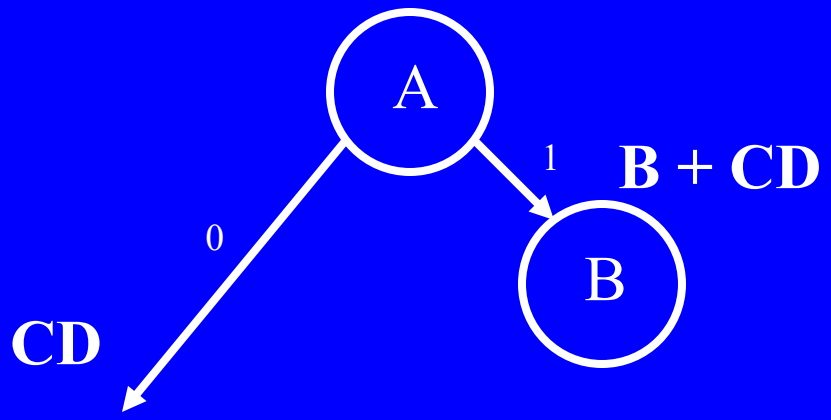
F = AB + CD
A<B<C<D

Example



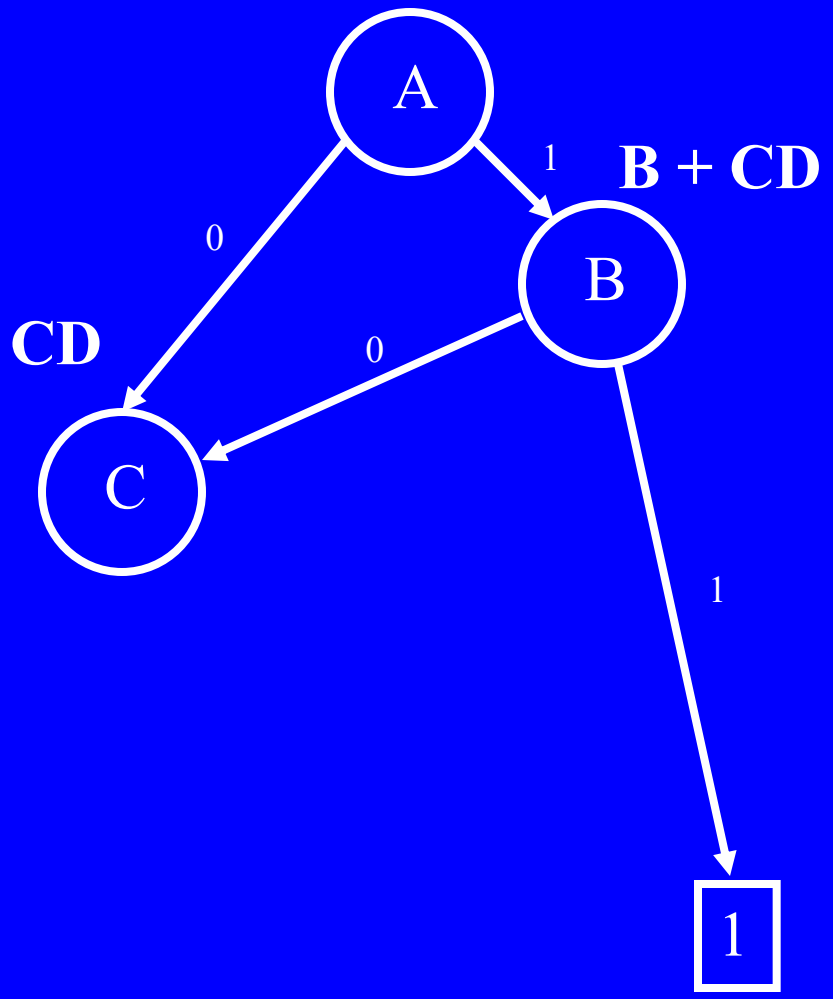
F = AB + CD
A<B<C<D

Example



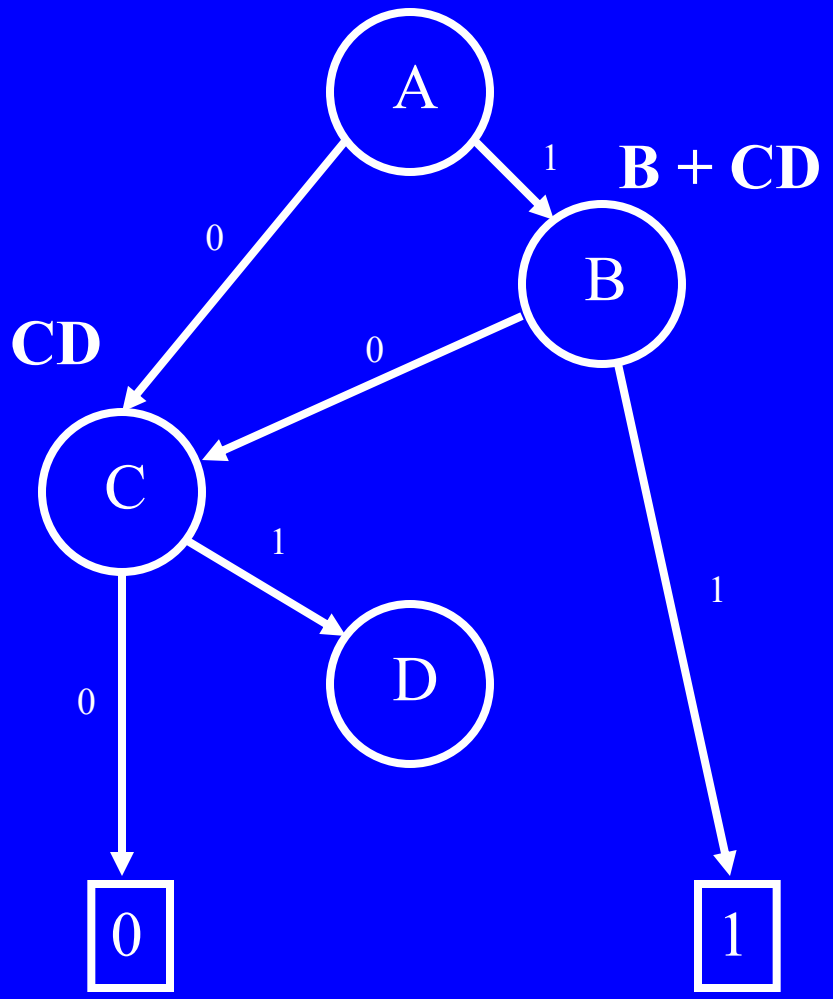
F = AB + CD
A<B<C<D

Example



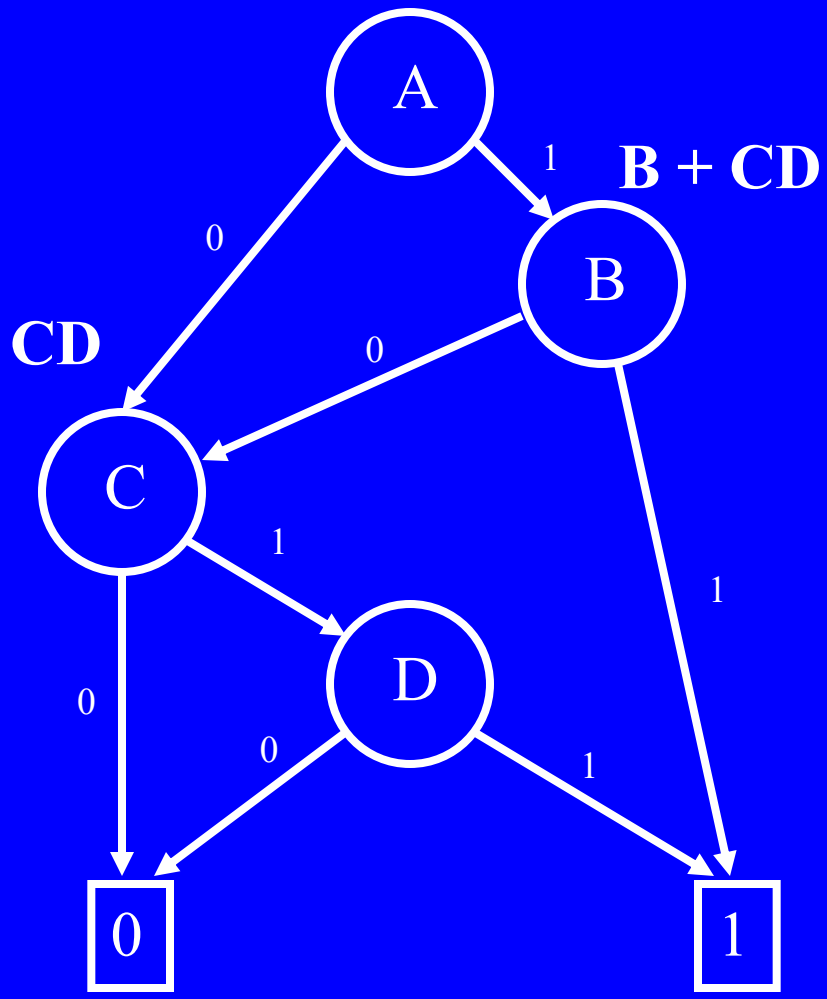
F = AB + CD
A<B<C<D

Example



F = AB + CD
A<B<C<D

Example



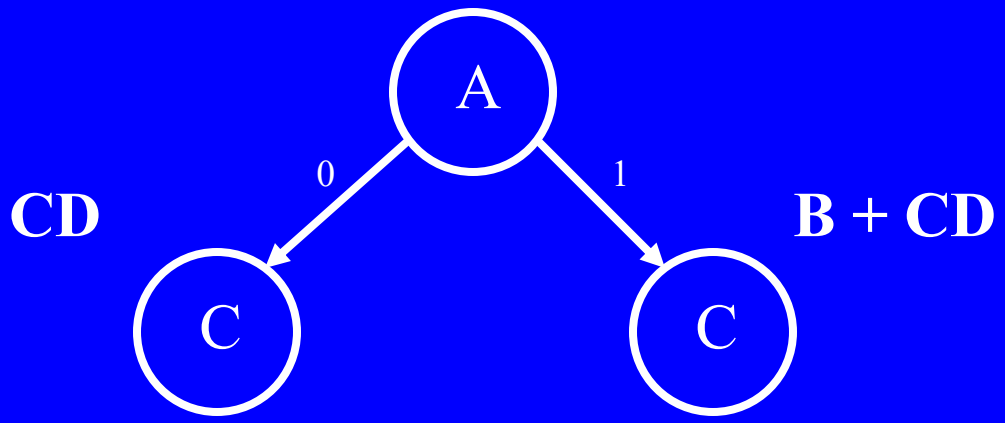
F = AB + CD
A<B<C<D

Another example



F = AB + CD
A<C<B<D

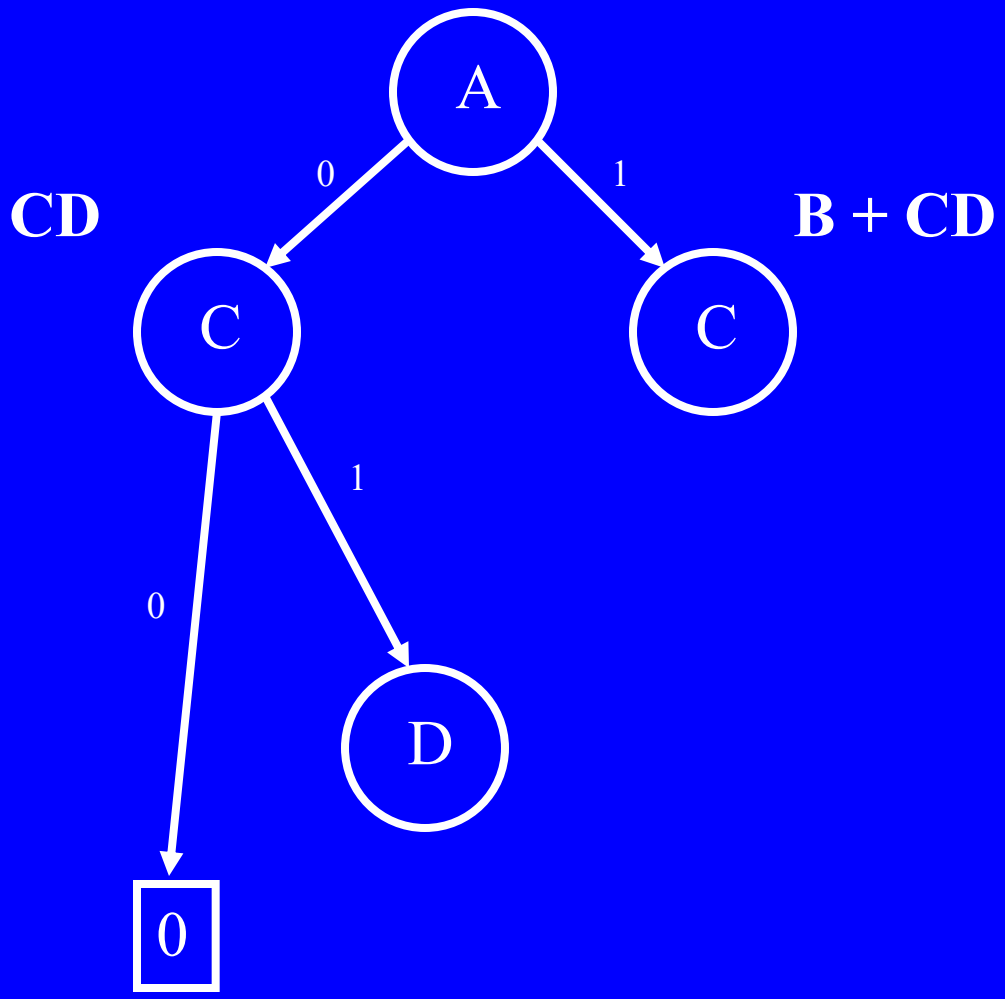
Another example



F = AB + CD

A<C<B<D

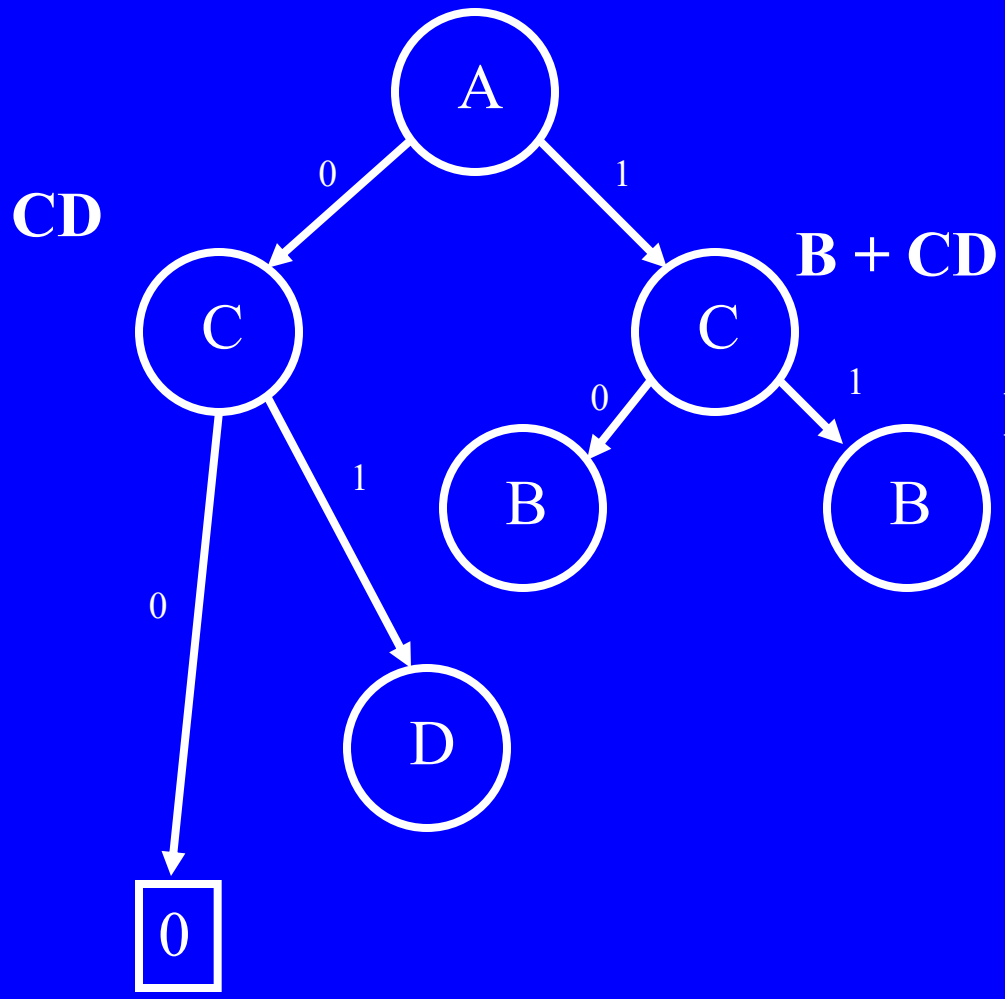
Another example



F = AB + CD

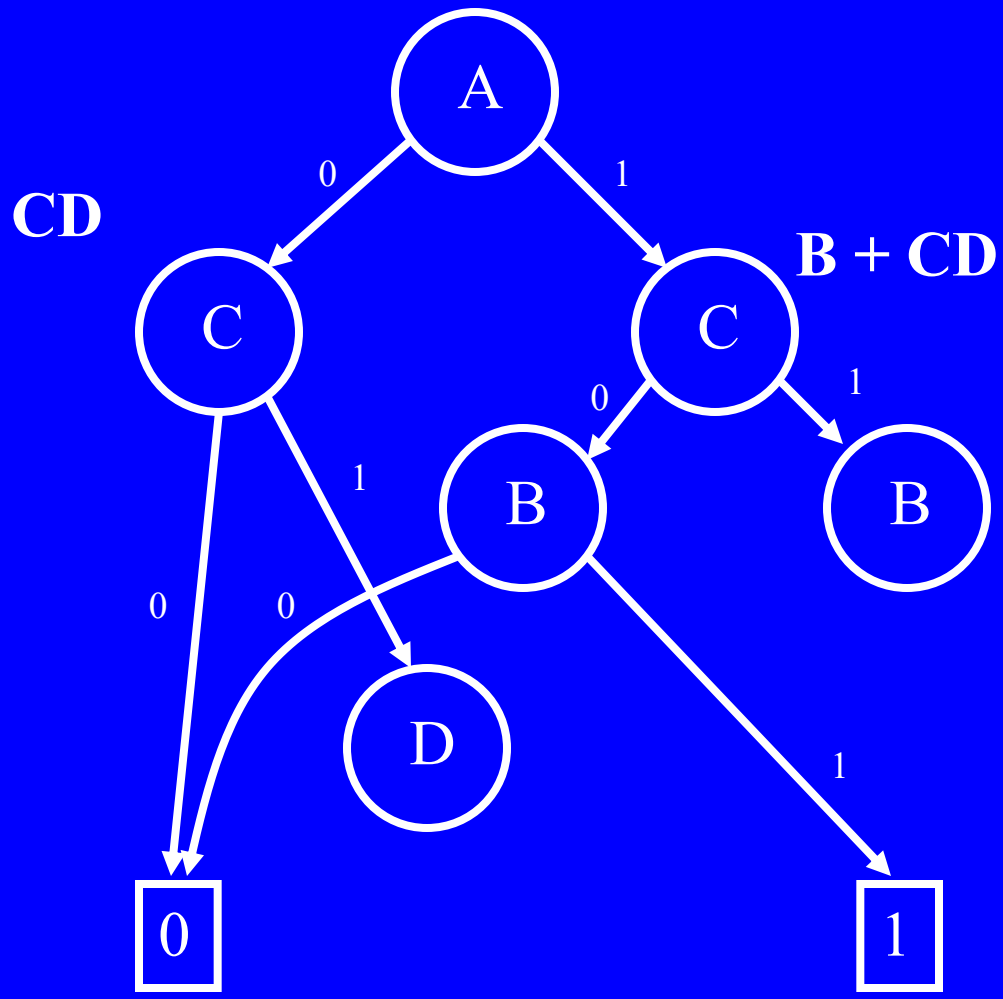
A<C<B<D

Another example



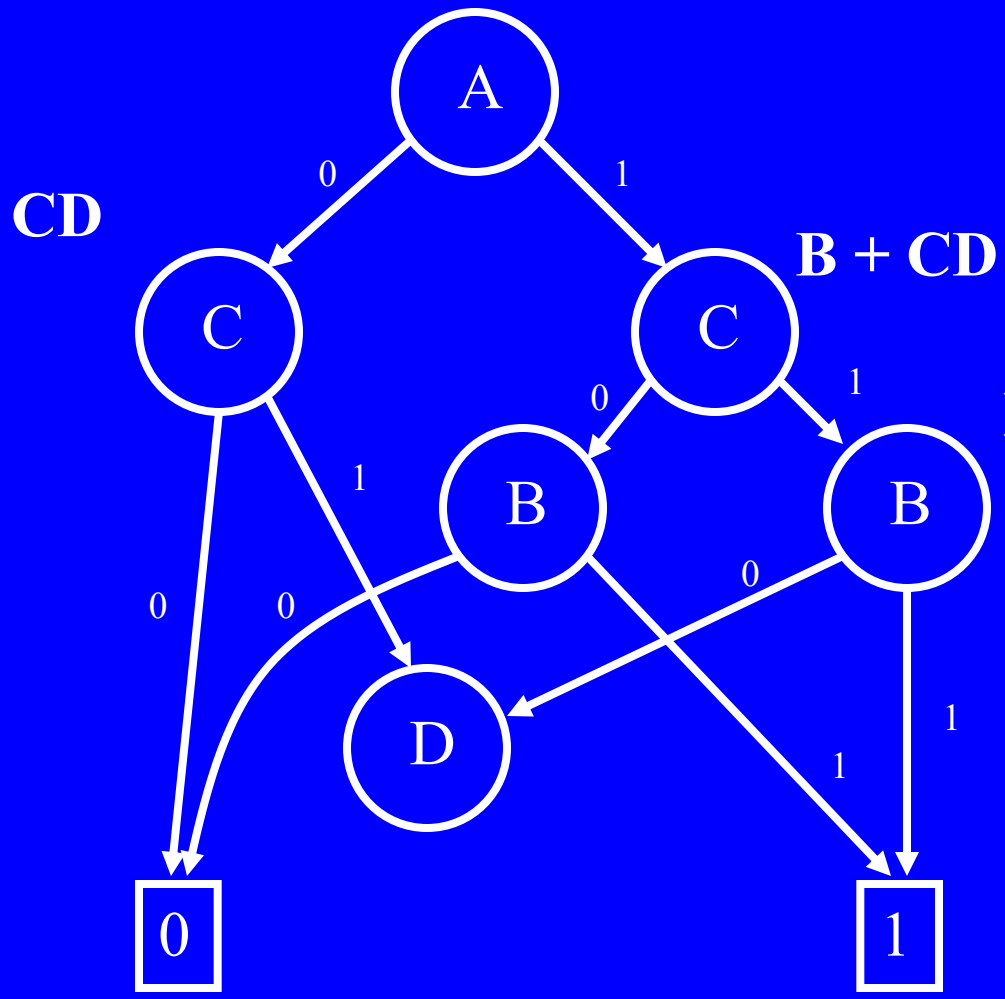
F = AB + CD
A<C<B<D

Another example



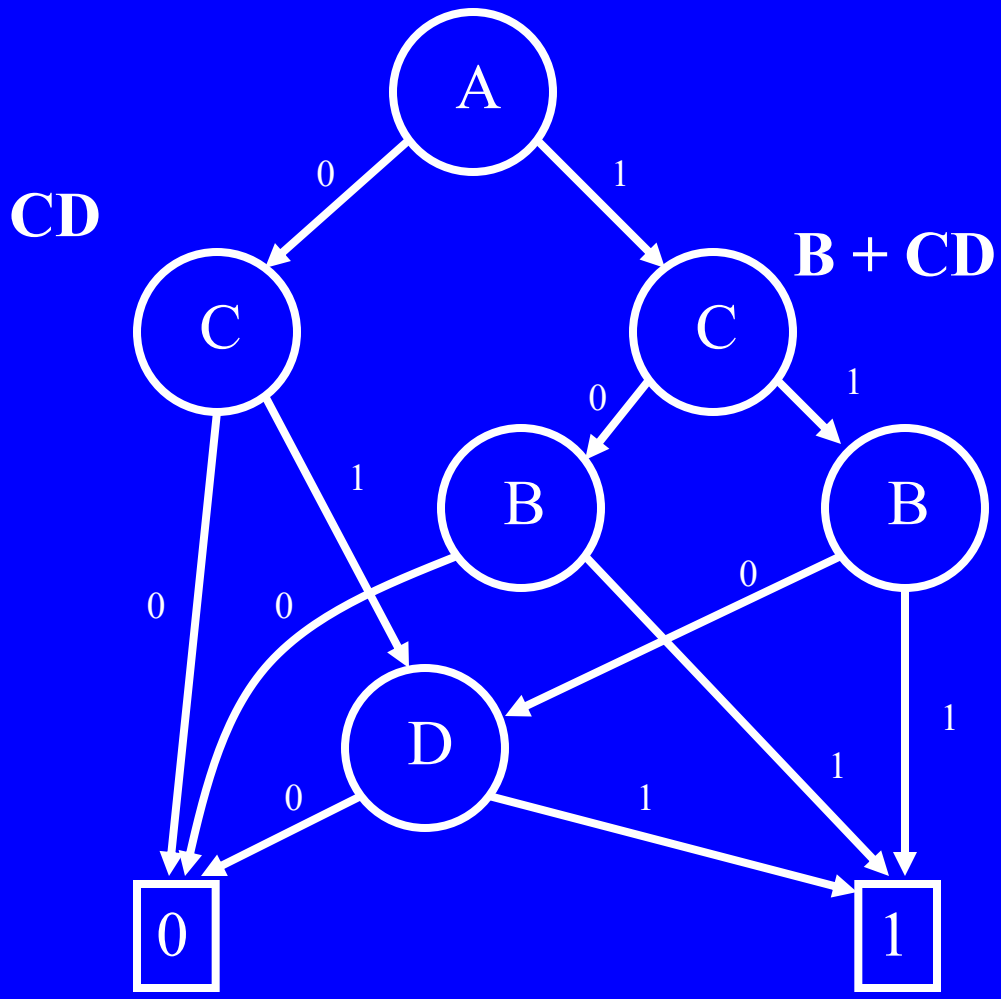
F = AB + CD
A<C<B<D

Another example



F = AB + CD
A<C<B<D

Another example



F = AB + CD
A<C<B<D