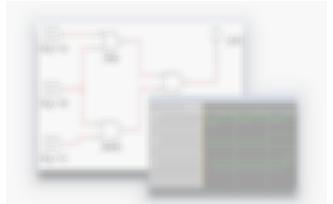


# ECE119 – Ψηφιακή Σχεδίαση

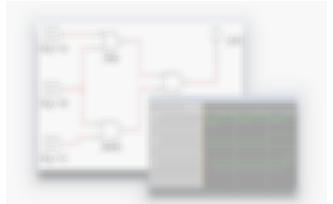
Διδάσκοντες Εργαστηρίου: Δ. Καραμπερόπουλος  
Δ. Γαρυφάλλου

## ➤ Lab 3: Logic Gates Explored and Boolean Algebra



# Περιεχόμενα Εργαστηριακού Μαθήματος

- Εισαγωγή
- Lab 1: Multisim Circuit Simulation and Basic Gates
- Lab 2: Truth Tables and Basic Logic Gates
- **Lab 3: Logic Gates Explored and Boolean Algebra**
- Lab 4: Karnaugh Maps
- Lab 5: Binary Conversion and Adders
- Lab 6: Encoders and Decoders
- Lab 7: Multiplexers and Demultiplexers
- Lab 8: Latches and Sequential Logic Circuits
- Lab 9: Flip-Flops
- Lab 10: Sequential Circuits - FSM



## Logic Gates Explored and Boolean Algebra

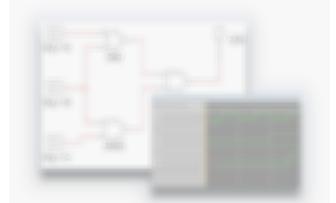
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In the previous lab, we were introduced to the two basic logic gates – **AND** and **OR** in detail. There is also **NOT (Inverter)** gate.

Building on these, we can create a few other types of logic gates.

These are: **NAND**, **NOR**, **XOR**, and **XNOR**.

Let's take a look at each one in greater detail.

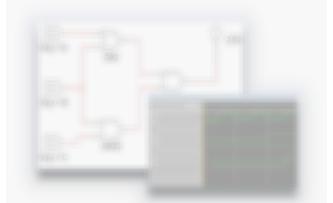


## Learning Objectives

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In this lab, students will:

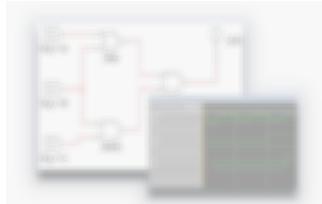
- Explore the function of various different **logic gates**
- Create circuits with varying logic gates in theory and in practice.
- Calculate and build combinational logic circuits from **Sum-of-Products** and **Product-of-Sums** derived from truth tables.
- Learn how to write a **Combinational Logic Circuit (CLC)** in Verilog.
- Learn how to **test a module** and take True Table in Verilog.



## Expected Deliverables

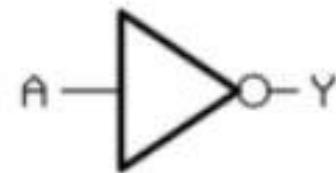
In this lab, you will collect the following deliverables:

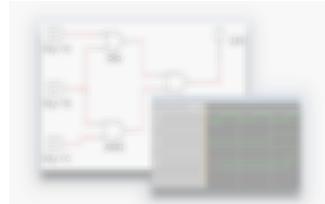
- SOP and POS Boolean expressions
- Design circuits
- Truth Tables
- Multisim Files
- Conclusion questions
- Verilog File



## Inverters

- Inverters are also known as **NOT** gates.
- They have only **one input** and **one output**.
- The truth table for an inverter is simple. The output is always the opposite of the input.
- For example, if the input is **1**, the output will be **0** and vice versa. Visually this is depicted by a **circle** at the input and/or output ends of the logic gates.
- In this situation, the circle is at the output, which means that the output is inverted. If it was at the input, then it is the input that would be inverted.



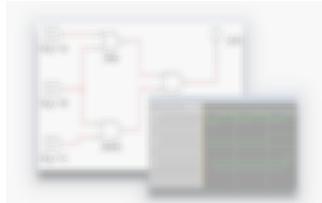


## NAND Logic Gates

- **NAND** gates invert the output of the AND gate.
- The inputs do not change from those of the AND truth table, but the output is the opposite.
- As a rule, if any of the inputs are **0**, the output will always be **1**.
- See the truth table and the symbol.

A	B	O
0	0	1
0	1	1
1	0	1
1	1	0

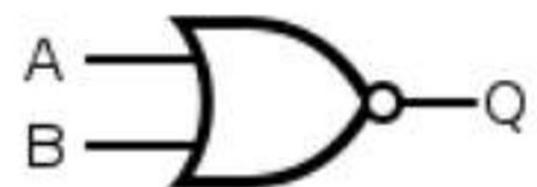


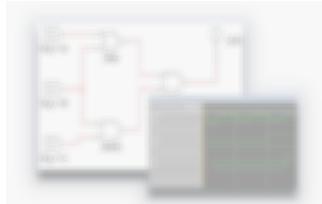


## NOR Logic Gates

- The **NOR** logic gate inverts the output of the OR gate.
- The inputs of the truth table for the OR gate do not change, but the output is the opposite.
- As a rule, if any of the inputs are 1, the output will always be 0.
- See the truth table and symbol.

A	B	O
0	0	1
0	1	0
1	0	0
1	1	0



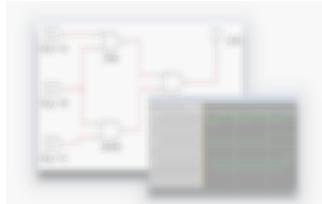


## XOR Logic Gates

- An **XOR** gate is also known as an **eXclusive OR** gate.
- The output will be 1 if only one of the inputs is 1. The output will be 0 if both inputs are 0 or both are 1.
- See the truth table and symbol.

A	B	O
0	0	0
0	1	1
1	0	1
1	1	0



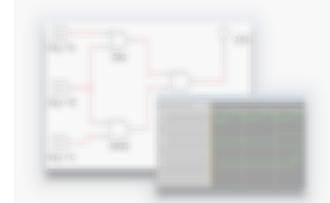


## XNOR Logic Gates

- The **XNOR** gate does the opposite of the XOR gate.
- The output will be 1 if the inputs are the same and the output will be 0 if the inputs are not the same.
- See the truth table.

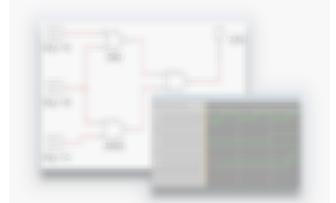
A	B	O
0	0	1
0	1	0
1	0	0
1	1	1

# Combinational Logic Circuits (CLCs), Συνδυαστικά κύκλωματα



- CLCs are a classification of circuits whose output is only **dependent on the current inputs** and are implemented by Boolean circuits.
- Using combinations of logic gates, different results can be achieved.
- A truth table is often used to define the behavior of a CLC, but sometimes we start with a truth table and need to design a CLC.

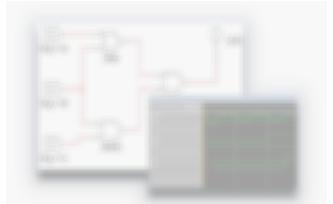




## Boolean Algebra (1/2)

**Boolean algebra** is an algebraic system where two values are used to represent the properties of bi-stable electrical switching circuits, namely **on** and **off**, or simply **1** and **0**. The rules for the two **binary operators** (addition and multiplication) and complement ('') for a two-valued Boolean algebraic expression are presented in the tables on the next slide.

- It can be seen that the binary addition, multiplication and complement are the same as the OR, AND and NOT logic operations.
- For the **complement**, several notations are used: apostrophe after the variable, exclamation mark, tilde or the word NOT before the variable or an over-bar on top of it.
- Because it works with digital systems with only the values 0 and 1, the algebra used is simply called "**binary logic**".



## Boolean Algebra (2/2)

- Any logic function, no matter how complex it is, can be implemented using only the three basic logic operations.
- A function represented by a truth table can be expressed using different methods.
- Knowing the logic expression and the function, the circuit can be then realized.

x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

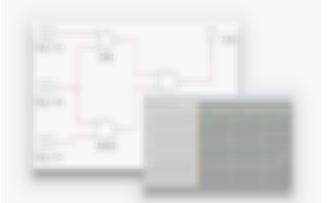
Binary Multiplication (AND logic operation)

x	y	$x + y$
0	0	0
0	1	1
1	0	1
1	1	1

Binary Addition (OR logic operation)

x	$x'$
0	1
1	0

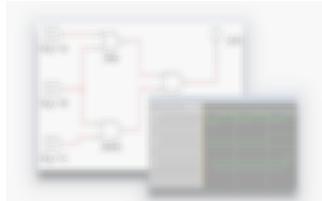
Compliment (NOT logic operation)



## Sum-of-Products (άθροισμα γινομένων)

A simple method for **converting a truth table into a CLC** is found in a standard form of Boolean expression called the **Sum-of-Products (SOP)**.

- An SOP expression is literally a sum of Boolean terms called **minterms (ελαχιστόροι)**.
- A minterm is a multiplicative combination of Boolean variables whose output equals 1.
- An example of an SOP expression is  $ABC + AB'C'$ , where  $ABC$ ,  $AB'C'$  are minterms.
- SOP expressions may be generated from truth tables using the following steps:
  1. Determine which rows of the table have an output of 1.
  2. Derive each row's minterm, such that the output is 1 given that row's input state.
  3. Sum the minterms.



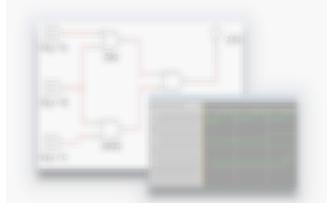
## Sum-of-Products (άθροισμα γινομένων)

Below is an example of a **truth table conversion to an SOP expression**.

A	B	C	O
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$$O = A'B'C + AB'C' + AB'C + ABC'$$

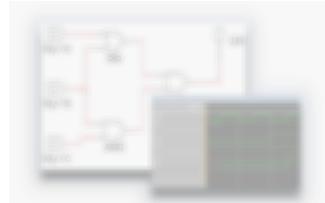
$A'B'C$   
 $AB'C'$   
 $AB'C$   
 $ABC'$



## Product-of-Sums (γινόμενο αθροισμάτων)

**Product-of-Sums (POS)** expressions are another way of **representing truth tables**.

- A POS expression is a product of Boolean terms called **maxterms (μεγιστόροι)**.
- A maxterm is a summation of Boolean variables whose output equals 0.
- To generate a POS expression from a truth table, perform the following steps:
  1. Determine which rows of the table have an output of 0.
  2. Derive each row's maxterm, such that the output is 0 given that row's input state.
  3. Multiply the maxterms.



## Product-of-Sums (γινόμενο αθροισμάτων)

Below is an example of a **truth table conversion to an POS expression**.

A	B	C	O
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$$A + B + C$$

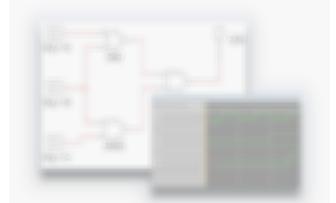
$$A + B' + C$$

$$A + B' + C'$$

$$A' + B' + C'$$

$$O = (A + B + C)(A + B' + C)(A + B' + C')(A' + B' + C')$$

## SOP and POS

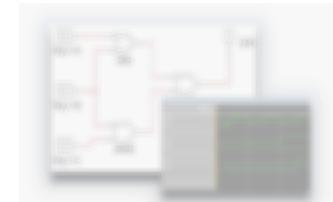


The **SOP** and **POS** standard Boolean forms are powerful tools when applied to truth tables.

They can be used to derive a Boolean expression—and ultimately, an actual logic circuit.

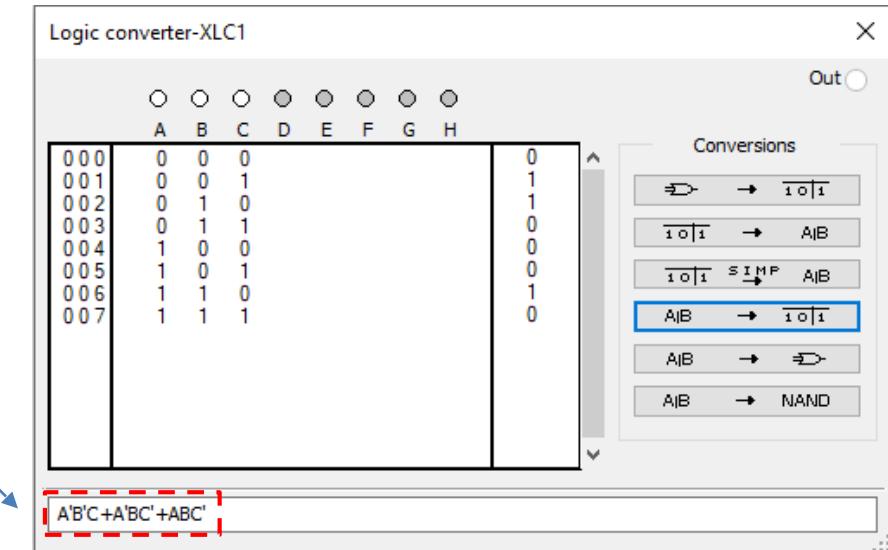
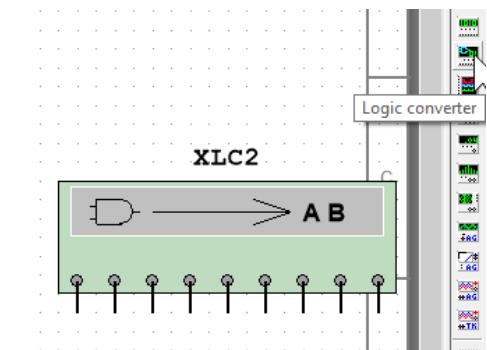
- When creating a circuit from **SOPs**, it would be constructed of **AND gates feeding into an OR gate**.
- When creating a circuit from **POSs**, it would be constructed of **OR gates feeding into an AND gate**.

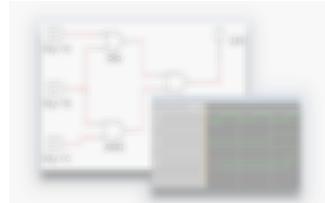
# MultiSim - Logic Converter (Expression to Truth Table)



The **Logic Converter** is a great tool for checking truth tables and logic expressions. To build a Logic Converter circuit:

- Place the **Logic Converter** from the instruments toolbar on the right screen onto the circuit.
- Double click the Logic Converter to open its user interface.
- Enter the **SOP expression** that you want in the text field at the bottom of the window.
- Click the fourth button, **Expression to Truth Table**.

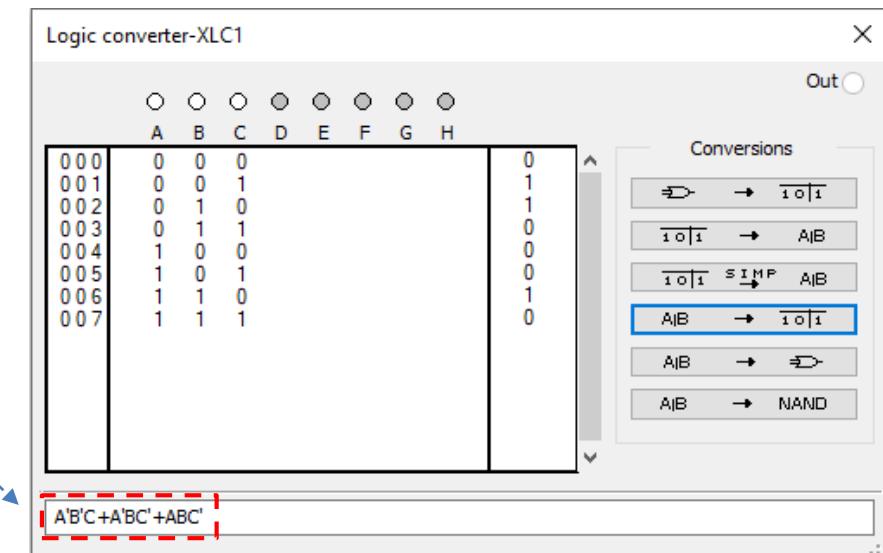




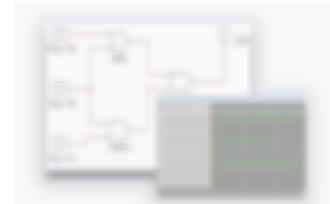
## MultiSim - Logic Converter (Simplified Expression)

The **Logic Converter** can also **generate circuits** from POS and SOP expressions. This can save some time from doing the work manually.

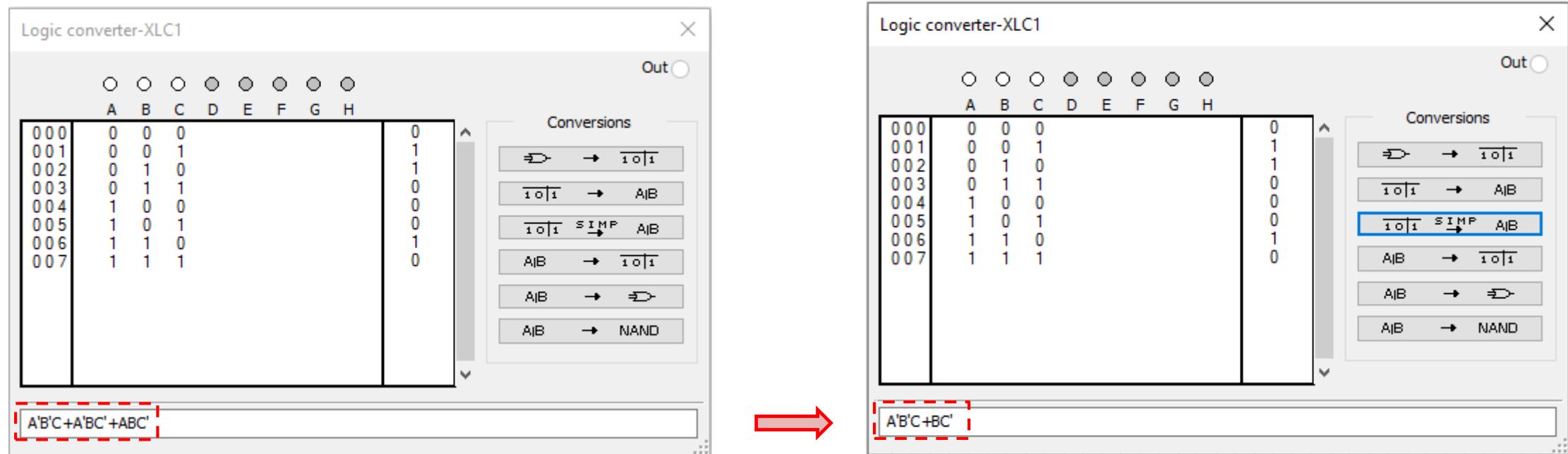
- Enter the **SOP expression** that you want in the text field at the bottom of the window.
- Click the fourth button, **Expression to Truth Table**.



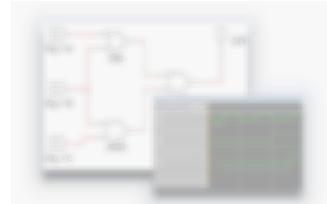
# MultiSim - Logic Converter (Simplified Expression)



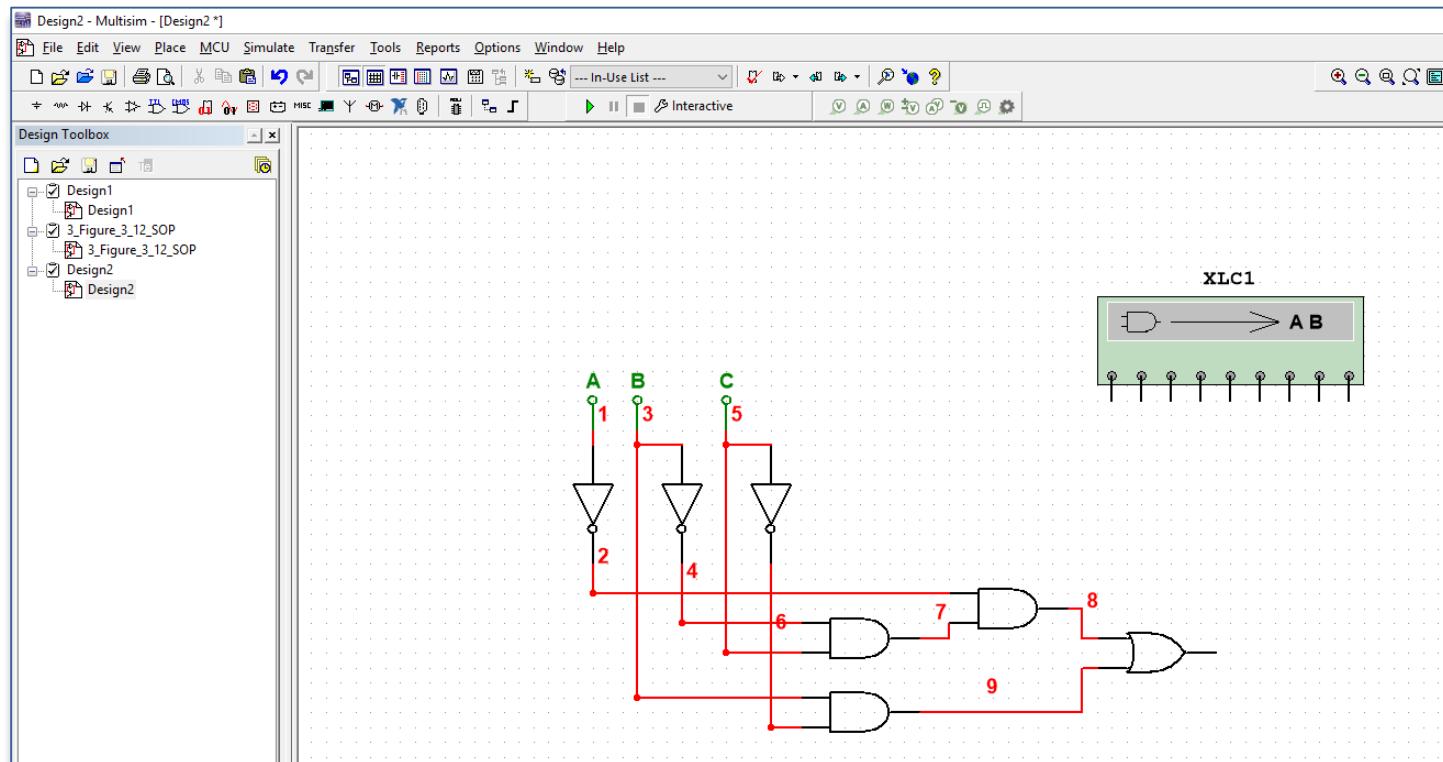
- Click the third button, **Truth Table to Simplified Expression**. This will simplify the expression if it can be simplified.

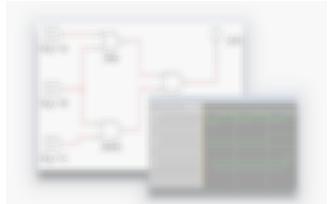


# MultiSim - Logic Converter (Expression to Circuit)



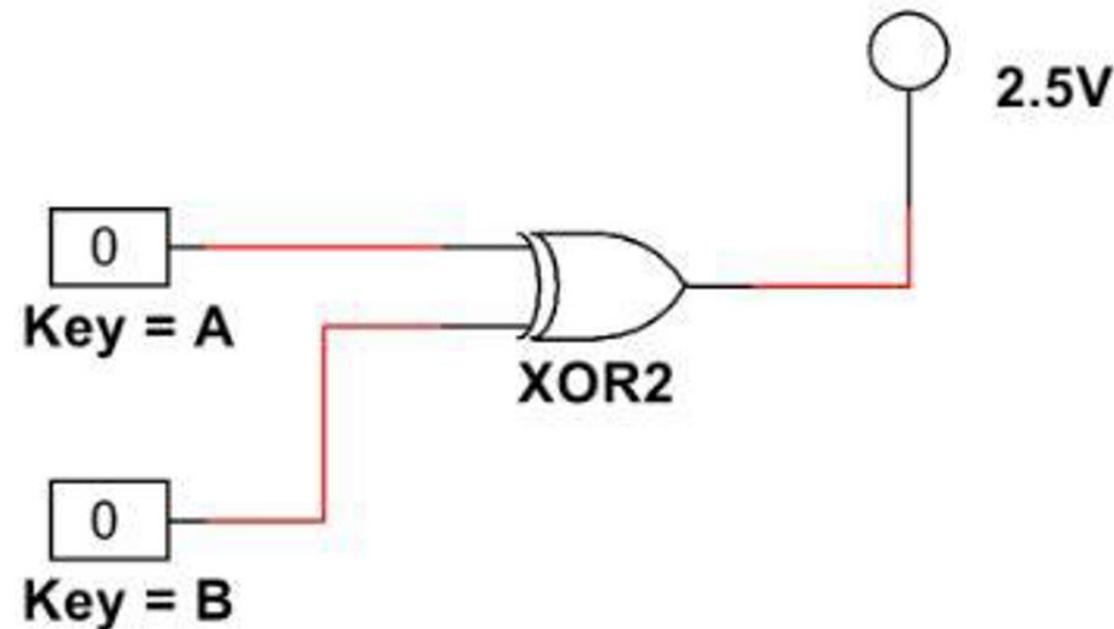
- Next, click the fifth button, **Expression to Circuit**.
- Place the circuit that it generates.

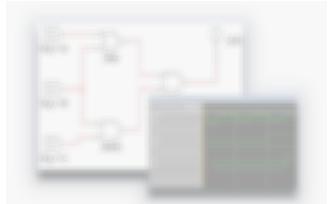




## Exercise: Building an XOR Logic Gate in Multisim

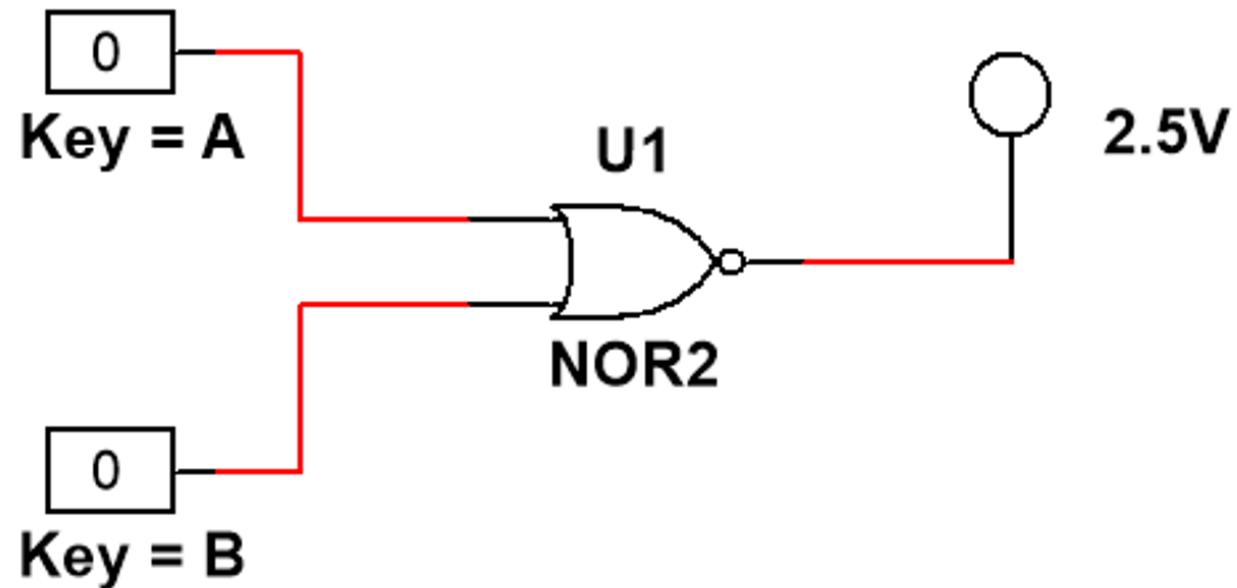
- Build the following circuit using an XOR gate:





## Exercise: Building a NOR Logic Gate in Multisim

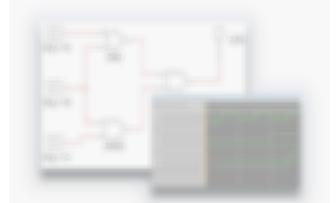
- Build the following circuit using an NOR gate:





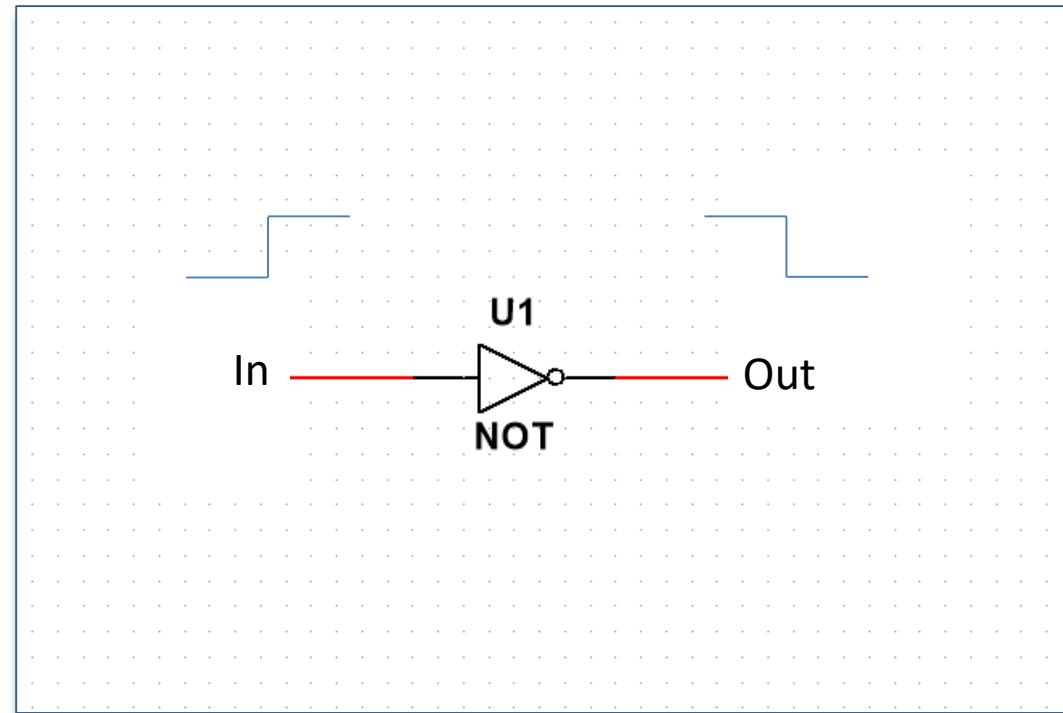
## Exercise: NOT - Gate

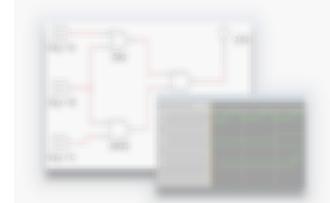
- Θέλουμε να μετρήσουμε και να συγκρίνουμε την **καθυστέρηση διάδοσης** του σήματος από την είσοδο μέχρι την έξοδο δύο αντιστροφέων διαφορετικής τεχνολογίας.
- Οι αντιστροφείς που θα συγκριθούν είναι οι εξής:
  - CMOS / CMOS\_5V / **4009BD\_5V**
  - TTL / 74STD / **7404N**
- Συνδέστε στην είσοδο και των δύο πυλών ένα ρολόι παραγωγής τετραγωνικών παλμών με συχνότητα 1 MHz.
- Χρησιμοποιώντας έναν **παλμογράφο** παρατηρείστε την είσοδο του παλμού στο κύκλωμα και την έξοδο της κάθε πύλης χρησιμοποιώντας τρία κανάλια της συσκευής.
- Υπολογίστε τον χρόνο καθυστέρησης στη διάδοση του σήματος κάθε αντιστροφέα, για την μετάβασή του από  $0 \rightarrow 1$  καθώς και για μετάβαση από  $1 \rightarrow 0$ .
- Η προσομοίωση να γίνει με το Multisim.



## Multisim - Παλμογράφος

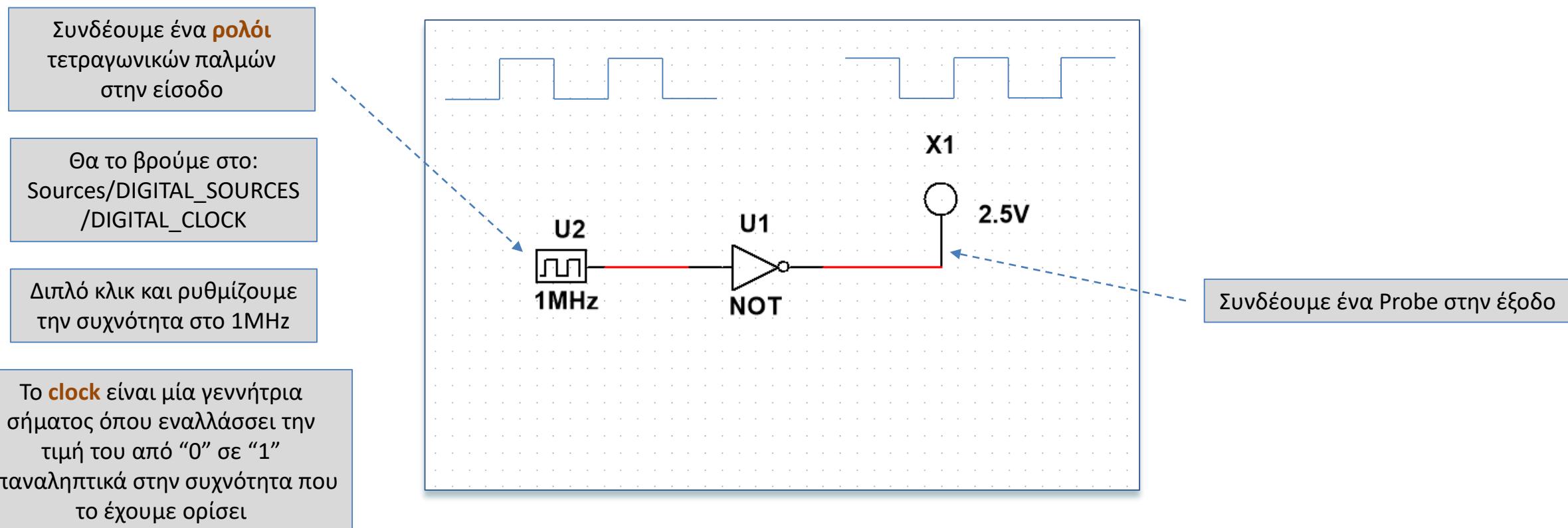
- Θέλουμε να μετρήσουμε την **καθυστέρηση διάδοσης** του σήματος από την είσοδο ενός αντιστροφέα στην έξοδό του.



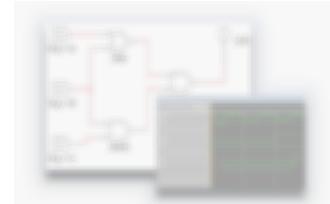


## Multisim - Παλμογράφος

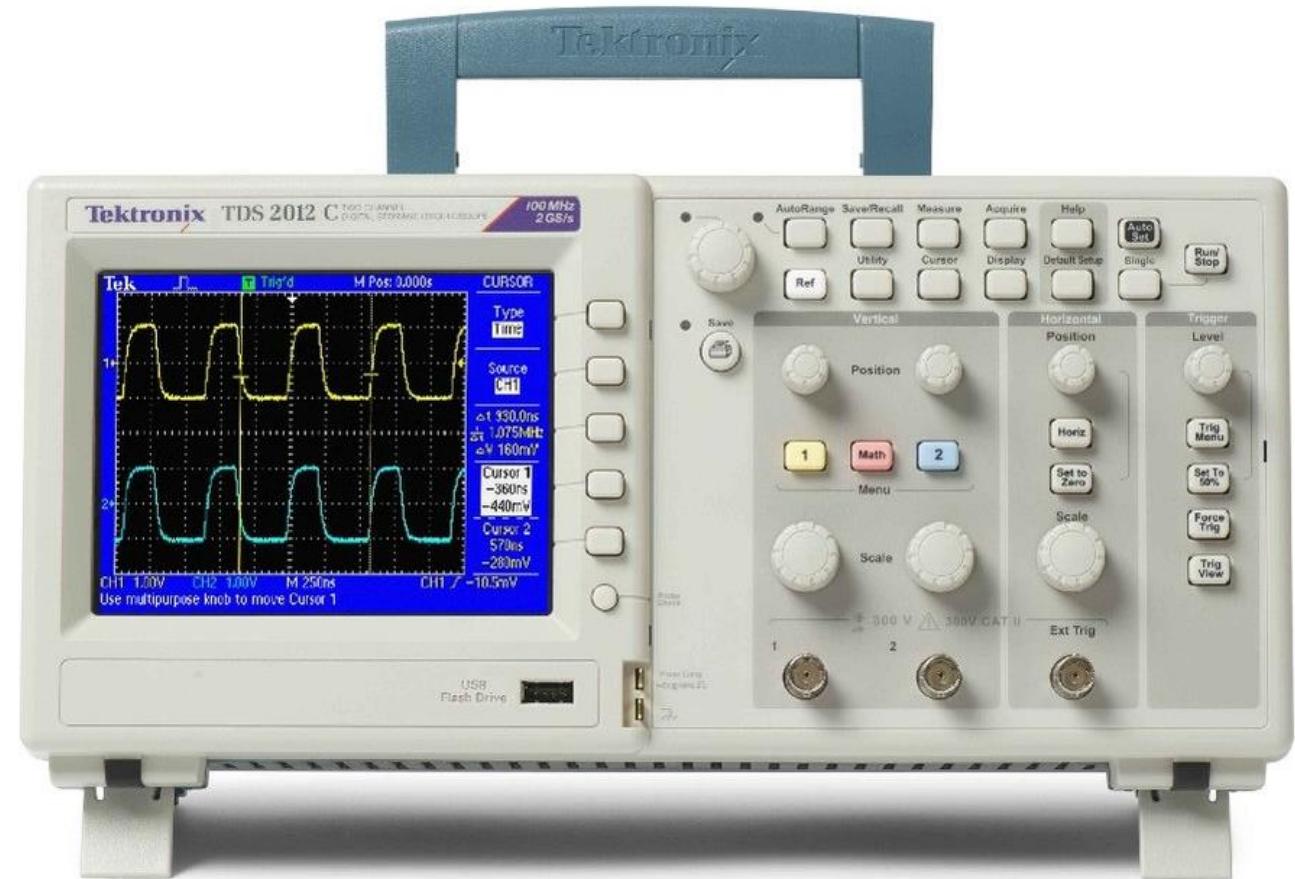
- Θέλουμε να μετρήσουμε την **καθυστέρηση διάδοσης** του σήματος από την είσοδο ενός αντιστροφέα στην έξοδό του.



# Multisim - Παλμογράφος

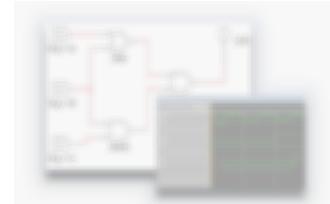
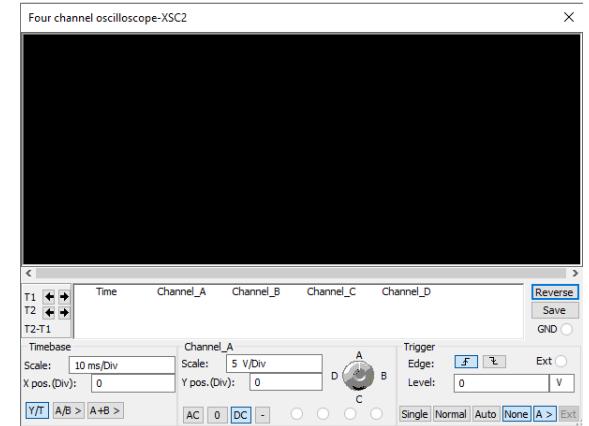
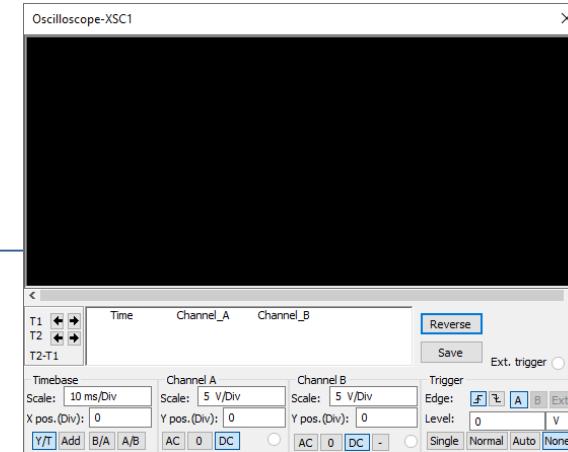
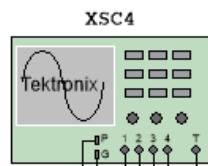
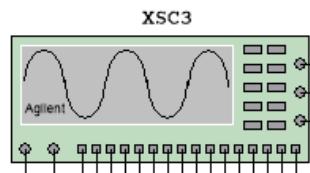
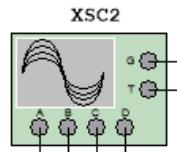
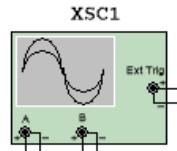


➤ Ο **Παλμογράφος** είναι ένα εργαστηριακό όργανο για την μέτρηση όχι μόνο της τιμής μεγεθών σε ηλεκτρικά/ηλεκτρονικά κυκλώματα αλλά και της χρονικής τους εξέλιξης τους.

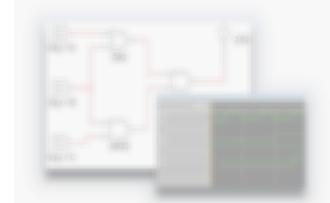


# Multisim - Παλμογράφος

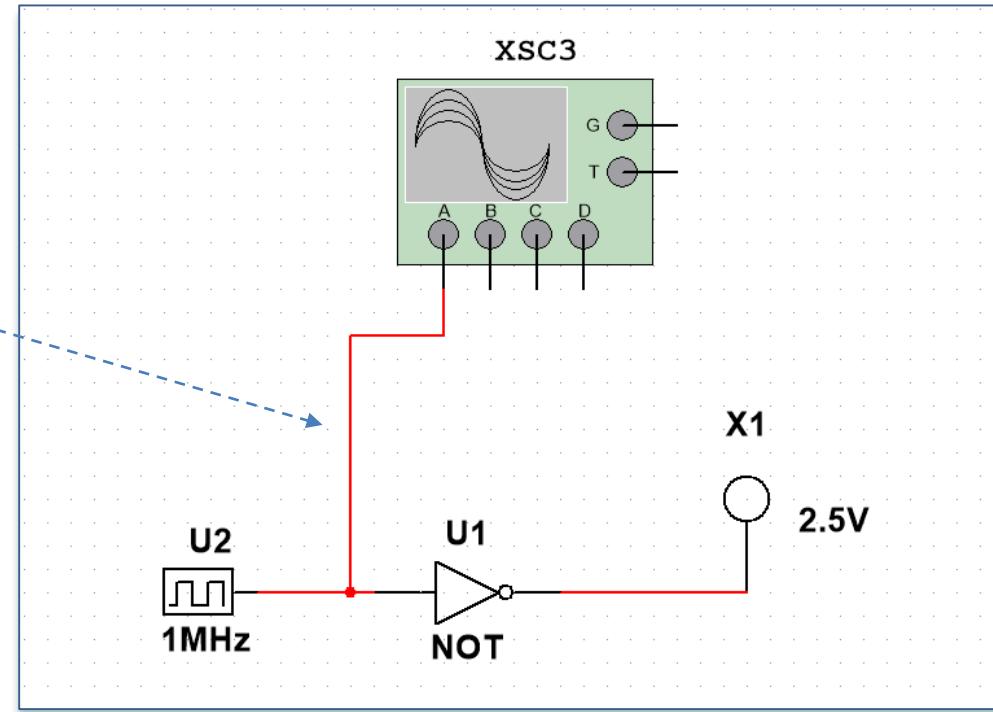
- Στο Multisim υπάρχουν **4 διαφορετικοί παλμογράφοι**
- Βρίσκονται όλοι στην δεξιά μπάρα των οργάνων
- Με 2 κανάλια (Virtual)
- Με 4 κανάλια (Virtual)
- Agilent oscilloscope – XSC3
- Tektronix oscilloscope – XSC4



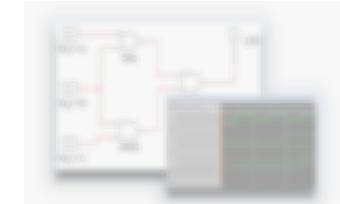
# Multisim - Παλμογράφος



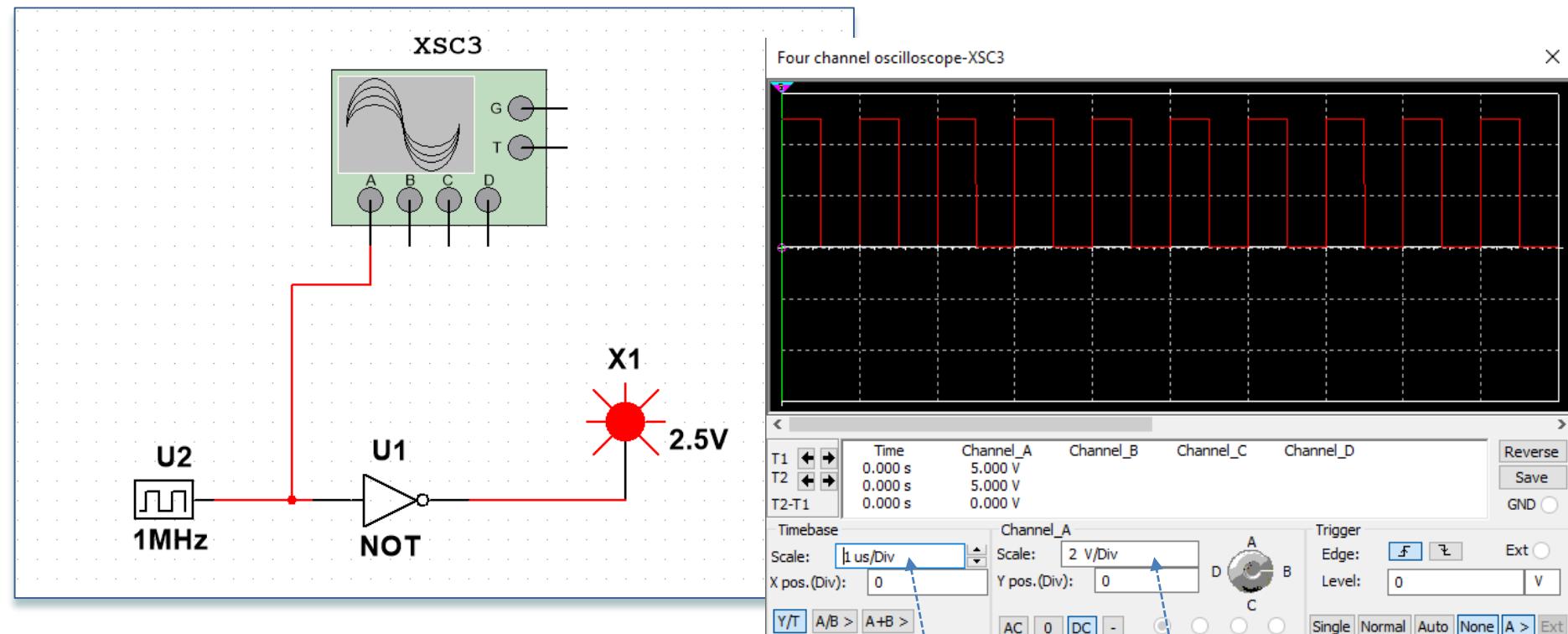
Συνδέουμε το Α κανάλι  
του παλμογράφου στην  
είσοδο του αντιστροφέα



# Multisim - Παλμογράφος



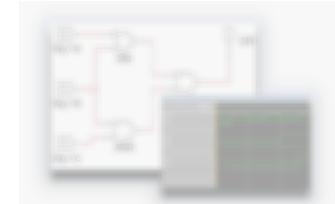
Πιέζουμε **Run** και έπειτα από λίγο **Stop**



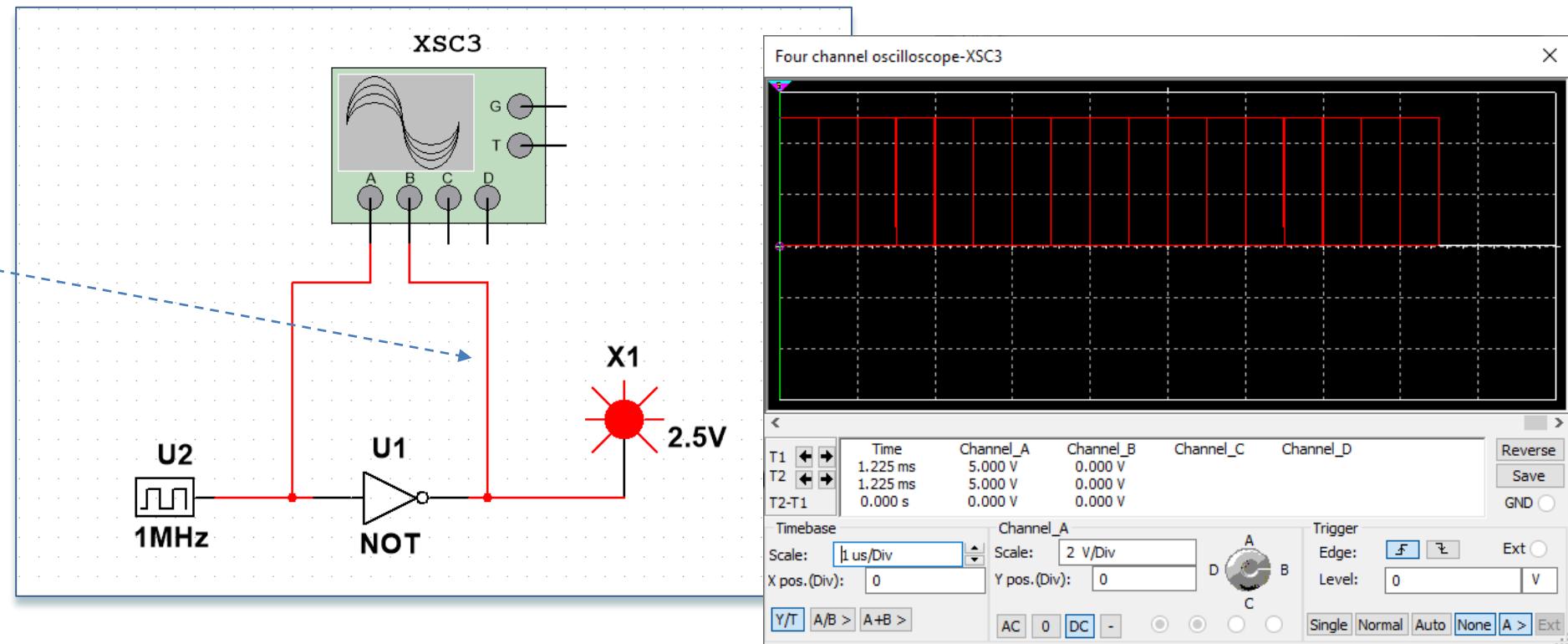
Ρύθμιση κλίμακας χρόνου στον άξονα X

Ρύθμιση κλίμακας τάσης στον άξονα Y

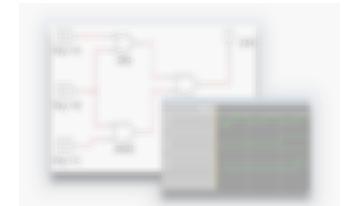
# Multisim - Παλμογράφος



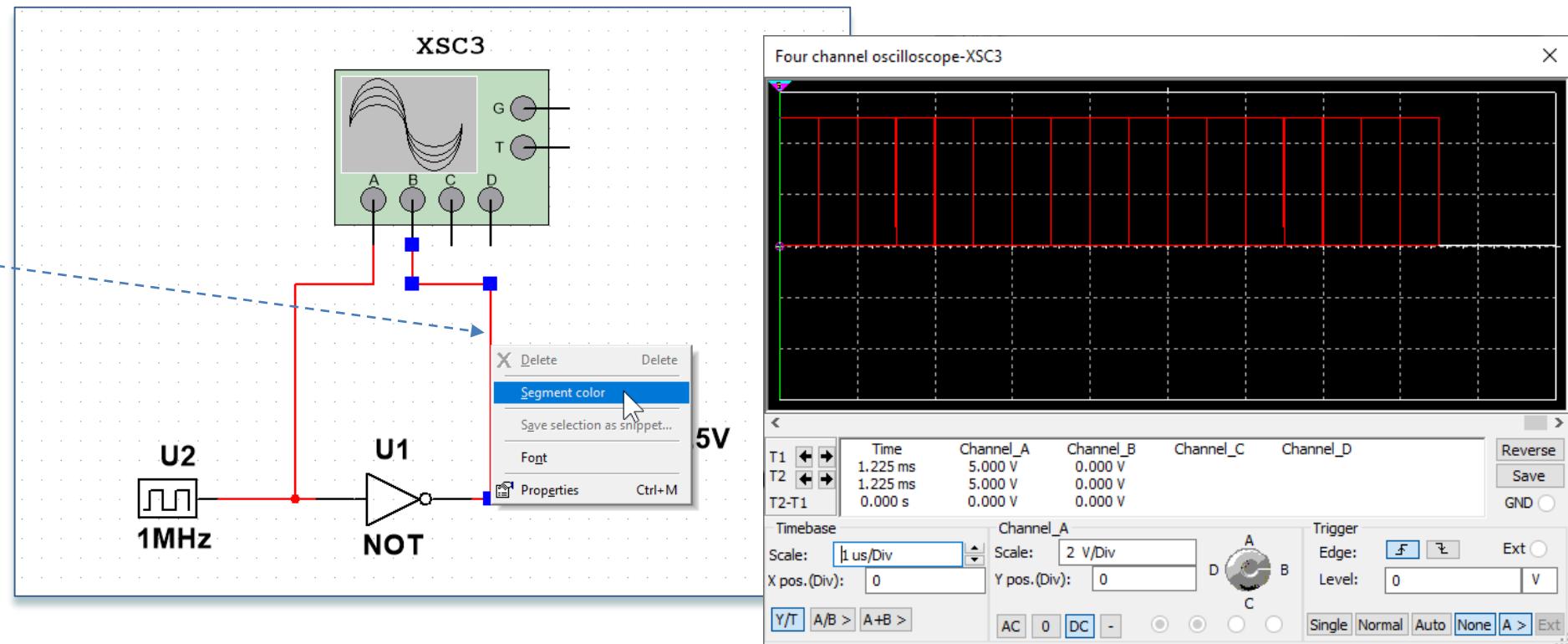
Συνδέουμε και το κανάλι B  
του παλμογράφου στην  
έξοδο του αντιστροφέα



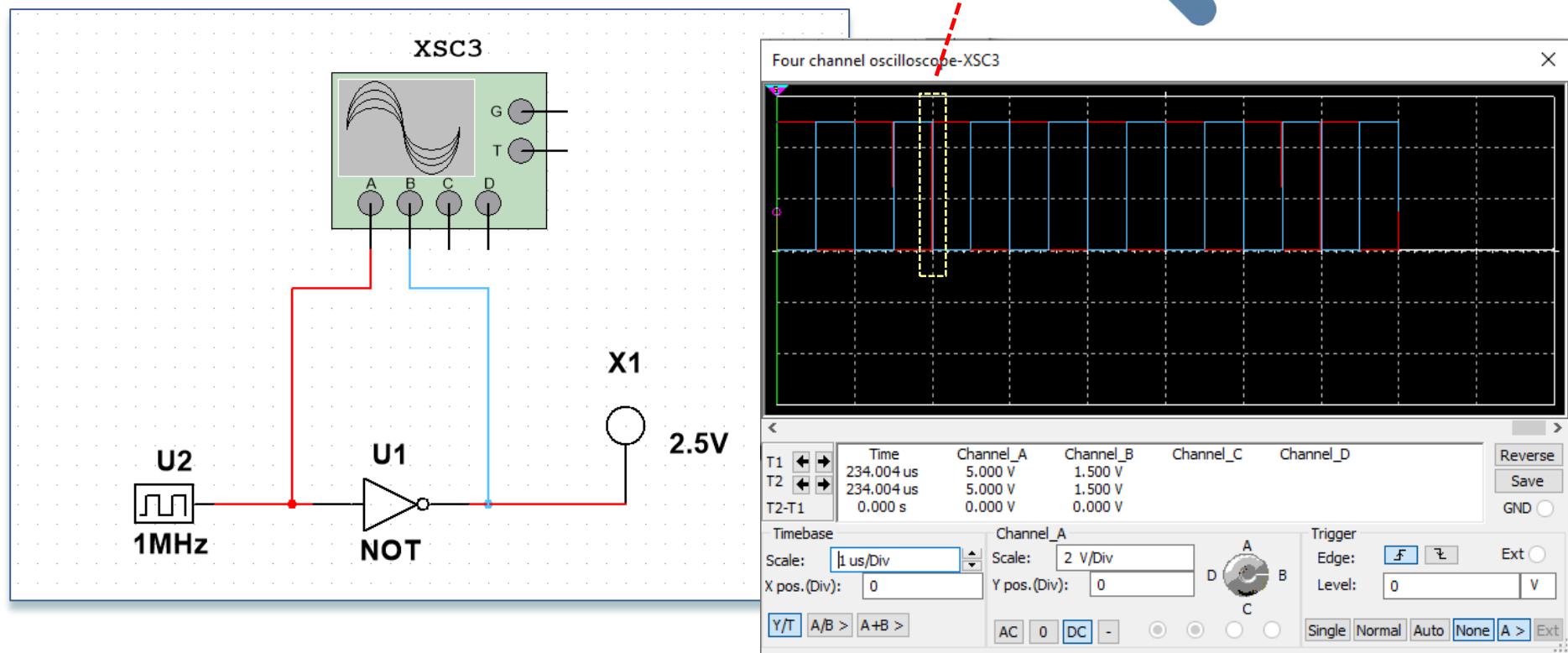
# Multisim - Παλμογράφος



Τροποποιούμε το χρώμα απεικόνισης του Β καναλιού.

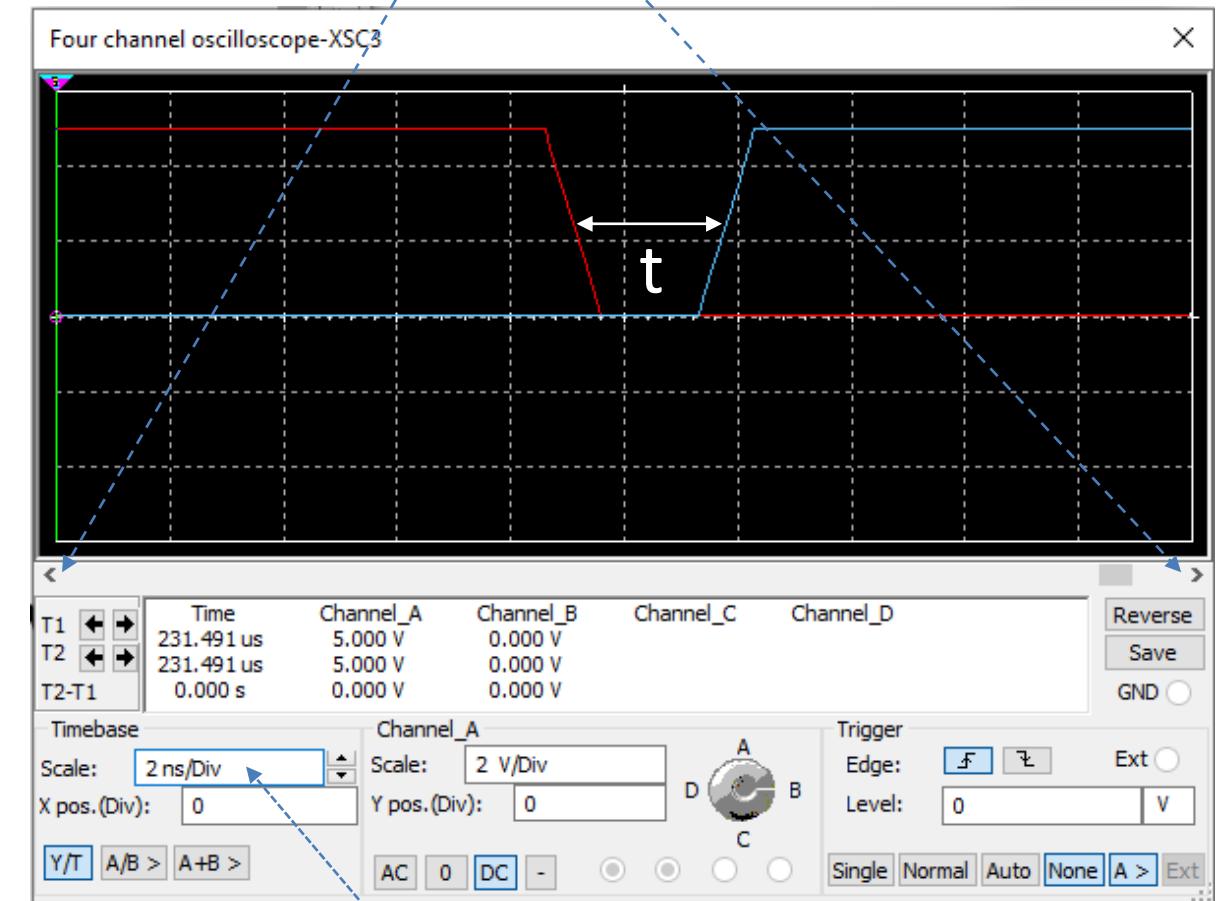
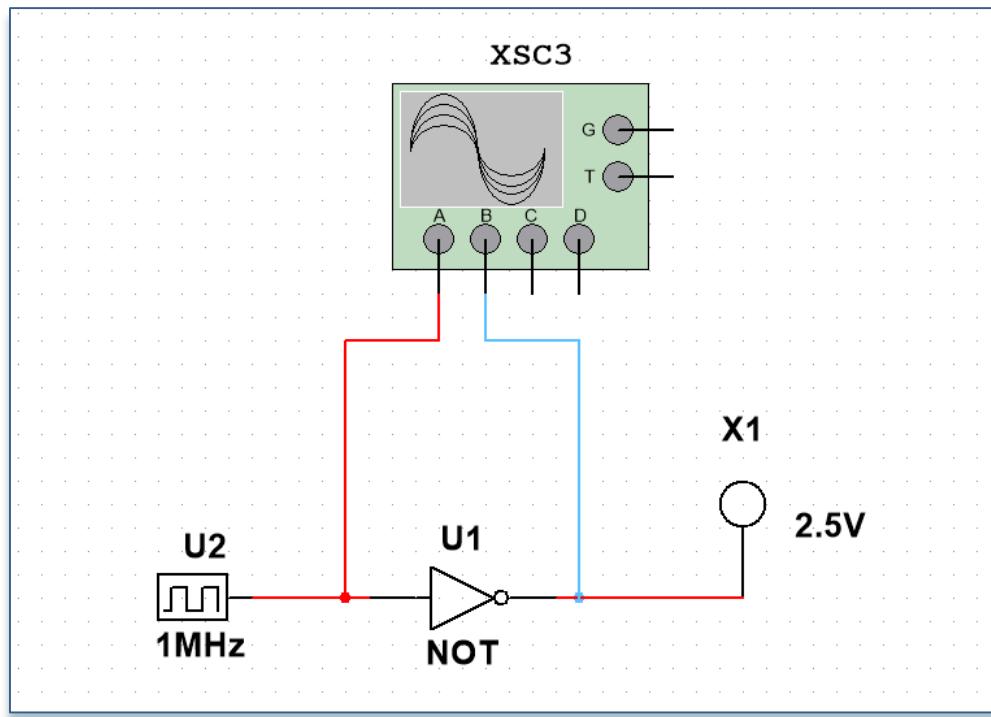
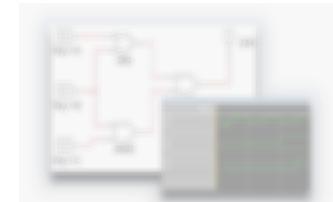


# Multisim - Παλμογράφος



# Multisim - Παλμογράφος

Μετακινούμε το γράφημα δεξιά ή αριστερά  
ώστε να βρούμε το σημείο της μετάβασης



Μειώνουμε την κλίμακα στον άξονα X (χρόνος)  
ώστε να γίνει ορατή η πολύ μικρή  
χρονοκαθυστέρηση.



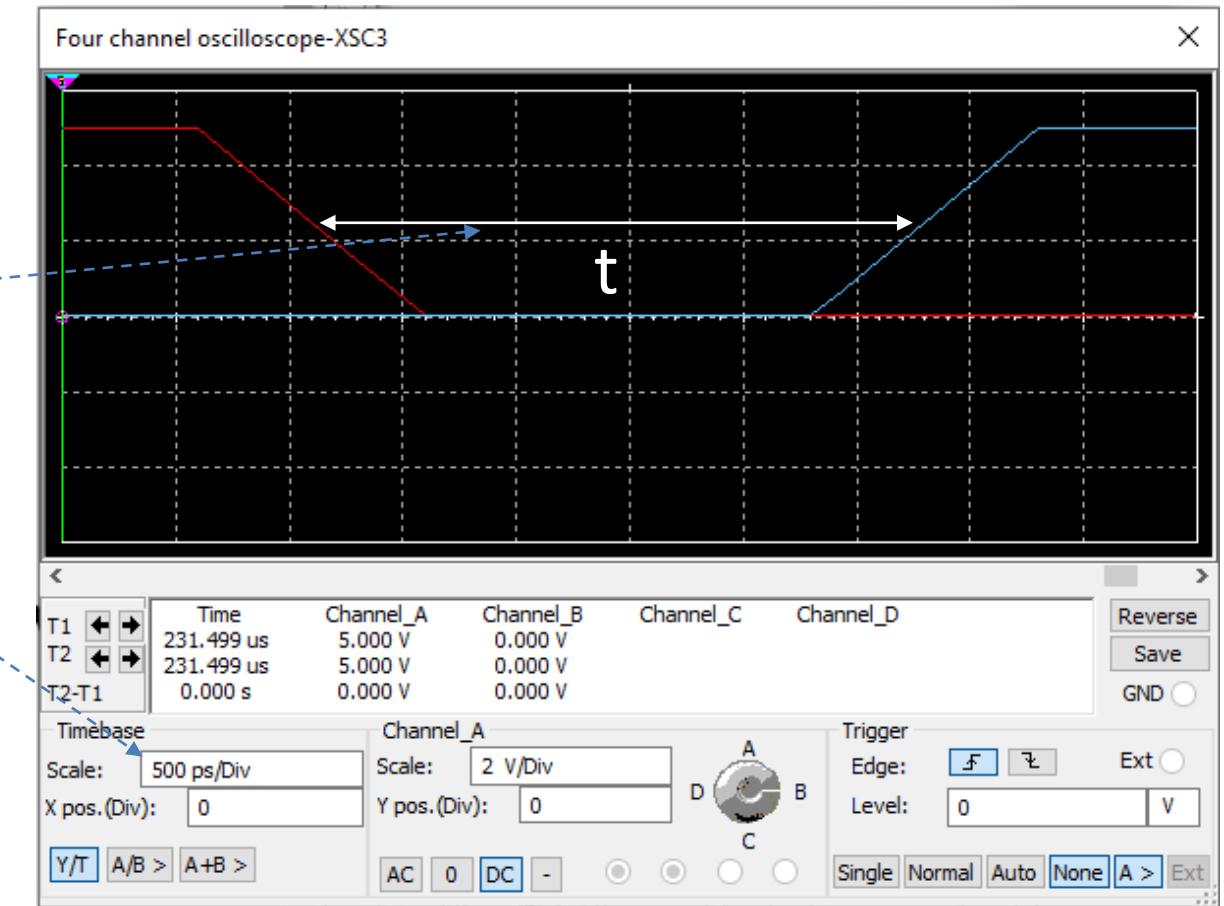
# Multisim - Παλμογράφος

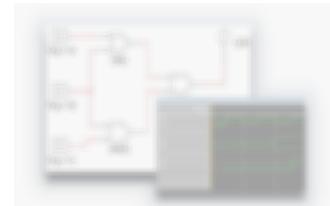
Μειώνουμε την κλίμακα τόσο ώστε να βρίσκονται και οι δύο μεταβάσεις μέσα στο διάγραμμα

Μετράμε περίπου 5,3 "κουτάκια"

Η κλίμακα είναι 500ps/Div οπότε οπτικά για μετάβαση του αντιστροφέα από 0→1 υπολογίζουμε περίπου καθυστέρηση μετάβασης  $t = 5,3 \times 500\text{ps} = 2,65 \text{ nsec}$

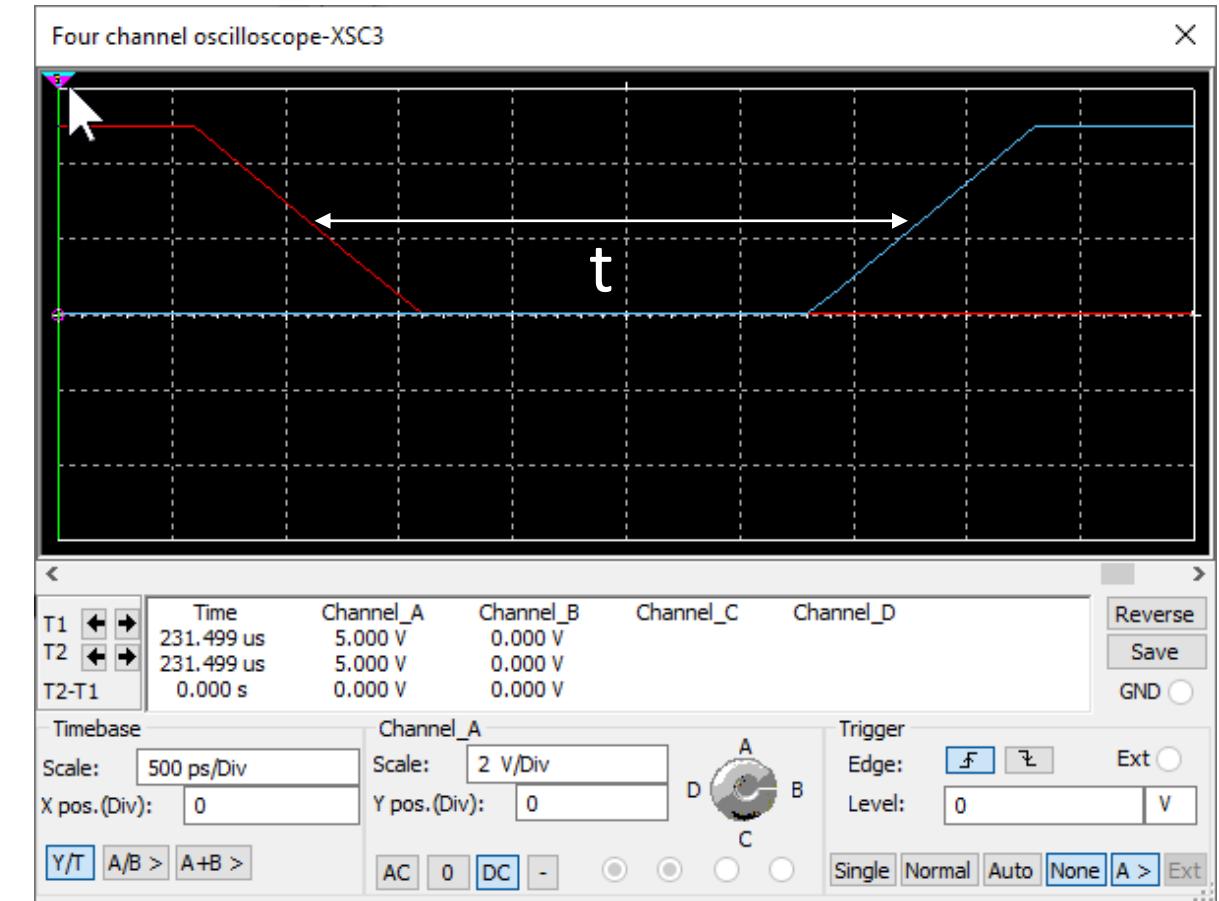
**Σημείωση:** Μετράμε από το μέσο της μεταβολής του σήματος εισόδου μέχρι το μέσο της μεταβολής του σήματος εξόδου.





# Multisim - Παλμογράφος

Για ακριβέστερη μέτρηση μπορούμε να χρησιμοποιήσουμε τους cursors που διαθέτει ο παλμογράφος.



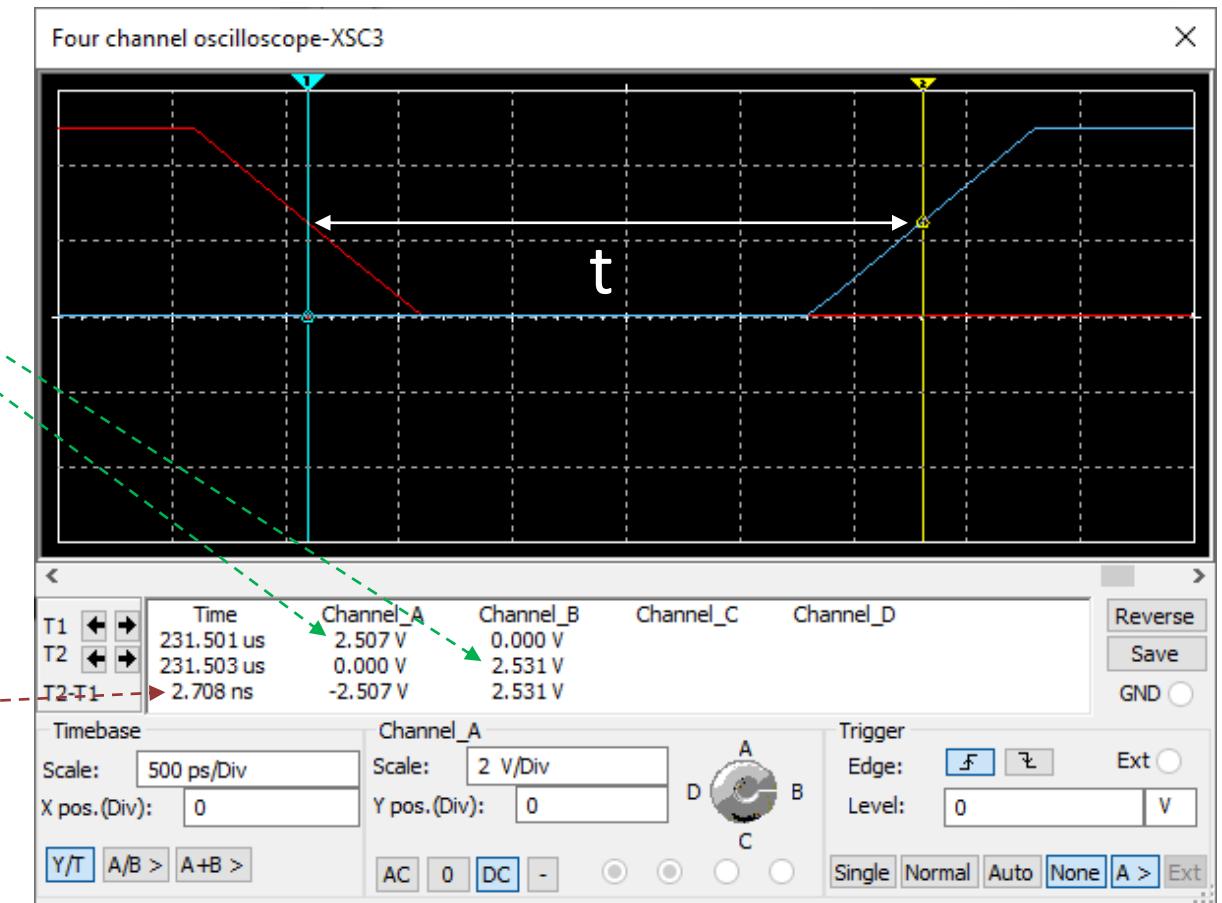


# Multisim - Παλμογράφος

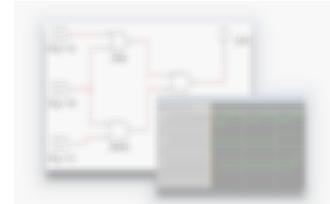
Μετακινούμε τους cursors ώστε να βρίσκονται όσο πιο κοντά στο μέσο της μετάβασης τους, το οποίο βρίσκεται στα 2,5 Volt

Ο παλμογράφος υπολογίζει την χρονική διαφορά μεταξύ των δύο cursors.

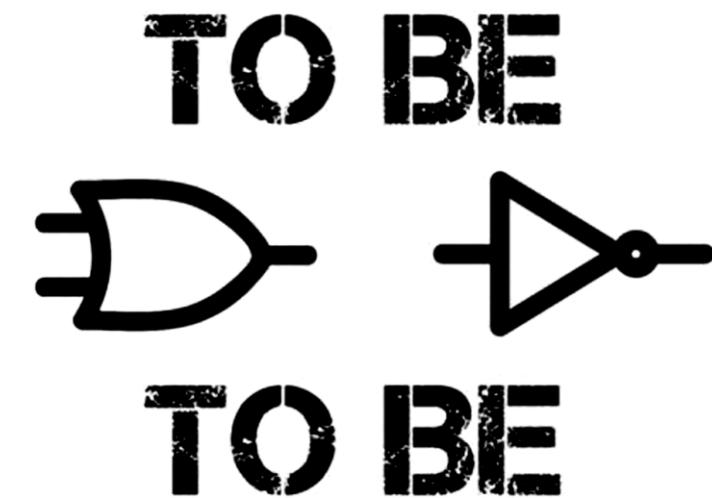
Η χρονική αυτή διαφορά είναι η καθυστέρηση διάδοσης του αντιστροφέα μας για μετάβαση 0→1 και είναι **t=2,708 nsec**



Ευχαριστώ για την προσοχή σας!



➤ Ερωτήσεις / Απορίες ;



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